Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.2-d-x-^m-a+b-arctan-c-x^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [166]. This is test number [148].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 94.58 (157)	% 5.42 (9)
Mathematica	% 98.19 (163)	% 1.81 (3)
Maple	% 86.75 (144)	% 13.25 (22)
Maxima	% 52.41 (87)	% 47.59 (79)
Fricas	% 54.82 (91)	% 45.18 (75)
Sympy	% 50.6 (84)	% 49.4 (82)
Giac	% 59.04 (98)	% 40.96 (68)

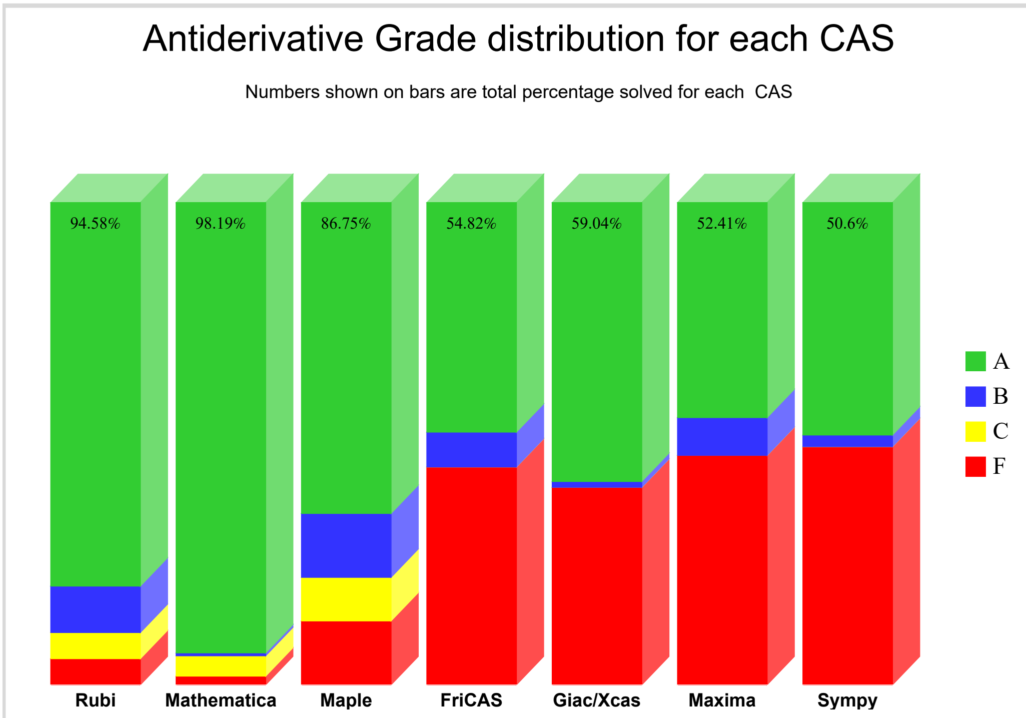
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

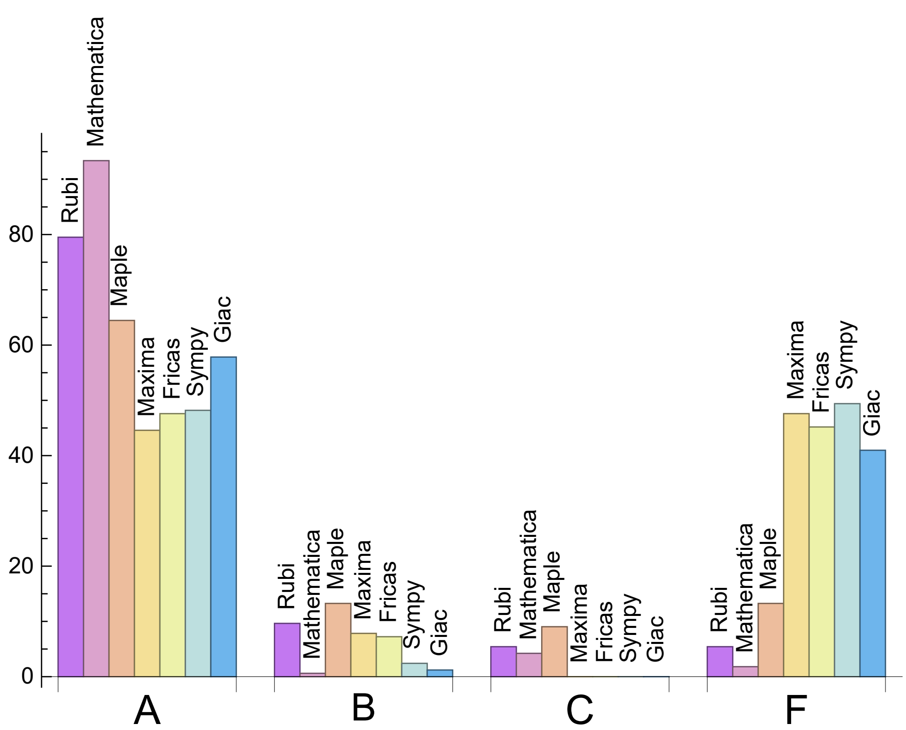
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	79.52	9.64	5.42	5.42
Mathematica	93.37	0.6	4.22	1.81
Maple	64.46	13.25	9.04	13.25
Maxima	44.58	7.83	0.	47.59
Fricas	47.59	7.23	0.	45.18
Sympy	48.19	2.41	0.	49.4
Giac	57.83	1.2	0.	40.96

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.45	199.46	1.46	59.	1.
Mathematica	0.71	163.15	0.95	65.	1.04
Maple	0.18	309.48	2.12	56.	0.95
Maxima	1.17	111.48	1.3	62.	1.35
Fricas	2.07	223.77	2.43	105.	2.39
Sympy	26.	245.51	2.87	48.	1.06
Giac	0.83	73.54	0.99	58.	1.27

1.4 list of integrals that has no closed form antiderivative

{35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 57, 58, 59, 91, 92, 94, 95, 127, 128, 130, 131}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {74, 75, 76, 77, 79, 80, 86, 87, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 140, 141, 142, 143, 145, 146, 152}

Mathematica {14, 16, 18, 20, 22, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 75, 77, 79, 82, 83, 86, 87, 89, 90, 114, 116, 118, 120, 121, 122, 123, 125, 126, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command AboluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is ValueError. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception ValueError then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

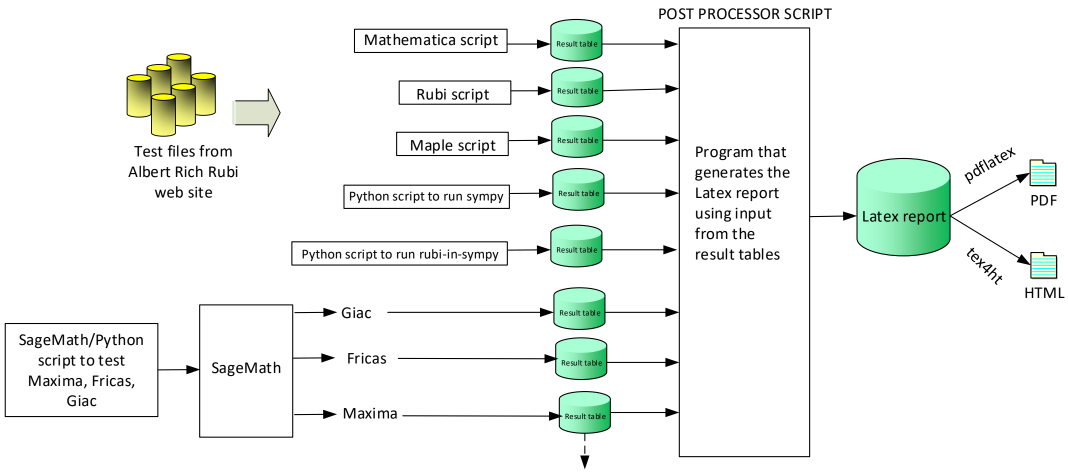
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 78, 81, 82, 83, 84, 85, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 117, 124, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 151, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade: { 75, 77, 79, 86, 87, 114, 116, 118, 120, 121, 122, 123, 141, 143, 145, 152 }

C grade: { 74, 76, 80, 113, 115, 119, 140, 142, 146 }

F grade: { 89, 90, 125, 126, 147, 148, 149, 150, 153}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166 }

B grade: { 83 }

C grade: { 9, 11, 66, 82, 102, 158, 159 }

F grade: { 81, 84, 85}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 18, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 80, 91, 92, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 119, 127, 128, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 145, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 7, 14, 16, 20, 22, 24, 26, 28, 29, 32, 34, 87, 123, 136, 141, 143, 147, 149, 152, 153, 157, 166 }

C grade: { 19, 25, 27, 30, 31, 33, 64, 86, 100, 122, 144, 148, 150, 151, 165 }

F grade: { 56, 75, 78, 79, 81, 82, 83, 84, 85, 88, 89, 90, 93, 114, 117, 118, 120, 121, 124, 125, 126, 129 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 53, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 74, 76, 80, 94, 95, 96, 97, 98, 99, 101, 102, 103, 105, 107, 109, 111, 113, 115, 119, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164

B grade: { 68, 69, 70, 71, 72, 73, 104, 106, 108, 110, 112, 157, 165 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 166 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 53, 54, 55, 57, 58, 60, 61, 62, 63, 65, 66, 67, 74, 76, 80, 91, 92, 94, 95, 96, 97, 98, 99, 101, 102, 103, 105, 107, 109, 111, 113, 115, 119, 127, 128, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 68, 69, 70, 71, 72, 73, 104, 106, 108, 110, 112, 166 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 59, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 80, 98, 99, 101, 102, 104, 105, 106, 109, 110, 111, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 161, 162, 163 }

B grade: { 158, 159, 160, 164 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 59, 64, 74, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 103, 107, 108, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 67, 68, 69, 70, 71, 72, 73, 74, 76, 91, 92, 94, 95, 96, 97, 98, 99, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 127, 128, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 66, 102 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	64	53	77	131	63	89
normalized size	1	1.	1.08	0.9	1.31	2.22	1.07	1.51
time (sec)	N/A	0.032	0.003	0.006	1.46	2.364	2.023	1.223

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	61	52	76	134	60	80
normalized size	1	1.	1.09	0.93	1.36	2.39	1.07	1.43
time (sec)	N/A	0.042	0.013	0.006	0.977	2.29	1.446	1.249

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	44	65	105	53	66
normalized size	1	1.	1.1	0.92	1.35	2.19	1.1	1.38
time (sec)	N/A	0.027	0.003	0.005	1.443	2.442	1.108	1.353

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	50	43	62	111	49	66
normalized size	1	1.	1.11	0.96	1.38	2.47	1.09	1.47
time (sec)	N/A	0.032	0.008	0.006	0.982	2.306	0.765	1.203

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	35	50	80	42	61
normalized size	1	1.	1.14	0.95	1.35	2.16	1.14	1.65
time (sec)	N/A	0.016	0.002	0.004	1.484	2.22	0.561	1.194

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	42	81	26	42
normalized size	1	1.	1.	0.97	1.45	2.79	0.9	1.45
time (sec)	N/A	0.011	0.003	0.003	0.985	2.232	0.253	1.335

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	74	0	0	0	0
normalized size	1	1.	1.	2.11	0.	0.	0.	0.
time (sec)	N/A	0.029	0.002	0.015	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	38	39	53	100	37	50
normalized size	1	1.	1.09	1.11	1.51	2.86	1.06	1.43
time (sec)	N/A	0.023	0.003	0.006	0.969	2.492	0.889	1.323

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	46	35	42	70	37	74
normalized size	1	1.	1.24	0.95	1.14	1.89	1.	2.
time (sec)	N/A	0.02	0.003	0.007	1.468	2.565	0.744	1.298

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	51	69	123	61	68
normalized size	1	1.	1.02	0.96	1.3	2.32	1.15	1.28
time (sec)	N/A	0.033	0.016	0.007	0.969	2.481	1.536	1.807

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	46	44	62	95	46	88
normalized size	1	1.	0.96	0.92	1.29	1.98	0.96	1.83
time (sec)	N/A	0.025	0.003	0.009	1.459	2.376	1.205	1.735

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	60	84	147	71	80
normalized size	1	1.	1.	0.94	1.31	2.3	1.11	1.25
time (sec)	N/A	0.038	0.017	0.01	0.975	2.528	2.591	1.406

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	138	171	220	351	199	243
normalized size	1	1.	0.96	1.19	1.53	2.44	1.38	1.69
time (sec)	N/A	0.31	0.13	0.013	1.509	2.543	3.607	1.203

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	169	334	0	0	0	0
normalized size	1	1.	0.99	1.96	0.	0.	0.	0.
time (sec)	N/A	0.289	0.485	0.044	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	111	135	184	266	155	181
normalized size	1	1.	0.99	1.21	1.64	2.38	1.38	1.62
time (sec)	N/A	0.209	0.079	0.016	1.52	2.407	2.089	1.187

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	131	298	0	0	0	0
normalized size	1	1.	0.95	2.16	0.	0.	0.	0.
time (sec)	N/A	0.194	0.28	0.014	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	97	140	189	107	140
normalized size	1	1.	0.99	1.28	1.84	2.49	1.41	1.84
time (sec)	N/A	0.106	0.065	0.012	1.531	2.441	0.935	1.197

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	90	128	0	0	0	0
normalized size	1	1.	1.08	1.54	0.	0.	0.	0.
time (sec)	N/A	0.098	0.076	0.071	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	144	1128	0	0	0	0
normalized size	1	1.	1.09	8.55	0.	0.	0.	0.
time (sec)	N/A	0.25	0.072	0.394	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	102	323	0	0	0	0
normalized size	1	1.	1.24	3.94	0.	0.	0.	0.
time (sec)	N/A	0.147	0.141	0.013	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	90	110	132	221	119	0
normalized size	1	1.	1.14	1.39	1.67	2.8	1.51	0.
time (sec)	N/A	0.128	0.065	0.011	1.497	2.54	1.452	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	153	399	0	0	0	0
normalized size	1	1.	1.09	2.85	0.	0.	0.	0.
time (sec)	N/A	0.23	0.364	0.014	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	128	147	205	297	170	0
normalized size	1	1.	1.1	1.27	1.77	2.56	1.47	0.
time (sec)	N/A	0.219	0.088	0.017	1.538	2.75	2.615	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	291	528	0	0	0	0
normalized size	1	1.	1.14	2.07	0.	0.	0.	0.
time (sec)	N/A	0.948	0.764	0.017	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	396	3053	0	0	0	0
normalized size	1	1.	1.46	11.27	0.	0.	0.	0.
time (sec)	N/A	0.756	0.83	3.677	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	225	445	0	0	0	0
normalized size	1	1.	1.16	2.29	0.	0.	0.	0.
time (sec)	N/A	0.545	0.524	0.019	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	206	269	2020	0	0	0	0
normalized size	1	1.	1.31	9.81	0.	0.	0.	0.
time (sec)	N/A	0.434	0.566	1.106	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	152	352	0	0	0	0
normalized size	1	1.	1.16	2.69	0.	0.	0.	0.
time (sec)	N/A	0.239	0.284	0.089	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	192	270	0	0	0	0
normalized size	1	1.	1.61	2.27	0.	0.	0.	0.
time (sec)	N/A	0.209	0.092	0.134	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	212	2309	0	0	0	0
normalized size	1	1.	1.03	11.21	0.	0.	0.	0.
time (sec)	N/A	0.427	0.136	0.301	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	214	2159	0	0	0	0
normalized size	1	1.	1.84	18.61	0.	0.	0.	0.
time (sec)	N/A	0.267	0.369	0.341	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	176	457	0	0	0	0
normalized size	1	1.	1.32	3.44	0.	0.	0.	0.
time (sec)	N/A	0.285	0.323	0.043	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	321	5974	0	0	0	0
normalized size	1	1.	1.51	28.05	0.	0.	0.	0.
time (sec)	N/A	0.476	0.812	1.735	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	265	550	0	0	0	0
normalized size	1	1.	1.34	2.78	0.	0.	0.	0.
time (sec)	N/A	0.601	0.656	0.1	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.611	0.189	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.003	0.011	0.148	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.425	0.12	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.591	0.368	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.003	0.881	0.149	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.01	0.135	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	1.959	0.246	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.003	1.741	0.131	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.246	0.26	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	0.948	0.206	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.003	2.149	0.135	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.069	0.25	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	1.232	0.225	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.003	0.02	0.132	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	1.13	0.223	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	1.477	0.239	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.003	2.531	0.135	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	2.72	0.183	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	108	74	116	404	104	116
normalized size	1	1.	0.92	0.63	0.99	3.45	0.89	0.99
time (sec)	N/A	0.067	0.025	0.023	1.485	2.794	8.17	1.212

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	3.731	1.937	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	2.433	1.633	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.031	1.434	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.253	0.824	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.437	0.41	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.328	0.986	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	50	73	111	58	81
normalized size	1	1.	1.09	0.93	1.35	2.06	1.07	1.5
time (sec)	N/A	0.035	0.008	0.024	1.458	2.628	128.876	1.097

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	65	115	80	63
normalized size	1	1.	1.11	0.96	1.38	2.45	1.7	1.34
time (sec)	N/A	0.031	0.013	0.024	0.984	2.582	76.17	1.125

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	58	85	48	58
normalized size	1	1.	1.12	0.95	1.35	1.98	1.12	1.35
time (sec)	N/A	0.027	0.005	0.025	1.519	2.605	40.632	1.154

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	51	89	70	54
normalized size	1	1.	1.14	1.	1.42	2.47	1.94	1.5
time (sec)	N/A	0.014	0.006	0.019	1.027	2.932	20.899	1.154

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	63	0	0	0	0
normalized size	1	1.	1.	1.62	0.	0.	0.	0.
time (sec)	N/A	0.045	0.005	0.082	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	44	39	55	111	593	81
normalized size	1	1.	1.13	1.	1.41	2.85	15.21	2.08
time (sec)	N/A	0.024	0.006	0.027	0.998	2.749	45.408	1.226

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	48	39	47	76	42	100
normalized size	1	1.	1.17	0.95	1.15	1.85	1.02	2.44
time (sec)	N/A	0.025	0.007	0.024	1.501	2.681	36.268	1.201

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	51	72	130	784	93
normalized size	1	1.	1.09	0.93	1.31	2.36	14.25	1.69
time (sec)	N/A	0.032	0.014	0.029	1.002	2.822	140.139	1.142

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	179	140	375	894	184	228
normalized size	1	1.	1.11	0.87	2.33	5.55	1.14	1.42
time (sec)	N/A	0.113	0.048	0.037	1.527	2.852	55.237	1.305

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	177	138	354	807	1207	223
normalized size	1	1.	1.11	0.87	2.23	5.08	7.59	1.4
time (sec)	N/A	0.101	0.033	0.026	1.532	2.803	28.332	1.239

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	107	125	348	768	146	201
normalized size	1	1.	0.76	0.89	2.49	5.49	1.04	1.44
time (sec)	N/A	0.105	0.042	0.023	1.519	2.826	15.067	1.144

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	158	125	339	790	1105	186
normalized size	1	1.	1.1	0.87	2.37	5.52	7.73	1.3
time (sec)	N/A	0.086	0.042	0.025	1.529	2.757	32.019	1.228

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	177	132	369	900	704	215
normalized size	1	1.	1.11	0.83	2.32	5.66	4.43	1.35
time (sec)	N/A	0.104	0.052	0.029	1.525	2.858	55.576	1.268

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	177	138	366	873	1265	203
normalized size	1	1.	1.11	0.87	2.3	5.49	7.96	1.28
time (sec)	N/A	0.103	0.05	0.03	1.52	2.831	107.518	1.364

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	124	731	121	151	228	301	0	196
normalized size	1	5.9	0.98	1.22	1.84	2.43	0.	1.58
time (sec)	N/A	1.627	0.091	0.036	1.779	2.993	0.	1.196

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	154	647	141	0	0	0	0	0
normalized size	1	4.2	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	1.366	0.289	180.	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	90	612	85	113	170	227	529	135
normalized size	1	6.8	0.94	1.26	1.89	2.52	5.88	1.5
time (sec)	N/A	1.042	0.064	0.033	1.752	2.773	63.552	1.197

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	101	255	107	146	0	0	0	0
normalized size	1	2.52	1.06	1.45	0.	0.	0.	0.
time (sec)	N/A	0.554	0.087	0.087	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	165	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.321	0.1	0.167	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	97	290	127	0	0	0	0	0
normalized size	1	2.99	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.653	0.149	0.337	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	87	419	98	118	149	258	1187	0
normalized size	1	4.82	1.13	1.36	1.71	2.97	13.64	0.
time (sec)	N/A	1.136	0.077	0.037	1.837	2.732	86.816	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1393	1393	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.602	4.849	0.207	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1191	1191	5466	0	0	0	0	0
normalized size	1	1.	4.59	0.	0.	0.	0.	0.
time (sec)	N/A	1.778	31.777	0.188	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1164	1164	5434	0	0	0	0	0
normalized size	1	1.	4.67	0.	0.	0.	0.	0.
time (sec)	N/A	1.622	31.403	0.191	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1360	1360	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.227	2.674	0.2	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1444	1444	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.427	2.7	0.208	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	149	951	170	690	0	0	0	0
normalized size	1	6.38	1.14	4.63	0.	0.	0.	0.
time (sec)	N/A	4.736	0.311	0.733	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	144	545	224	306	0	0	0	0
normalized size	1	3.78	1.56	2.12	0.	0.	0.	0.
time (sec)	N/A	2.545	0.102	0.125	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	245	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.529	0.195	0.199	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-2)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	138	0	239	0	0	0	0	0
normalized size	1	0.	1.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.831	0.43	0.332	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-2)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	149	0	196	0	0	0	0	0
normalized size	1	0.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.659	0.334	0.661	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	1.788	0.237	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	1.175	0.213	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.059	0.209	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.312	0.145	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.329	0.171	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	50	73	113	0	81
normalized size	1	1.	1.09	0.93	1.35	2.09	0.	1.5
time (sec)	N/A	0.039	0.008	0.026	1.502	2.619	0.	1.117

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	65	115	0	63
normalized size	1	1.	1.11	0.96	1.38	2.45	0.	1.34
time (sec)	N/A	0.036	0.013	0.023	1.052	2.656	0.	1.137

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	58	85	48	58
normalized size	1	1.	1.12	0.95	1.35	1.98	1.12	1.35
time (sec)	N/A	0.029	0.005	0.023	1.483	2.587	166.108	1.105

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	51	89	808	54
normalized size	1	1.	1.14	1.	1.42	2.47	22.44	1.5
time (sec)	N/A	0.021	0.007	0.021	1.003	2.661	83.144	1.161

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	63	0	0	0	0
normalized size	1	1.	1.	1.62	0.	0.	0.	0.
time (sec)	N/A	0.05	0.005	0.086	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	44	39	55	111	505	81
normalized size	1	1.	1.13	1.	1.41	2.85	12.95	2.08
time (sec)	N/A	0.025	0.007	0.026	0.993	3.279	155.503	1.111

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	48	39	47	76	42	100
normalized size	1	1.	1.17	0.95	1.15	1.85	1.02	2.44
time (sec)	N/A	0.027	0.006	0.028	1.505	3.132	153.635	1.166

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	51	72	130	0	93
normalized size	1	1.	1.09	0.93	1.31	2.36	0.	1.69
time (sec)	N/A	0.035	0.014	0.027	1.021	3.325	0.	1.113

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	179	165	396	948	1287	225
normalized size	1	1.	1.03	0.95	2.28	5.45	7.4	1.29
time (sec)	N/A	0.325	0.043	0.092	1.532	3.141	111.885	1.235

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	131	98	143	632	1703	128
normalized size	1	1.	1.3	0.97	1.42	6.26	16.86	1.27
time (sec)	N/A	0.098	0.04	0.023	1.494	3.306	55.892	1.152

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	170	148	381	1224	311	185
normalized size	1	1.	1.03	0.9	2.31	7.42	1.88	1.12
time (sec)	N/A	0.294	0.048	0.056	1.521	3.584	124.381	1.244

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	183	105	144	336	0	146
normalized size	1	1.	1.59	0.91	1.25	2.92	0.	1.27
time (sec)	N/A	0.091	0.045	0.03	1.541	2.738	0.	1.205

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	181	170	432	1069	0	231
normalized size	1	1.	1.03	0.97	2.45	6.07	0.	1.31
time (sec)	N/A	0.431	0.075	0.069	1.503	3.25	0.	1.394

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	185	113	162	362	1640	161
normalized size	1	1.	1.58	0.97	1.38	3.09	14.02	1.38
time (sec)	N/A	0.096	0.027	0.028	1.521	3.246	160.661	1.22

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	170	154	412	971	1620	212
normalized size	1	1.	1.03	0.93	2.5	5.88	9.82	1.28
time (sec)	N/A	0.393	0.025	0.056	1.557	2.54	67.111	1.307

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	170	104	151	258	1975	123
normalized size	1	1.	1.63	1.	1.45	2.48	18.99	1.18
time (sec)	N/A	0.085	0.037	0.026	1.54	2.66	92.538	1.23

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	179	159	425	1385	0	217
normalized size	1	1.	1.03	0.91	2.44	7.96	0.	1.25
time (sec)	N/A	0.413	0.05	0.059	1.535	2.963	0.	1.714

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	124	731	121	151	228	281	0	196
normalized size	1	5.9	0.98	1.22	1.84	2.27	0.	1.58
time (sec)	N/A	1.655	0.095	0.037	1.928	2.711	0.	1.14

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	154	647	141	0	0	0	0	0
normalized size	1	4.2	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	1.394	0.293	180.	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	90	612	85	113	170	200	0	135
normalized size	1	6.8	0.94	1.26	1.89	2.22	0.	1.5
time (sec)	N/A	1.052	0.065	0.037	1.871	2.619	0.	1.158

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	104	255	107	148	0	0	0	0
normalized size	1	2.45	1.03	1.42	0.	0.	0.	0.
time (sec)	N/A	0.644	0.071	0.09	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	167	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.316	0.116	0.198	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	100	290	125	0	0	0	0	0
normalized size	1	2.9	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.661	0.155	0.408	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	F(-1)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	87	419	98	118	149	232	0	0
normalized size	1	4.82	1.13	1.36	1.71	2.67	0.	0.
time (sec)	N/A	1.122	0.077	0.04	1.947	2.843	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	154	536	167	0	0	0	0	0
normalized size	1	3.48	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	1.45	0.374	0.288	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	240	1867	346	0	0	0	0	0
normalized size	1	7.78	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	7.034	0.505	0.279	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	147	951	170	867	0	0	0	0
normalized size	1	6.47	1.16	5.9	0.	0.	0.	0.
time (sec)	N/A	4.733	0.166	0.917	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	139	545	224	303	0	0	0	0
normalized size	1	3.92	1.61	2.18	0.	0.	0.	0.
time (sec)	N/A	2.702	0.1	0.129	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	248	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.523	0.195	0.194	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	133	0	240	0	0	0	0	0
normalized size	1	0.	1.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.86	0.397	0.381	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	146	0	196	0	0	0	0	0
normalized size	1	0.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	1.67	0.329	0.557	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	1.798	0.261	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	1.192	0.23	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.061	0.239	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.314	0.158	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.331	0.205	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	55	46	61	101	46	81
normalized size	1	1.	1.1	0.92	1.22	2.02	0.92	1.62
time (sec)	N/A	0.031	0.01	0.032	1.479	2.21	0.827	1.161

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	55	58	103	41	54
normalized size	1	1.	1.12	1.28	1.35	2.4	0.95	1.26
time (sec)	N/A	0.03	0.007	0.033	0.989	2.182	0.545	1.125

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	44	37	49	77	36	69
normalized size	1	1.	1.13	0.95	1.26	1.97	0.92	1.77
time (sec)	N/A	0.017	0.008	0.03	1.487	2.331	0.385	1.164

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	38	36	65	22	36
normalized size	1	1.	1.	1.41	1.33	2.41	0.81	1.33
time (sec)	N/A	0.012	0.003	0.029	0.987	2.126	0.22	1.121

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	94	0	0	0	0
normalized size	1	1.	1.	2.41	0.	0.	0.	0.
time (sec)	N/A	0.045	0.006	0.036	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	37	36	51	103	36	54
normalized size	1	1.	1.09	1.06	1.5	3.03	1.06	1.59
time (sec)	N/A	0.019	0.008	0.021	1.026	2.227	0.977	1.116

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	57	84	44	84
normalized size	1	1.	1.12	0.95	1.33	1.95	1.02	1.95
time (sec)	N/A	0.024	0.01	0.026	1.525	2.068	1.237	1.145

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	45	73	132	60	74
normalized size	1	1.	1.09	0.82	1.33	2.4	1.09	1.35
time (sec)	N/A	0.036	0.01	0.026	1.026	2.232	1.637	1.135

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	122	862	111	157	177	290	144	228
normalized size	1	7.07	0.91	1.29	1.45	2.38	1.18	1.87
time (sec)	N/A	1.791	0.09	0.043	1.604	2.19	1.148	1.176

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	152	787	152	445	0	0	0	0
normalized size	1	5.18	1.	2.93	0.	0.	0.	0.
time (sec)	N/A	1.49	0.328	0.104	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	82	663	73	116	136	201	97	173
normalized size	1	8.09	0.89	1.41	1.66	2.45	1.18	2.11
time (sec)	N/A	1.126	0.054	0.04	1.54	2.282	0.587	1.175

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	83	478	105	357	0	0	0	0
normalized size	1	5.76	1.27	4.3	0.	0.	0.	0.
time (sec)	N/A	0.44	0.106	0.09	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	148	1249	0	0	0	0
normalized size	1	1.	1.	8.44	0.	0.	0.	0.
time (sec)	N/A	0.294	0.085	0.394	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	96	259	107	147	0	0	0	0
normalized size	1	2.7	1.11	1.53	0.	0.	0.	0.
time (sec)	N/A	0.528	0.111	0.088	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	A	A	A	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	84	836	99	110	158	239	117	0
normalized size	1	9.95	1.18	1.31	1.88	2.85	1.39	0.
time (sec)	N/A	1.294	0.074	0.034	1.62	2.312	1.438	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	214	0	253	608	0	0	0	0
normalized size	1	0.	1.18	2.84	0.	0.	0.	0.
time (sec)	N/A	4.629	0.591	0.108	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	229	0	330	6441	0	0	0	0
normalized size	1	0.	1.44	28.13	0.	0.	0.	0.
time (sec)	N/A	3.52	0.678	1.027	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	145	0	174	507	0	0	0	0
normalized size	1	0.	1.2	3.5	0.	0.	0.	0.
time (sec)	N/A	2.352	0.274	0.104	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	119	0	215	2363	0	0	0	0
normalized size	1	0.	1.81	19.86	0.	0.	0.	0.
time (sec)	N/A	0.739	0.257	0.33	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	219	2542	0	0	0	0
normalized size	1	1.	0.95	11.05	0.	0.	0.	0.
time (sec)	N/A	0.494	0.207	0.276	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	136	551	222	306	0	0	0	0
normalized size	1	4.05	1.63	2.25	0.	0.	0.	0.
time (sec)	N/A	2.359	0.119	0.125	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	F(-1)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	147	0	178	396	0	0	0	0
normalized size	1	0.	1.21	2.69	0.	0.	0.	0.
time (sec)	N/A	2.255	0.283	0.095	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	34	32	42	88	39	42
normalized size	1	1.	0.67	0.63	0.82	1.73	0.76	0.82
time (sec)	N/A	0.015	0.013	0.023	1.506	2.213	4.159	1.138

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	28	27	35	72	32	35
normalized size	1	1.	0.67	0.64	0.83	1.71	0.76	0.83
time (sec)	N/A	0.01	0.011	0.025	1.494	2.201	2.021	1.117

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	17	22	47	19	22
normalized size	1	1.	0.82	0.77	1.	2.14	0.86	1.
time (sec)	N/A	0.006	0.006	0.02	1.468	2.209	1.358	1.105

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	61	47	0	0	0
normalized size	1	1.	1.	1.97	1.52	0.	0.	0.
time (sec)	N/A	0.035	0.004	0.036	1.534	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	30	22	28	54	94	28
normalized size	1	1.	1.11	0.81	1.04	2.	3.48	1.04
time (sec)	N/A	0.012	0.01	0.026	1.466	2.132	3.14	1.345

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	34	27	35	80	160	35
normalized size	1	1.	0.81	0.64	0.83	1.9	3.81	0.83
time (sec)	N/A	0.014	0.011	0.027	1.453	2.192	8.434	1.245

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	32	88	92	32
normalized size	1	1.	0.83	0.69	0.89	2.44	2.56	0.89
time (sec)	N/A	0.015	0.015	0.023	0.977	2.225	7.674	1.109

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	26	73	24	26
normalized size	1	1.	0.86	0.69	0.9	2.52	0.83	0.9
time (sec)	N/A	0.012	0.01	0.023	0.977	2.179	1.633	1.182

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	54	17	22
normalized size	1	1.	1.	0.85	1.1	2.7	0.85	1.1
time (sec)	N/A	0.008	0.007	0.023	0.966	2.163	0.452	1.118

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	78	20	24
normalized size	1	1.	1.	0.86	1.09	3.55	0.91	1.09
time (sec)	N/A	0.009	0.011	0.026	1.15	2.185	1.919	1.11

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	26	34	96	143	38
normalized size	1	1.	0.84	0.7	0.92	2.59	3.86	1.03
time (sec)	N/A	0.014	0.018	0.029	0.996	2.192	8.499	1.158

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	57	84	0	0	0
normalized size	1	1.	1.	1.73	2.55	0.	0.	0.
time (sec)	N/A	0.036	0.006	0.088	1.469	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	32	94	0	181	0	0
normalized size	1	1.	0.82	2.41	0.	4.64	0.	0.
time (sec)	N/A	0.036	0.013	0.033	0.	2.562	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [142] had the largest ratio of [2.286]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	12	0.25
2	A	4	3	1.	12	0.25
3	A	4	3	1.	12	0.25
4	A	4	3	1.	12	0.25
5	A	3	3	1.	10	0.3
6	A	3	2	1.	8	0.25
7	A	3	2	1.	12	0.167
8	A	5	5	1.	12	0.417
9	A	3	3	1.	12	0.25
10	A	4	3	1.	12	0.25
11	A	4	3	1.	12	0.25
12	A	4	3	1.	12	0.25
13	A	16	7	1.	14	0.5
14	A	14	9	1.	14	0.643
15	A	11	7	1.	14	0.5
16	A	9	8	1.	14	0.571
17	A	6	5	1.	12	0.417
18	A	5	5	1.	10	0.5
19	A	6	5	1.	14	0.357
20	A	4	4	1.	14	0.286
21	A	8	7	1.	14	0.5
22	A	8	7	1.	14	0.5
23	A	13	8	1.	14	0.571
24	A	33	11	1.	14	0.786
25	A	24	11	1.	14	0.786
26	A	18	10	1.	14	0.714
27	A	12	9	1.	14	0.643
28	A	8	8	1.	12	0.667
29	A	5	6	1.	10	0.6
30	A	8	6	1.	14	0.429
31	A	5	6	1.	14	0.429
32	A	7	6	1.	14	0.429
33	A	14	11	1.	14	0.786
34	A	16	8	1.	14	0.571

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
35	A	0	0	0.	0	0.
36	A	0	0	0.	0	0.
37	A	0	0	0.	0	0.
38	A	0	0	0.	0	0.
39	A	0	0	0.	0	0.
40	A	0	0	0.	0	0.
41	A	0	0	0.	0	0.
42	A	0	0	0.	0	0.
43	A	0	0	0.	0	0.
44	A	0	0	0.	0	0.
45	A	0	0	0.	0	0.
46	A	0	0	0.	0	0.
47	A	0	0	0.	0	0.
48	A	0	0	0.	0	0.
49	A	0	0	0.	0	0.
50	A	0	0	0.	0	0.
51	A	0	0	0.	0	0.
52	A	0	0	0.	0	0.
53	A	12	9	1.	8	1.125
54	A	0	0	0.	0	0.
55	A	0	0	0.	0	0.
56	A	2	2	1.	14	0.143
57	A	0	0	0.	0	0.
58	A	0	0	0.	0	0.
59	A	0	0	0.	0	0.
60	A	5	4	1.	14	0.286
61	A	4	3	1.	14	0.214
62	A	4	4	1.	14	0.286
63	A	2	2	1.	12	0.167
64	A	4	3	1.	14	0.214
65	A	5	5	1.	14	0.357
66	A	4	4	1.	14	0.286
67	A	4	3	1.	14	0.214
68	A	11	8	1.	14	0.571
69	A	11	8	1.	14	0.571
70	A	11	7	1.	10	0.7
71	A	10	7	1.	14	0.5
72	A	11	8	1.	14	0.571
73	A	11	8	1.	14	0.571
74	C	62	19	5.9	16	1.187
75	B	53	19	4.2	16	1.187
76	C	44	16	6.8	16	1.
77	B	28	12	2.52	14	0.857
78	A	7	6	1.	16	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
79	B	24	13	2.99	16	0.812
80	C	46	23	4.82	16	1.438
81	A	86	27	1.	16	1.687
82	A	69	23	1.	12	1.917
83	A	47	23	1.	16	1.438
84	A	64	25	1.	16	1.562
85	A	77	25	1.	16	1.562
86	B	155	30	6.38	16	1.875
87	B	82	23	3.78	14	1.643
88	A	9	7	1.	16	0.438
89	F	0	0	N/A	0	N/A
90	F	0	0	N/A	0	N/A
91	A	0	0	0.	0	0.
92	A	0	0	0.	0	0.
93	A	3	3	1.	16	0.188
94	A	0	0	0.	0	0.
95	A	0	0	0.	0	0.
96	A	5	4	1.	14	0.286
97	A	4	3	1.	14	0.214
98	A	4	4	1.	14	0.286
99	A	2	2	1.	14	0.143
100	A	4	3	1.	14	0.214
101	A	5	5	1.	14	0.357
102	A	4	4	1.	14	0.286
103	A	4	3	1.	14	0.214
104	A	12	8	1.	14	0.571
105	A	9	8	1.	10	0.8
106	A	11	7	1.	14	0.5
107	A	9	9	1.	14	0.643
108	A	12	8	1.	14	0.571
109	A	9	9	1.	14	0.643
110	A	11	7	1.	12	0.583
111	A	8	8	1.	14	0.571
112	A	12	8	1.	14	0.571
113	C	62	19	5.9	16	1.187
114	B	53	19	4.2	16	1.187
115	C	44	16	6.8	16	1.
116	B	28	12	2.45	16	0.75
117	A	7	6	1.	16	0.375
118	B	24	13	2.9	16	0.812
119	C	46	23	4.82	16	1.438
120	B	59	24	3.48	16	1.5
121	B	239	32	7.78	16	2.
122	B	155	30	6.47	16	1.875

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
123	B	82	23	3.92	16	1.438
124	A	9	7	1.	16	0.438
125	F	0	0	N/A	0	N/A
126	F	0	0	N/A	0	N/A
127	A	0	0	0.	0	0.
128	A	0	0	0.	0	0.
129	A	3	3	1.	16	0.188
130	A	0	0	0.	0	0.
131	A	0	0	0.	0	0.
132	A	5	4	1.	14	0.286
133	A	5	4	1.	14	0.286
134	A	4	4	1.	12	0.333
135	A	4	3	1.	10	0.3
136	A	4	3	1.	14	0.214
137	A	2	2	1.	14	0.143
138	A	4	4	1.	14	0.286
139	A	5	4	1.	14	0.286
140	C	88	34	7.07	16	2.125
141	B	73	34	5.18	16	2.125
142	C	58	32	8.09	14	2.286
143	B	31	14	5.76	12	1.167
144	A	7	6	1.	16	0.375
145	B	28	12	2.7	16	0.75
146	C	66	23	9.95	16	1.438
147	F	0	0	N/A	0	N/A
148	F	0	0	N/A	0	N/A
149	F	0	0	N/A	0	N/A
150	F	0	0	N/A	0	N/A
151	A	9	7	1.	16	0.438
152	B	82	23	4.05	16	1.438
153	F	0	0	N/A	0	N/A
154	A	6	4	1.	10	0.4
155	A	5	4	1.	8	0.5
156	A	4	4	1.	6	0.667
157	A	4	3	1.	10	0.3
158	A	4	4	1.	10	0.4
159	A	5	4	1.	10	0.4
160	A	3	2	1.	12	0.167
161	A	3	2	1.	12	0.167
162	A	2	2	1.	12	0.167
163	A	4	4	1.	12	0.333
164	A	3	2	1.	12	0.167
165	A	4	3	1.	10	0.3
166	A	4	3	1.	10	0.3

Chapter 3

Listing of integrals

3.1 $\int x^5 \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=59

$$\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)+\frac{bx^3}{18c^3}-\frac{bx}{6c^5}+\frac{b\tan^{-1}(cx)}{6c^6}-\frac{bx^5}{30c}$$

```
[Out] -(b*x)/(6*c^5) + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (b*ArcTan[c*x])/(6*c^6
) + (x^6*(a + b*ArcTan[c*x]))/6
```

Rubi [A] time = 0.0316717, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4852, 302, 203}

$$\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)+\frac{bx^3}{18c^3}-\frac{bx}{6c^5}+\frac{b\tan^{-1}(cx)}{6c^6}-\frac{bx^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x)/(6*c^5) + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (b*ArcTan[c*x])/(6*c^6
) + (x^6*(a + b*ArcTan[c*x]))/6
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a+b\tan^{-1}(cx)\right)dx &= \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\int\frac{x^6}{1+c^2x^2}\,dx\\ &= \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\int\left(\frac{1}{c^6}-\frac{x^2}{c^4}+\frac{x^4}{c^2}-\frac{1}{c^6\left(1+c^2x^2\right)}\right)dx\\ &= -\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)+\frac{b\int\frac{1}{1+c^2x^2}\,dx}{6c^5}\\ &= -\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{b\tan^{-1}(cx)}{6c^6}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.0031498, size = 64, normalized size = 1.08

$$\frac{ax^6}{6}+\frac{bx^3}{18c^3}-\frac{bx}{6c^5}+\frac{b\tan^{-1}(cx)}{6c^6}-\frac{bx^5}{30c}+\frac{1}{6}bx^6\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x)/(6*c^5) + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (a*x^6)/6 + (b*ArcTan[
c*x])/(6*c^6) + (b*x^6*ArcTan[c*x])/6
```

Maple [A] time = 0.006, size = 53, normalized size = 0.9

$$\frac{x^6a}{6}+\frac{bx^6\arctan(cx)}{6}-\frac{bx^5}{30\,c}+\frac{bx^3}{18\,c^3}-\frac{bx}{6\,c^5}+\frac{b\arctan(cx)}{6\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctan(c*x)-1/30*b*x^5/c+1/18*b*x^3/c^3-1/6*b*x/c^5+1/6
*b*arctan(c*x)/c^6
```

Maxima [A] time = 1.46044, size = 77, normalized size = 1.31

$$\frac{1}{6}ax^6+\frac{1}{90}\left(15\,x^6\arctan(cx)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^
6 - 15*arctan(c*x)/c^7))*b
```

Fricas [A] time = 2.36432, size = 131, normalized size = 2.22

$$\frac{15\,ac^{6}x^{6}-3\,bc^{5}x^{5}+5\,bc^{3}x^{3}-15\,bcx+15\left(bc^{6}x^{6}+b\right)\arctan\left(cx\right)}{90\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^6*x^6 - 3*b*c^5*x^5 + 5*b*c^3*x^3 - 15*b*c*x + 15*(b*c^6*x^6 +
 b)*arctan(c*x))/c^6
```

Sympy [A] time = 2.02291, size = 63, normalized size = 1.07

$$\begin{cases}\frac{ax^{6}}{6}+\frac{bx^{6}\operatorname{atan}\left(cx\right)}{6}-\frac{bx^{5}}{30c}+\frac{bx^{3}}{18c^{3}}-\frac{bx}{6c^{5}}+\frac{b\operatorname{atan}\left(cx\right)}{6c^{6}} & \text{for}\: c\neq 0\\ \frac{ax^{6}}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x)/6 - b*x**5/(30*c) + b*x**3/(18*c**3)
 - b*x/(6*c**5) + b*atan(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

Giac [A] time = 1.22262, size = 89, normalized size = 1.51

$$\frac{15\,bc^{6}x^{6}\arctan\left(cx\right)+15\,ac^{6}x^{6}-3\,bc^{5}x^{5}+5\,bc^{3}x^{3}-15\,\pi b\operatorname{sgn}\left(c\right)\operatorname{sgn}\left(x\right)-15\,bcx+15\,b\arctan\left(cx\right)}{90\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/90*(15*b*c^6*x^6*arctan(c*x) + 15*a*c^6*x^6 - 3*b*c^5*x^5 + 5*b*c^3*x^3 -
 15*pi*b*sgn(c)*sgn(x) - 15*b*c*x + 15*b*arctan(c*x))/c^6
```

3.2 $\int x^4 \left(a + b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=56

$$\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)+\frac{bx^2}{10c^3}-\frac{b\log\left(c^2x^2+1\right)}{10c^5}-\frac{bx^4}{20c}$$

```
[Out] (b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (x^5*(a + b*ArcTan[c*x]))/5 - (b*Log[1
+ c^2*x^2])/(10*c^5)
```

Rubi [A] time = 0.0417646, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4852, 266, 43}

$$\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)+\frac{bx^2}{10c^3}-\frac{b\log\left(c^2x^2+1\right)}{10c^5}-\frac{bx^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (x^5*(a + b*ArcTan[c*x]))/5 - (b*Log[1
+ c^2*x^2])/(10*c^5)
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4 \left(a+b\tan^{-1}(cx)\right) dx &= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{5}(bc)\int \frac{x^5}{1+c^2x^2}\,dx \\
&= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{10}(bc)\,\text{Subst}\left(\int \frac{x^2}{1+c^2x}\,dx,x,x^2\right) \\
&= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{10}(bc)\,\text{Subst}\left(\int \left(-\frac{1}{c^4}+\frac{x}{c^2}+\frac{1}{c^4\left(1+c^2x\right)}\right)dx,x,x^2\right) \\
&= \frac{bx^2}{10c^3}-\frac{bx^4}{20c}+\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{b\log\left(1+c^2x^2\right)}{10c^5}
\end{aligned}
$$

Mathematica [A] time = 0.0129023, size = 61, normalized size = 1.09

$$
\frac{ax^5}{5}+\frac{bx^2}{10c^3}-\frac{b\log\left(c^2x^2+1\right)}{10c^5}-\frac{bx^4}{20c}+\frac{1}{5}bx^5\tan^{-1}(cx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x])/5 - (b*
Log[1 + c^2*x^2])/(10*c^5)
```

Maple [A] time = 0.006, size = 52, normalized size = 0.9

$$
\frac{ax^5}{5}+\frac{x^5b\arctan(cx)}{5}-\frac{bx^4}{20\,c}+\frac{bx^2}{10\,c^3}-\frac{b\ln\left(c^2x^2+1\right)}{10\,c^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x)),x)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arctan(c*x)-1/20*b*x^4/c+1/10*b*x^2/c^3-1/10*b*ln(c^2*x
^2+1)/c^5
```

Maxima [A] time = 0.977194, size = 76, normalized size = 1.36

$$
\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\arctan(cx)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)b
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*
x^2 + 1)/c^6))*b
```

Fricas [A] time = 2.29035, size = 134, normalized size = 2.39

$$
\frac{4\,bc^5x^5\arctan(cx)+4\,ac^5x^5-bc^4x^4+2\,bc^2x^2-2\,b\log\left(c^2x^2+1\right)}{20\,c^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^5*x^5*arctan(c*x) + 4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*b
*log(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 1.44594, size = 60, normalized size = 1.07

$$\begin{cases} \frac{ax^5}{5} + \frac{bx^5 \operatorname{atan}(cx)}{5} - \frac{bx^4}{20c} + \frac{bx^2}{10c^3} - \frac{b\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for } c \neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atan(c*x)/5 - b*x**4/(20*c) + b*x**2/(10*c**3)
 - b*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*x**5/5, True))
```

Giac [A] time = 1.24879, size = 80, normalized size = 1.43

$$\frac{4\,bc^5x^5\arctan(cx) + 4\,ac^5x^5 - bc^4x^4 + 2\,bc^2x^2 - 2\,b\log\left(c^2x^2+1\right)}{20\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/20*(4*b*c^5*x^5*arctan(c*x) + 4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*b
*log(c^2*x^2 + 1))/c^5
```

3.3 $\int x^3 \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=48

$$\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)+\frac{bx}{4c^3}-\frac{b\tan^{-1}(cx)}{4c^4}-\frac{bx^3}{12c}$$

```
[Out] (b*x)/(4*c^3) - (b*x^3)/(12*c) - (b*ArcTan[c*x])/(4*c^4) + (x^4*(a + b*ArcT
an[c*x]))/4
```

Rubi [A] time = 0.0271138, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4852, 302, 203}

$$\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)+\frac{bx}{4c^3}-\frac{b\tan^{-1}(cx)}{4c^4}-\frac{bx^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) - (b*x^3)/(12*c) - (b*ArcTan[c*x])/(4*c^4) + (x^4*(a + b*ArcT
an[c*x]))/4
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tan^{-1}(cx)\right)dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}(bc)\int\frac{x^4}{1+c^2x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)-\frac{1}{4}(bc)\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)dx\\
&= \frac{bx}{4c^3}-\frac{bx^3}{12c}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)-\frac{b\int\frac{1}{1+c^2x^2}\,dx}{4c^3}\\
&= \frac{bx}{4c^3}-\frac{bx^3}{12c}-\frac{b\tan^{-1}(cx)}{4c^4}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] time = 0.0026207, size = 53, normalized size = 1.1

$$\frac{ax^4}{4}+\frac{bx}{4c^3}-\frac{b\tan^{-1}(cx)}{4c^4}-\frac{bx^3}{12c}+\frac{1}{4}bx^4\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) - (b*x^3)/(12*c) + (a*x^4)/4 - (b*ArcTan[c*x])/(4*c^4) + (b*x
^4*ArcTan[c*x])/4
```

Maple [A] time = 0.005, size = 44, normalized size = 0.9

$$\frac{x^4a}{4}+\frac{x^4b\arctan(cx)}{4}-\frac{bx^3}{12c}+\frac{bx}{4c^3}-\frac{b\arctan(cx)}{4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x)),x)
```

```
[Out] 1/4*x^4*a+1/4*x^4*b*arctan(c*x)-1/12*b*x^3/c+1/4*b*x/c^3-1/4*b*arctan(c*x)/
c^4
```

Maxima [A] time = 1.44273, size = 65, normalized size = 1.35

$$\frac{1}{4}ax^4+\frac{1}{12}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x
)/c^5))*b
```

Fricas [A] time = 2.44174, size = 105, normalized size = 2.19

$$\frac{3ac^4x^4-bc^3x^3+3bcx+3\left(bc^4x^4-b\right)\arctan(cx)}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x))/c^
4
```

Sympy [A] time = 1.10828, size = 53, normalized size = 1.1

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atan}(cx)}{4} - \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b \operatorname{atan}(cx)}{4c^4} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x)/4 - b*x**3/(12*c) + b*x/(4*c**3) - b
*atan(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))
```

Giac [A] time = 1.35318, size = 66, normalized size = 1.38

$$\frac{3\, bc^4 x^4 \arctan(cx) + 3\, ac^4 x^4 - bc^3 x^3 + 3\, bcx - 3\, b \arctan(cx)}{12\, c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*c^4*x^4*arctan(c*x) + 3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x - 3*b*arc
tan(c*x))/c^4
```

3.4 $\int x^2 \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=45

$$\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)+\frac{b\log\left(c^2x^2+1\right)}{6c^3}-\frac{bx^2}{6c}$$

```
[Out] -(b*x^2)/(6*c) + (x^3*(a + b*ArcTan[c*x]))/3 + (b*Log[1 + c^2*x^2])/(6*c^3)
```

Rubi [A] time = 0.0318549, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4852, 266, 43}

$$\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)+\frac{b\log\left(c^2x^2+1\right)}{6c^3}-\frac{bx^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x^2)/(6*c) + (x^3*(a + b*ArcTan[c*x]))/3 + (b*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\tan^{-1}(cx)\right)dx &= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{3}(bc)\int\frac{x^3}{1+c^2x^2}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\,\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\,\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)dx,x,x^2\right)\\
&= -\frac{bx^2}{6c}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)+\frac{b\log\left(1+c^2x^2\right)}{6c^3}
\end{aligned}$$

Mathematica [A] time = 0.0082896, size = 50, normalized size = 1.11

$$\frac{ax^3}{3}+\frac{b\log\left(c^2x^2+1\right)}{6c^3}-\frac{bx^2}{6c}+\frac{1}{3}bx^3\tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x^2)/(6*c) + (a*x^3)/3 + (b*x^3*ArcTan[c*x])/3 + (b*Log[1 + c^2*x^2])/(
6*c^3)
```

Maple [A] time = 0.006, size = 43, normalized size = 1.

$$\frac{x^3a}{3}+\frac{bx^3\arctan(cx)}{3}-\frac{bx^2}{6c}+\frac{b\ln\left(c^2x^2+1\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctan(c*x)-1/6*b*x^2/c+1/6*b*ln(c^2*x^2+1)/c^3
```

Maxima [A] time = 0.982194, size = 62, normalized size = 1.38

$$\frac{1}{3}ax^3+\frac{1}{6}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b
```

Fricas [A] time = 2.30553, size = 111, normalized size = 2.47

$$\frac{2bc^3x^3\arctan(cx)+2ac^3x^3-bc^2x^2+b\log\left(c^2x^2+1\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^3*x^3*arctan(c*x) + 2*a*c^3*x^3 - b*c^2*x^2 + b*log(c^2*x^2 + 1)
)/c^3
```

Sympy [A] time = 0.765492, size = 49, normalized size = 1.09

$$\begin{cases}\frac{ax^3}{3}+\frac{bx^3\operatorname{atan}(cx)}{3}-\frac{bx^2}{6c}+\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for } c\neq 0\\ \frac{ax^3}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x)/3 - b*x**2/(6*c) + b*log(x**2 + c**(
-2))/(6*c**3), Ne(c, 0)), (a*x**3/3, True))
```

Giac [A] time = 1.20281, size = 66, normalized size = 1.47

$$\frac{2\,bc^3x^3\arctan\left(cx\right)+2\,ac^3x^3-bc^2x^2+b\log\left(c^2x^2+1\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^3*x^3*arctan(c*x) + 2*a*c^3*x^3 - b*c^2*x^2 + b*log(c^2*x^2 + 1)
)/c^3
```

3.5 $\int x\left(a+b\tan^{-1}(cx)\right)dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)+\frac{b\tan^{-1}(cx)}{2c^2}-\frac{bx}{2c}$$

```
[Out] -(b*x)/(2*c) + (b*ArcTan[c*x])/(2*c^2) + (x^2*(a + b*ArcTan[c*x]))/2
```

Rubi [A] time = 0.0156177, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4852, 321, 203}

$$\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)+\frac{b\tan^{-1}(cx)}{2c^2}-\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x)/(2*c) + (b*ArcTan[c*x])/(2*c^2) + (x^2*(a + b*ArcTan[c*x]))/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tan^{-1}(cx)\right)dx&=\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\int\frac{x^2}{1+c^2x^2}\,dx\\&=-\frac{bx}{2c}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)+\frac{b\int\frac{1}{1+c^2x^2}\,dx}{2c}\\&=-\frac{bx}{2c}+\frac{b\tan^{-1}(cx)}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.0022397, size = 42, normalized size = 1.14

$$\frac{ax^2}{2}+\frac{b\tan^{-1}(cx)}{2c^2}+\frac{1}{2}bx^2\tan^{-1}(cx)-\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*x)/(2*c) + (a*x^2)/2 + (b*ArcTan[c*x])/(2*c^2) + (b*x^2*ArcTan[c*x])/2
```

Maple [A] time = 0.004, size = 35, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^2\arctan(cx)}{2}-\frac{bx}{2c}+\frac{b\arctan(cx)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctan(c*x)-1/2*b*x/c+1/2*b*arctan(c*x)/c^2
```

Maxima [A] time = 1.48385, size = 50, normalized size = 1.35

$$\frac{1}{2}ax^2+\frac{1}{2}\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
```

Fricas [A] time = 2.21952, size = 80, normalized size = 2.16

$$\frac{ac^2x^2-bcx+\left(bc^2x^2+b\right)\arctan(cx)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*x^2 - b*c*x + (b*c^2*x^2 + b)*arctan(c*x))/c^2
```

Sympy [A] time = 0.561452, size = 42, normalized size = 1.14

$$\begin{cases}\frac{ax^2}{2}+\frac{bx^2\operatorname{atan}(cx)}{2}-\frac{bx}{2c}+\frac{b\operatorname{atan}(cx)}{2c^2} & \text{for } c\neq 0\\ \frac{ax^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x)/2 - b*x/(2*c) + b*atan(c*x)/(2*c**2)
, Ne(c, 0)), (a*x**2/2, True))
```

Giac [A] time = 1.19385, size = 61, normalized size = 1.65

$$\frac{bc^2x^2\arctan(cx) + ac^2x^2 - \pi b\,\mathrm{sgn}(c)\,\mathrm{sgn}(x) - bcx + b\arctan(cx)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*(b*c^2*x^2*arctan(c*x) + a*c^2*x^2 - pi*b*sgn(c)*sgn(x) - b*c*x + b*arc
tan(c*x))/c^2
```

3.6 $\int \left(a+b \tan^{-1}(cx)\right) \, dx$

Optimal. Leaf size=29

$$ax-\frac{b \log \left(c^2 x^2+1\right)}{2 c}+b x \tan^{-1}(c x)$$

```
[Out] a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Rubi [A] time = 0.0113169, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4846, 260}

$$ax-\frac{b \log \left(c^2 x^2+1\right)}{2 c}+b x \tan^{-1}(c x)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c*x],x]
```

```
[Out] a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b \tan^{-1}(cx)\right) \, dx &= ax+b \int \tan^{-1}(cx) \, dx\\
&= ax+bx \tan^{-1}(cx)-(bc) \int \frac{x}{1+c^2x^2} \, dx\\
&= ax+bx \tan^{-1}(cx)-\frac{b \log \left(1+c^2x^2\right)}{2c}
\end{aligned}$$

Mathematica [A] time = 0.0029365, size = 29, normalized size = 1.

$$ax-\frac{b \log \left(c^2 x^2+1\right)}{2 c}+b x \tan^{-1}(c x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c*x],x]
```

```
[Out] a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Maple [A] time = 0.003, size = 28, normalized size = 1.

$$ax + bx\arctan(cx) - \frac{b\ln(c^2x^2+1)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c*x),x)
```

```
[Out] a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

Maxima [A] time = 0.985285, size = 42, normalized size = 1.45

$$ax + \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

Fricas [A] time = 2.23199, size = 81, normalized size = 2.79

$$\frac{2\,bcx\arctan(cx) + 2\,acx - b\log(c^2x^2+1)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c*x*arctan(c*x) + 2*a*c*x - b*log(c^2*x^2 + 1))/c
```

Sympy [A] time = 0.252711, size = 26, normalized size = 0.9

$$ax + b\left(\begin{cases} x\,\mathrm{atan}(cx) - \frac{\log(c^2x^2+1)}{2c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*atan(c*x) - log(c**2*x**2 + 1)/(2*c), Ne(c, 0)), (0, T
rue))
```

Giac [A] time = 1.33481, size = 42, normalized size = 1.45

$$ax + \frac{\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2 + 1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x),x, algorithm="giac")
```

```
[Out] a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

3.7 $\int \frac{a+b\tan^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}ib\text{PolyLog}(2,-icx)-\frac{1}{2}ib\text{PolyLog}(2,icx)+a\log(x)$$

```
[Out] a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Rubi [A] time = 0.028752, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4848, 2391}

$$\frac{1}{2}ib\text{PolyLog}(2,-icx)-\frac{1}{2}ib\text{PolyLog}(2,icx)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x,x]
```

```
[Out] a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx)}{x}\,dx &= a\log(x)+\frac{1}{2}(ib)\int \frac{\log(1-icx)}{x}\,dx-\frac{1}{2}(ib)\int \frac{\log(1+icx)}{x}\,dx\\ &= a\log(x)+\frac{1}{2}ib\text{Li}_2(-icx)-\frac{1}{2}ib\text{Li}_2(icx)\end{aligned}$$

Mathematica [A] time = 0.0021458, size = 35, normalized size = 1.

$$\frac{1}{2}ib\text{PolyLog}(2,-icx)-\frac{1}{2}ib\text{PolyLog}(2,icx)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x,x]
```

```
[Out] a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Maple [B] time = 0.015, size = 74, normalized size = 2.1

$$a\ln(cx)+b\ln(cx)\arctan(cx)+\frac{i}{2}b\ln(cx)\ln(1+icx)-\frac{i}{2}b\ln(cx)\ln(1-icx)+\frac{i}{2}b\mathrm{dilog}(1+icx)-\frac{i}{2}b\mathrm{dilog}(1-icx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x,x)
```

```
[Out] a*ln(c*x)+b*ln(c*x)*arctan(c*x)+1/2*I*b*ln(c*x)*ln(1+I*c*x)-1/2*I*b*ln(c*x)
*ln(1-I*c*x)+1/2*I*b*dilog(1+I*c*x)-1/2*I*b*dilog(1-I*c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\arctan(cx)}{x}\,dx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x)/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\arctan(cx)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{atan}(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\arctan(cx)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/x, x)
```

3.8 $\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{a+b\tan^{-1}(cx)}{x}-\frac{1}{2}bc\log\left(c^2x^2+1\right)+bc\log(x)$$

```
[Out] -((a + b*ArcTan[c*x])/x) + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

Rubi [A] time = 0.0230129, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4852, 266, 36, 29, 31}

$$-\frac{a+b\tan^{-1}(cx)}{x}-\frac{1}{2}bc\log\left(c^2x^2+1\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x])/x) + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx = -\frac{a+b\tan^{-1}(cx)}{x} + (bc)\int \frac{1}{x\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}(cx)}{x} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tan^{-1}(cx)}{x} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tan^{-1}(cx)}{x} + bc\log(x) - \frac{1}{2}bc\log\left(1+c^2x^2\right)$$

Mathematica [A] time = 0.002613, size = 38, normalized size = 1.09

$$-\frac{a}{x} - \frac{1}{2}bc\log\left(c^2x^2+1\right) + bc\log(x) - \frac{b\tan^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c*x])/x + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

Maple [A] time = 0.006, size = 39, normalized size = 1.1

$$-\frac{a}{x} - \frac{b\arctan(cx)}{x} - \frac{bc\ln\left(c^2x^2+1\right)}{2} + cb\ln(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2,x)
```

```
[Out] -a/x-b/x*arctan(c*x)-1/2*b*c*ln(c^2*x^2+1)+c*b*ln(c*x)
```

Maxima [A] time = 0.969497, size = 53, normalized size = 1.51

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b - a/x
```

Fricas [A] time = 2.49191, size = 100, normalized size = 2.86

$$-\frac{bcx\log\left(c^2x^2+1\right) - 2\,bcx\log(x) + 2\,b\arctan(cx) + 2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*x*log(c^2*x^2 + 1) - 2*b*c*x*log(x) + 2*b*arctan(c*x) + 2*a)/x
```

Sympy [A] time = 0.88915, size = 37, normalized size = 1.06

$$\begin{cases} -\frac{a}{x} + bc\log{(x)} - \frac{bc\log{\left(x^2+\frac{1}{c^2}\right)}}{2} - \frac{b\,\mathrm{atan}\,(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c*log(x) - b*c*log(x**2 + c**(-2))/2 - b*atan(c*x)/x, N
e(c, 0)), (-a/x, True))
```

Giac [A] time = 1.32274, size = 50, normalized size = 1.43

$$-\frac{bcx\log\left(c^2x^2+1\right) - 2\,bcx\log\left(x\right) + 2\,b\arctan\left(cx\right) + 2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*c*x*log(c^2*x^2 + 1) - 2*b*c*x*log(x) + 2*b*arctan(c*x) + 2*a)/x
```

3.9 $\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{a+b\tan^{-1}(cx)}{2x^2}-\frac{1}{2}bc^2\tan^{-1}(cx)-\frac{bc}{2x}$$

```
[Out] -(b*c)/(2*x) - (b*c^2*ArcTan[c*x])/2 - (a + b*ArcTan[c*x])/(2*x^2)
```

Rubi [A] time = 0.0195559, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4852, 325, 203}

$$-\frac{a+b\tan^{-1}(cx)}{2x^2}-\frac{1}{2}bc^2\tan^{-1}(cx)-\frac{bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^3,x]
```

```
[Out] -(b*c)/(2*x) - (b*c^2*ArcTan[c*x])/2 - (a + b*ArcTan[c*x])/(2*x^2)
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx &= -\frac{a+b\tan^{-1}(cx)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx\\
&= -\frac{bc}{2x}-\frac{a+b\tan^{-1}(cx)}{2x^2}-\frac{1}{2}\left(bc^3\right)\int \frac{1}{1+c^2x^2}\,dx\\
&= -\frac{bc}{2x}-\frac{1}{2}bc^2\tan^{-1}(cx)-\frac{a+b\tan^{-1}(cx)}{2x^2}
\end{aligned}$$

Mathematica [C] time = 0.0028617, size = 46, normalized size = 1.24

$$-\frac{bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x}-\frac{a}{2x^2}-\frac{b\tan^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*ArcTan[c*x])/(2*x^2) - (b*c*Hypergeometric2F1[-1/2, 1, 1/2,
 -(c^2*x^2)])/(2*x)
```

Maple [A] time = 0.007, size = 35, normalized size = 1.

$$-\frac{a}{2x^2}-\frac{b\arctan(cx)}{2x^2}-\frac{bc^2\arctan(cx)}{2}-\frac{bc}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c*x)-1/2*b*c^2*arctan(c*x)-1/2*b*c/x
```

Maxima [A] time = 1.46763, size = 42, normalized size = 1.14

$$-\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)b-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b - 1/2*a/x^2
```

Fricas [A] time = 2.56519, size = 70, normalized size = 1.89

$$-\frac{bcx+\left(bc^2x^2+b\right)\arctan(cx)+a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)/x^2
```

Sympy [A] time = 0.744, size = 37, normalized size = 1.

$$-\frac{a}{2x^2}-\frac{bc^2\operatorname{atan}(cx)}{2}-\frac{bc}{2x}-\frac{b\operatorname{atan}(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) - b*c**2*atan(c*x)/2 - b*c/(2*x) - b*atan(c*x)/(2*x**2)
```

Giac [A] time = 1.29837, size = 74, normalized size = 2.

$$\frac{bc^2ix^2\log(cix+1) - bc^2ix^2\log(-cix+1) - 2\,bcx - 2\,b\arctan(cx) - 2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*c^2*i*x^2*log(c*i*x + 1) - b*c^2*i*x^2*log(-c*i*x + 1) - 2*b*c*x - 2
*b*arctan(c*x) - 2*a)/x^2
```

3.10 $\int \frac{a+b\tan^{-1}(cx)}{x^4} dx$

Optimal. Leaf size=53

$$-\frac{a+b\tan^{-1}(cx)}{3x^3}+\frac{1}{6}bc^3\log\left(c^2x^2+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{6x^2}$$

```
[Out] -(b*c)/(6*x^2) - (a + b*ArcTan[c*x])/(3*x^3) - (b*c^3*Log[x])/3 + (b*c^3*Lo
g[1 + c^2*x^2])/6
```

Rubi [A] time = 0.033227, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4852, 266, 44}

$$-\frac{a+b\tan^{-1}(cx)}{3x^3}+\frac{1}{6}bc^3\log\left(c^2x^2+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^4,x]
```

```
[Out] -(b*c)/(6*x^2) - (a + b*ArcTan[c*x])/(3*x^3) - (b*c^3*Log[x])/3 + (b*c^3*Lo
g[1 + c^2*x^2])/6
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx = -\frac{a+b\tan^{-1}(cx)}{3x^3} + \frac{1}{3}(bc)\int \frac{1}{x^3\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}(cx)}{3x^3} + \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x\right)}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tan^{-1}(cx)}{3x^3} + \frac{1}{6}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{c^2}{x} + \frac{c^4}{1+c^2x}\right)dx, x, x^2\right)$$
$$= -\frac{bc}{6x^2} - \frac{a+b\tan^{-1}(cx)}{3x^3} - \frac{1}{3}bc^3\log(x) + \frac{1}{6}bc^3\log\left(1+c^2x^2\right)$$

Mathematica [A] time = 0.015549, size = 54, normalized size = 1.02

$$-\frac{a}{3x^3} + \frac{1}{6}bc\left(c^2\log\left(c^2x^2+1\right) - 2c^2\log(x) - \frac{1}{x^2}\right) - \frac{b\tan^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^4,x]
```

```
[Out] -a/(3*x^3) - (b*ArcTan[c*x])/(3*x^3) + (b*c*(-x^(-2) - 2*c^2*Log[x] + c^2*L
og[1 + c^2*x^2]))/6
```

Maple [A] time = 0.007, size = 51, normalized size = 1.

$$-\frac{a}{3\,x^3} - \frac{b\arctan(cx)}{3\,x^3} + \frac{bc^3\ln\left(c^2x^2+1\right)}{6} - \frac{bc}{6\,x^2} - \frac{c^3b\ln(cx)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c*x)+1/6*b*c^3*ln(c^2*x^2+1)-1/6*b*c/x^2-1/3*c^
3*b*ln(c*x)
```

Maxima [A] time = 0.969048, size = 69, normalized size = 1.3

$$\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)b - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
 - 1/3*a/x^3
```

Fricas [A] time = 2.48142, size = 123, normalized size = 2.32

$$\frac{bc^3x^3\log\left(c^2x^2+1\right) - 2\,bc^3x^3\log(x) - bcx - 2\,b\arctan(cx) - 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*c^3*x^3*log(c^2*x^2 + 1) - 2*b*c^3*x^3*log(x) - b*c*x - 2*b*arctan(c
*x) - 2*a)/x^3
```

Sympy [A] time = 1.5361, size = 61, normalized size = 1.15

$$\begin{cases} -\frac{a}{3x^3} - \frac{bc^3\log(x)}{3} + \frac{bc^3\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bc}{6x^2} - \frac{b\,\mathrm{atan}\,(cx)}{3x^3} & \text{for } c \neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) - b*c**3*log(x)/3 + b*c**3*log(x**2 + c**(-2))/6 - b
*c/(6*x**2) - b*atan(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), True))
```

Giac [A] time = 1.80703, size = 68, normalized size = 1.28

$$\frac{bc^3x^3\log\left(c^2x^2+1\right) - 2\,bc^3x^3\log(x) - bcx - 2\,b\arctan(cx) - 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(b*c^3*x^3*log(c^2*x^2 + 1) - 2*b*c^3*x^3*log(x) - b*c*x - 2*b*arctan(c
*x) - 2*a)/x^3
```

3.11 $\int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx$

Optimal. Leaf size=48

$$-\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\tan^{-1}(cx)-\frac{bc}{12x^3}$$

```
[Out] -(b*c)/(12*x^3) + (b*c^3)/(4*x) + (b*c^4*ArcTan[c*x])/4 - (a + b*ArcTan[c*x
])/(4*x^4)
```

Rubi [A] time = 0.0250794, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4852, 325, 203}

$$-\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\tan^{-1}(cx)-\frac{bc}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^5,x]
```


```
[Out] -(b*c)/(12*x^3) + (b*c^3)/(4*x) + (b*c^4*ArcTan[c*x])/4 - (a + b*ArcTan[c*x
])/(4*x^4)
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx &= -\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{1}{4}(bc)\int \frac{1}{x^4\left(1+c^2x^2\right)}\,dx \\
&= -\frac{bc}{12x^3}-\frac{a+b\tan^{-1}(cx)}{4x^4}-\frac{1}{4}\left(bc^3\right)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx \\
&= -\frac{bc}{12x^3}+\frac{bc^3}{4x}-\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{1}{4}\left(bc^5\right)\int \frac{1}{1+c^2x^2}\,dx \\
&= -\frac{bc}{12x^3}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\tan^{-1}(cx)-\frac{a+b\tan^{-1}(cx)}{4x^4}
\end{aligned}
$$

Mathematica [C] time = 0.002897, size = 46, normalized size = 0.96

$$
-\frac{bc\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3}-\frac{a}{4x^4}-\frac{b\tan^{-1}(cx)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^5,x]
```

```
[Out] -a/(4*x^4) - (b*ArcTan[c*x])/(4*x^4) - (b*c*Hypergeometric2F1[-3/2, 1, -1/2
, -(c^2*x^2)])/(12*x^3)
```

Maple [A] time = 0.009, size = 44, normalized size = 0.9

$$
-\frac{a}{4\,x^4}-\frac{b\arctan(cx)}{4\,x^4}+\frac{bc^4\arctan(cx)}{4}-\frac{bc}{12\,x^3}+\frac{bc^3}{4\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctan(c*x)+1/4*b*c^4*arctan(c*x)-1/12*b*c/x^3+1/4*b*c
^3/x
```

Maxima [A] time = 1.45929, size = 62, normalized size = 1.29

$$
\frac{1}{12}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan(cx)}{x^4}\right)b-\frac{a}{4\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b -
1/4*a/x^4
```

Fricas [A] time = 2.37558, size = 95, normalized size = 1.98

$$
\frac{3\,bc^3x^3-bcx+3\left(bc^4x^4-b\right)\arctan(cx)-3\,a}{12\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^3*x^3 - b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x) - 3*a)/x^4
```

Sympy [A] time = 1.20462, size = 46, normalized size = 0.96

$$-\frac{a}{4x^4}+\frac{bc^4\,\mathrm{atan}\,(cx)}{4}+\frac{bc^3}{4x}-\frac{bc}{12x^3}-\frac{b\,\mathrm{atan}\,(cx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**5,x)
```

```
[Out] -a/(4*x**4) + b*c**4*atan(c*x)/4 + b*c**3/(4*x) - b*c/(12*x**3) - b*atan(c*
x)/(4*x**4)
```

Giac [A] time = 1.7354, size = 88, normalized size = 1.83

$$\frac{3\,bc^4ix^4\log\left(cix-1\right)-3\,bc^4ix^4\log\left(-cix-1\right)+6\,bc^3x^3-2\,bcx-6\,b\arctan\left(cx\right)-6\,a}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*b*c^4*i*x^4*log(c*i*x - 1) - 3*b*c^4*i*x^4*log(-c*i*x - 1) + 6*b*c^
3*x^3 - 2*b*c*x - 6*b*arctan(c*x) - 6*a)/x^4
```

3.12 $\int \frac{a+b\tan^{-1}(cx)}{x^6}\,dx$

Optimal. Leaf size=64

$$-\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(c^2x^2+1\right)+\frac{1}{5}bc^5\log(x)-\frac{bc}{20x^4}$$

```
[Out] -(b*c)/(20*x^4) + (b*c^3)/(10*x^2) - (a + b*ArcTan[c*x])/(5*x^5) + (b*c^5*L
og[x])/5 - (b*c^5*Log[1 + c^2*x^2])/10
```

Rubi [A] time = 0.0382962, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4852, 266, 44}

$$-\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(c^2x^2+1\right)+\frac{1}{5}bc^5\log(x)-\frac{bc}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^6,x]
```

```
[Out] -(b*c)/(20*x^4) + (b*c^3)/(10*x^2) - (a + b*ArcTan[c*x])/(5*x^5) + (b*c^5*L
og[x])/5 - (b*c^5*Log[1 + c^2*x^2])/10
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^6}\,dx = -\frac{a+b\tan^{-1}(cx)}{5x^5} + \frac{1}{5}(bc)\int \frac{1}{x^5\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}(cx)}{5x^5} + \frac{1}{10}(bc)\,\text{Subst}\left(\int \frac{1}{x^3\left(1+c^2x\right)}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tan^{-1}(cx)}{5x^5} + \frac{1}{10}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^3}-\frac{c^2}{x^2}+\frac{c^4}{x}-\frac{c^6}{1+c^2x}\right)dx, x, x^2\right)$$
$$= -\frac{bc}{20x^4} + \frac{bc^3}{10x^2} - \frac{a+b\tan^{-1}(cx)}{5x^5} + \frac{1}{5}bc^5\log(x) - \frac{1}{10}bc^5\log\left(1+c^2x^2\right)$$

Mathematica [A] time = 0.0168336, size = 64, normalized size = 1.

$$-\frac{a}{5x^5} + \frac{1}{10}bc\left(\frac{c^2}{x^2} - c^4\log\left(c^2x^2+1\right) + 2c^4\log(x) - \frac{1}{2x^4}\right) - \frac{b\tan^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^6,x]
```

```
[Out] -a/(5*x^5) - (b*ArcTan[c*x])/(5*x^5) + (b*c*(-1/(2*x^4) + c^2/x^2 + 2*c^4*L
og[x] - c^4*Log[1 + c^2*x^2]))/10
```

Maple [A] time = 0.01, size = 60, normalized size = 0.9

$$-\frac{a}{5\,x^5} - \frac{b\arctan(cx)}{5\,x^5} - \frac{bc^5\ln\left(c^2x^2+1\right)}{10} - \frac{bc}{20\,x^4} + \frac{c^5b\ln(cx)}{5} + \frac{bc^3}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctan(c*x)-1/10*b*c^5*ln(c^2*x^2+1)-1/20*b*c/x^4+1/5*
c^5*b*ln(c*x)+1/10*b*c^3/x^2
```

Maxima [A] time = 0.975209, size = 84, normalized size = 1.31

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right) - 2\,c^4\log\left(x^2\right) - \frac{2\,c^2x^2-1}{x^4}\right)c + \frac{4\arctan(cx)}{x^5}\right)b - \frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b - 1/5*a/x^5
```

Fricas [A] time = 2.52753, size = 147, normalized size = 2.3

$$-\frac{2\,bc^5x^5\log\left(c^2x^2+1\right) - 4\,bc^5x^5\log(x) - 2\,bc^3x^3 + bcx + 4\,b\arctan(cx) + 4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*b*c^5*x^5*log(c^2*x^2 + 1) - 4*b*c^5*x^5*log(x) - 2*b*c^3*x^3 + b*
c*x + 4*b*arctan(c*x) + 4*a)/x^5
```

Sympy [A] time = 2.5915, size = 71, normalized size = 1.11

$$\begin{cases} -\frac{a}{5x^5} + \frac{bc^5 \log(x)}{5} - \frac{bc^5 \log\left(x^2 + \frac{1}{c^2}\right)}{10} + \frac{bc^3}{10x^2} - \frac{bc}{20x^4} - \frac{b\,\mathrm{atan}\,(cx)}{5x^5} & \text{for}\ c \neq 0 \\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x**2 + c**(-2))/10 +
b*c**3/(10*x**2) - b*c/(20*x**4) - b*atan(c*x)/(5*x**5), Ne(c, 0)), (-a/(5*
x**5), True))
```

Giac [A] time = 1.40601, size = 80, normalized size = 1.25

$$-\frac{2\,bc^5x^5\log\left(c^2x^2+1\right) - 4\,bc^5x^5\log(x) - 2\,bc^3x^3 + bcx + 4\,b\arctan(cx) + 4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*(2*b*c^5*x^5*log(c^2*x^2 + 1) - 4*b*c^5*x^5*log(x) - 2*b*c^3*x^3 + b*
c*x + 4*b*arctan(c*x) + 4*a)/x^5
```

3.13 $\int x^5 \left(a+b\tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=144

$$\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{abx}{3c^5}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{6c^6}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c}+\frac{b^2x^4}{60c^2}-\frac{4b^2x^2}{45c^4}\cdot$$

```
[Out] -(a*b*x)/(3*c^5) - (4*b^2*x^2)/(45*c^4) + (b^2*x^4)/(60*c^2) - (b^2*x*ArcTa
n[c*x])/(3*c^5) + (b*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*x^5*(a + b*ArcTa
n[c*x]))/(15*c) + (a + b*ArcTan[c*x])^2/(6*c^6) + (x^6*(a + b*ArcTan[c*x])^
2)/6 + (23*b^2*Log[1 + c^2*x^2])/(90*c^6)
```

Rubi [A] time = 0.310175, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3}-\frac{abx}{3c^5}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{6c^6}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c}+\frac{b^2x^4}{60c^2}-\frac{4b^2x^2}{45c^4}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -(a*b*x)/(3*c^5) - (4*b^2*x^2)/(45*c^4) + (b^2*x^4)/(60*c^2) - (b^2*x*ArcTa
n[c*x])/(3*c^5) + (b*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*x^5*(a + b*ArcTa
n[c*x]))/(15*c) + (a + b*ArcTan[c*x])^2/(6*c^6) + (x^6*(a + b*ArcTan[c*x])^
2)/6 + (23*b^2*Log[1 + c^2*x^2])/(90*c^6)
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^5 \left(a+b\tan^{-1}(cx)\right)^2 dx = \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2 - \frac{1}{3}(bc)\int \frac{x^6\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx$$

$$= \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2 - \frac{b\int x^4\left(a+b\tan^{-1}(cx)\right)dx}{3c} + \frac{b\int \frac{x^4\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx}{3c}$$

$$= -\frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c} + \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{15}b^2\int \frac{x^5}{1+c^2x^2}dx + \frac{b\int x^2\left(a+b\tan\right.}{3c^3}$$

$$= \frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3} - \frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c} + \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2 + \frac{1}{30}b^2\,\mathrm{Subst}\left(\int\right.$$

$$= -\frac{abx}{3c^5} + \frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3} - \frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c} + \frac{\left(a+b\tan^{-1}(cx)\right)^2}{6c^6} + \frac{1}{6}x^6\left(a+b\right.$$

$$= -\frac{abx}{3c^5} - \frac{b^2x^2}{30c^4} + \frac{b^2x^4}{60c^2} - \frac{b^2x\tan^{-1}(cx)}{3c^5} + \frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3} - \frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c} + \left(\right.$$

$$= -\frac{abx}{3c^5} - \frac{4b^2x^2}{45c^4} + \frac{b^2x^4}{60c^2} - \frac{b^2x\tan^{-1}(cx)}{3c^5} + \frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{9c^3} - \frac{bx^5\left(a+b\tan^{-1}(cx)\right)}{15c} +$$

Mathematica [A] time = 0.129538, size = 138, normalized size = 0.96

$$\frac{cx\left(30a^2c^5x^5 - 4ab\left(3c^4x^4 - 5c^2x^2 + 15\right) + b^2cx\left(3c^2x^2 - 16\right)\right) + 4b\tan^{-1}(cx)\left(15a\left(c^6x^6+1\right) + bcx\left(-3c^4x^4 + 5c^2x^2 - 15\right)\right.}{180c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(30*a^2*c^5*x^5 + b^2*c*x*(-16 + 3*c^2*x^2) - 4*a*b*(15 - 5*c^2*x^2 +
3*c^4*x^4)) + 4*b*(b*c*x*(-15 + 5*c^2*x^2 - 3*c^4*x^4) + 15*a*(1 + c^6*x^6)
)*ArcTan[c*x] + 30*b^2*(1 + c^6*x^6)*ArcTan[c*x]^2 + 46*b^2*Log[1 + c^2*x^2
])/(180*c^6)
```

Maple [A] time = 0.013, size = 171, normalized size = 1.2

$$\frac{x^6a^2}{6}+\frac{b^2x^6(\arctan(cx))^2}{6}-\frac{b^2\arctan(cx)x^5}{15c}+\frac{b^2\arctan(cx)x^3}{9c^3}-\frac{b^2x\arctan(cx)}{3c^5}+\frac{b^2(\arctan(cx))^2}{6c^6}+\frac{b^2x^4}{60c^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/6*x^6*a^2+1/6*b^2*x^6*arctan(c*x)^2-1/15/c*b^2*arctan(c*x)*x^5+1/9/c^3*b^
2*arctan(c*x)*x^3-1/3*b^2*x*arctan(c*x)/c^5+1/6/c^6*b^2*arctan(c*x)^2+1/60*
b^2*x^4/c^2-4/45*b^2*x^2/c^4+23/90*b^2*ln(c^2*x^2+1)/c^6+1/3*a*b*x^6*arctan
(c*x)-1/15/c*x^5*a*b+1/9*a*b*x^3/c^3-1/3*a*b*x/c^5+1/3/c^6*a*b*arctan(c*x)
```

Maxima [A] time = 1.50921, size = 220, normalized size = 1.53

$$\frac{1}{6}b^2x^6\arctan(cx)^2+\frac{1}{6}a^2x^6+\frac{1}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)ab-\frac{1}{180}\left(4c\left(\frac{3c}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctan(c*x)^2 + 1/6*a^2*x^6 + 1/45*(15*x^6*arctan(c*x) - c*((3*
c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b - 1/180*(4*c*((3
*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) - (3*c^4
*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*b^2
```

Fricas [A] time = 2.54298, size = 351, normalized size = 2.44

$$\frac{30a^2c^6x^6-12abc^5x^5+3b^2c^4x^4+20abc^3x^3-16b^2c^2x^2-60abcx+30\left(b^2c^6x^6+b^2\right)\arctan(cx)^2+46b^2\log\left(c^2x^2+\right.}{180c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(30*a^2*c^6*x^6 - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20*a*b*c^3*x^3 - 1
6*b^2*c^2*x^2 - 60*a*b*c*x + 30*(b^2*c^6*x^6 + b^2)*arctan(c*x)^2 + 46*b^2*
log(c^2*x^2 + 1) + 4*(15*a*b*c^6*x^6 - 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 15*b
^2*c*x + 15*a*b)*arctan(c*x))/c^6
```

Sympy [A] time = 3.60678, size = 199, normalized size = 1.38

$$\begin{cases}\frac{a^2x^6}{6}+\frac{abx^6\operatorname{atan}(cx)}{3}-\frac{abx^5}{15c}+\frac{abx^3}{9c^3}-\frac{abx}{3c^5}+\frac{ab\operatorname{atan}(cx)}{3c^6}+\frac{b^2x^6\operatorname{atan}^2(cx)}{6}-\frac{b^2x^5\operatorname{atan}(cx)}{15c}+\frac{b^2x^4}{60c^2}+\frac{b^2x^3\operatorname{atan}(cx)}{9c^3}-\frac{4b^2x^2}{45c^4}-\frac{b^2x\operatorname{atan}}{3c^5} \\ \frac{a^2x^6}{6}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x)/3 - a*b*x**5/(15*c) + a*b*x**3/
(9*c**3) - a*b*x/(3*c**5) + a*b*atan(c*x)/(3*c**6) + b**2*x**6*atan(c*x)**2
/6 - b**2*x**5*atan(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atan(c*x)
/(9*c**3) - 4*b**2*x**2/(45*c**4) - b**2*x*atan(c*x)/(3*c**5) + 23*b**2*log
(x**2 + c**(-2))/(90*c**6) + b**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a**2*x
**6/6, True))
```

Giac [A] time = 1.20348, size = 243, normalized size = 1.69

$$\frac{30\,b^2c^6x^6\arctan\left(cx\right)^2+60\,abc^6x^6\arctan\left(cx\right)+30\,a^2c^6x^6-12\,b^2c^5x^5\arctan\left(cx\right)-12\,abc^5x^5+3\,b^2c^4x^4+20\,b^2c^3x^3\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/180*(30*b^2*c^6*x^6*arctan(c*x)^2 + 60*a*b*c^6*x^6*arctan(c*x) + 30*a^2*c
^6*x^6 - 12*b^2*c^5*x^5*arctan(c*x) - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20*b
^2*c^3*x^3*arctan(c*x) + 20*a*b*c^3*x^3 - 16*b^2*c^2*x^2 - 60*b^2*c*x*arcta
n(c*x) - 60*pi*a*b*sgn(c)*sgn(x) - 60*a*b*c*x + 30*b^2*arctan(c*x)^2 + 60*a
*b*arctan(c*x) + 46*b^2*log(c^2*x^2 + 1))/c^6
```

3.14 $\int x^4 \left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=170

$$\frac{ib^2\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{5c^5}+\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{5c^5}+\frac{1}{5}x^5\left(a\right.$$

```
[Out] (-3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) + (3*b^2*ArcTan[c*x])/(10*c^5) + (
b*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*x^4*(a + b*ArcTan[c*x]))/(10*c) + (
(I/5)*(a + b*ArcTan[c*x])^2)/c^5 + (x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + ((I/5)*b^2*PolyLog[2, 1 - 2/
(1 + I*c*x)])/c^5
```

Rubi [A] time = 0.288737, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {4852, 4916, 302, 203, 321, 4920, 4854, 2402, 2315}

$$\frac{ib^2\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{5c^5}+\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{5c^5}+\frac{1}{5}x^5\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] (-3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) + (3*b^2*ArcTan[c*x])/(10*c^5) + (
b*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*x^4*(a + b*ArcTan[c*x]))/(10*c) + (
(I/5)*(a + b*ArcTan[c*x])^2)/c^5 + (x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + ((I/5)*b^2*PolyLog[2, 1 - 2/
(1 + I*c*x)])/c^5
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{5}(2bc)\int\frac{x^5\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx$$

$$=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{(2b)\int x^3\left(a+b\tan^{-1}(cx)\right)dx}{5c}+\frac{(2b)\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx}{5c}$$

$$=-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{10}b^2\int\frac{x^4}{1+c^2x^2}\,dx+\frac{(2b)\int x\left(a+b\,\mathrm{t}\right.}{5c^3}$$

$$=\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{5c^5}+\frac{1}{5}x^5\left(a+b\tan^{-1}(c\right.$$

$$=-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{5c^5}+\frac{1}{5}$$

$$=-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\tan^{-1}(cx)}{10c^5}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\frac{i\left(a+b\right.}{}$$

$$=-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\tan^{-1}(cx)}{10c^5}+\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{5c^3}-\frac{bx^4\left(a+b\tan^{-1}(cx)\right)}{10c}+\frac{i\left(a+b\right.}{}$$

Mathematica [A] time = 0.484662, size = 169, normalized size = 0.99

$$\frac{-6ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+6a^2c^5x^5-3abc^4x^4+6abc^2x^2-6ab\log\left(c^2x^2+1\right)-3b\tan^{-1}(cx)\left(-4ac^5x^5+b\left(c^4x\right.\right.}{30c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (9*a*b - 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 - 3*a*b*c^4*x^4 + 6*a^2*c^
5*x^5 + 6*b^2*(-I + c^5*x^5)*ArcTan[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^5*x^5
+ b*(-3 - 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 6*a*
b*Log[1 + c^2*x^2] - (6*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

Maple [B] time = 0.044, size = 334, normalized size = 2.

$$\frac{x^5a^2}{5}+\frac{x^5b^2\left(\arctan\left(cx\right)\right)^2}{5}-\frac{b^2\arctan\left(cx\right)x^4}{10\,c}+\frac{b^2\arctan\left(cx\right)x^2}{5\,c^3}-\frac{b^2\arctan\left(cx\right)\ln\left(c^2x^2+1\right)}{5\,c^5}+\frac{b^2x^3}{30\,c^2}-\frac{3\,b^2x}{10\,c^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/5*x^5*a^2+1/5*x^5*b^2*arctan(c*x)^2-1/10/c*b^2*arctan(c*x)*x^4+1/5/c^3*b^
2*arctan(c*x)*x^2-1/5/c^5*b^2*arctan(c*x)*ln(c^2*x^2+1)+1/30*b^2*x^3/c^2-3/
10*b^2*x/c^4+3/10*b^2*arctan(c*x)/c^5+1/10*I/c^5*b^2*dilog(-1/2*I*(c*x+I))-
1/10*I/c^5*b^2*ln(c^2*x^2+1)*ln(c*x-I)+1/20*I/c^5*b^2*ln(c*x-I)^2+1/10*I/c^
5*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/10*I/c^5*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I
))-1/10*I/c^5*b^2*dilog(1/2*I*(c*x-I))+1/10*I/c^5*b^2*ln(c^2*x^2+1)*ln(c*x+
I)-1/20*I/c^5*b^2*ln(c*x+I)^2+2/5*x^5*a*b*arctan(c*x)-1/10/c*x^4*a*b+1/5*a*
b*x^2/c^3-1/5/c^5*a*b*ln(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{5}\,a^2x^5+\frac{1}{10}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)ab+\frac{1}{80}\left(4\,x^5\arctan\left(cx\right)^2-x^5\log\left(c^2x^2+1\right)^2+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^
2*x^2 + 1)/c^6))*a*b + 1/80*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2 +
 80*integrate(1/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 60*(
c^2*x^6 + x^4)*arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c^2*x
^2 + 1), x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^4\arctan\left(cx\right)^2+2\,abx^4\arctan\left(cx\right)+a^2x^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctan(c*x)^2 + 2*a*b*x^4*arctan(c*x) + a^2*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{atan}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**4*(a + b*atan(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(cx) + a)^2 x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^4, x)
```

3.15 $\int x^3 \left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=112

$$\frac{abx}{2c^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{b^2x^2}{12c^2}-\frac{b^2\log\left(c^2x^2+1\right)}{3c^4}+\frac{b^2x\tan^{-1}(c}{2c^3}$$

```
[Out] (a*b*x)/(2*c^3) + (b^2*x^2)/(12*c^2) + (b^2*x*ArcTan[c*x])/(2*c^3) - (b*x^3
*(a + b*ArcTan[c*x]))/(6*c) - (a + b*ArcTan[c*x])^2/(4*c^4) + (x^4*(a + b*A
rcTan[c*x])^2)/4 - (b^2*Log[1 + c^2*x^2])/(3*c^4)
```

Rubi [A] time = 0.208729, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{abx}{2c^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{b^2x^2}{12c^2}-\frac{b^2\log\left(c^2x^2+1\right)}{3c^4}+\frac{b^2x\tan^{-1}(c}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*b*x)/(2*c^3) + (b^2*x^2)/(12*c^2) + (b^2*x*ArcTan[c*x])/(2*c^3) - (b*x^3
*(a + b*ArcTan[c*x]))/(6*c) - (a + b*ArcTan[c*x])^2/(4*c^4) + (x^4*(a + b*A
rcTan[c*x])^2)/4 - (b^2*Log[1 + c^2*x^2])/(3*c^4)
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}(cx)\right)^2 dx = \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}(bc)\int\frac{x^4\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx$$
$$=\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{b\int x^2\left(a+b\tan^{-1}(cx)\right)dx}{2c}+\frac{b\int\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx}{2c}$$
$$=-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}b^2\int\frac{x^3}{1+c^2x^2}dx+\frac{b\int\left(a+b\tan^{-1}(\right.}{2c^3}$$
$$=\frac{abx}{2c^3}-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{12}b^2\,\text{Subst}$$
$$=\frac{abx}{2c^3}+\frac{b^2x\tan^{-1}(cx)}{2c^3}-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right.$$
$$=\frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\tan^{-1}(cx)}{2c^3}-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\,\text{ta}\right.$$

Mathematica [A] time = 0.0794119, size = 111, normalized size = 0.99

$$\frac{cx\left(3a^2c^3x^3-2abc^2x^2+6ab+b^2cx\right)-2b\tan^{-1}(cx)\left(a\left(3-3c^4x^4\right)+bcx\left(c^2x^2-3\right)\right)-4b^2\log\left(c^2x^2+1\right)+3b^2\left(c^4x^4-1\right)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(6*a*b + b^2*c*x - 2*a*b*c^2*x^2 + 3*a^2*c^3*x^3) - 2*b*(b*c*x*(-3 + c
^2*x^2) + a*(3 - 3*c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]
^2 - 4*b^2*Log[1 + c^2*x^2])/(12*c^4)
```

Maple [A] time = 0.016, size = 135, normalized size = 1.2

$$\frac{a^2x^4}{4}+\frac{b^2x^4(\arctan(cx))^2}{4}-\frac{b^2\arctan(cx)\,x^3}{6c}+\frac{b^2x\arctan(cx)}{2c^3}-\frac{b^2(\arctan(cx))^2}{4c^4}+\frac{b^2x^2}{12c^2}-\frac{b^2\ln\left(c^2x^2+1\right)}{3c^4}+\frac{x^4ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctan(c*x)^2-1/6/c*b^2*arctan(c*x)*x^3+1/2*b^2*x*a
rctan(c*x)/c^3-1/4/c^4*b^2*arctan(c*x)^2+1/12*b^2*x^2/c^2-1/3*b^2*ln(c^2*x^
2+1)/c^4+1/2*x^4*a*b*arctan(c*x)-1/6*a*b*x^3/c+1/2*a*b*x/c^3-1/2/c^4*a*b*ar
ctan(c*x)
```

Maxima [A] time = 1.51971, size = 184, normalized size = 1.64

$$\frac{1}{4}b^2x^4\arctan(cx)^2+\frac{1}{4}a^2x^4+\frac{1}{6}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)ab-\frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\ \mathrm{arc}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 +
 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^
2 + 1))/c^4)*b^2
```

Fricas [A] time = 2.40703, size = 266, normalized size = 2.38

$$\frac{3a^2c^4x^4-2abc^3x^3+b^2c^2x^2+6abcx+3\left(b^2c^4x^4-b^2\right)\arctan(cx)^2-4b^2\log\left(c^2x^2+1\right)+2\left(3abc^4x^4-b^2c^3x^3+3b\right.}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*c^4*x^4 - 2*a*b*c^3*x^3 + b^2*c^2*x^2 + 6*a*b*c*x + 3*(b^2*c^4*
x^4 - b^2)*arctan(c*x)^2 - 4*b^2*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 - b^2*
c^3*x^3 + 3*b^2*c*x - 3*a*b)*arctan(c*x))/c^4
```

Sympy [A] time = 2.08856, size = 155, normalized size = 1.38

$$\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\operatorname{atan}(cx)}{2}-\frac{abx^3}{6c}+\frac{abx}{2c^3}-\frac{ab\operatorname{atan}(cx)}{2c^4}+\frac{b^2x^4\operatorname{atan}^2(cx)}{4}-\frac{b^2x^3\operatorname{atan}(cx)}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2x\operatorname{atan}(cx)}{2c^3}-\frac{b^2\log\left(x^2+\frac{1}{c^2}\right)}{3c^4}-\frac{b^2\operatorname{atan}^2}{4c^4}\\\frac{a^2x^4}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x)/2 - a*b*x**3/(6*c) + a*b*x/(2*c
**3) - a*b*atan(c*x)/(2*c**4) + b**2*x**4*atan(c*x)**2/4 - b**2*x**3*atan(c
*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atan(c*x)/(2*c**3) - b**2*log(x**2
 + c**(-2))/(3*c**4) - b**2*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*x**4/4,
 True))
```

Giac [A] time = 1.18741, size = 181, normalized size = 1.62

$$\frac{3\,b^{2}c^{4}x^{4}\arctan\left(cx\right)^{2}+6\,abc^{4}x^{4}\arctan\left(cx\right)+3\,a^{2}c^{4}x^{4}-2\,b^{2}c^{3}x^{3}\arctan\left(cx\right)-2\,abc^{3}x^{3}+b^{2}c^{2}x^{2}+6\,b^{2}cx\arctan\left(cx\right)+}{12\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*arctan(c*x)^2 + 6*a*b*c^4*x^4*arctan(c*x) + 3*a^2*c^4*x
^4 - 2*b^2*c^3*x^3*arctan(c*x) - 2*a*b*c^3*x^3 + b^2*c^2*x^2 + 6*b^2*c*x*ar
ctan(c*x) + 6*a*b*c*x - 3*b^2*arctan(c*x)^2 - 6*a*b*arctan(c*x) - 4*b^2*log
(c^2*x^2 + 1))/c^4
```

3.16 $\int x^2 \left(a+b\tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=138

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^2(}{}$$

```
[Out] (b^2*x)/(3*c^2) - (b^2*ArcTan[c*x])/(3*c^3) - (b*x^2*(a + b*ArcTan[c*x]))/(
3*c) - ((I/3)*(a + b*ArcTan[c*x])^2)/c^3 + (x^3*(a + b*ArcTan[c*x])^2)/3 -
(2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) - ((I/3)*b^2*PolyLog[2
, 1 - 2/(1 + I*c*x)])/c^3
```

Rubi [A] time = 0.193661, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4852, 4916, 321, 203, 4920, 4854, 2402, 2315}

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{bx^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) - (b^2*ArcTan[c*x])/(3*c^3) - (b*x^2*(a + b*ArcTan[c*x]))/(
3*c) - ((I/3)*(a + b*ArcTan[c*x])^2)/c^3 + (x^3*(a + b*ArcTan[c*x])^2)/3 -
(2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) - ((I/3)*b^2*PolyLog[2
, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(2bc)\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{(2b)\int x\left(a+b\tan^{-1}(cx)\right)\,dx}{3c}+\frac{(2b)\int\frac{x\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{3c}\\
&= -\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}b^2\int\frac{x^2}{1+c^2x^2}\\
&= \frac{b^2x}{3c^2}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{2b\left(a+b\,\mathrm{ta}\right.}{}\\
&= \frac{b^2x}{3c^2}-\frac{b^2\tan^{-1}(cx)}{3c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right.\\
&= \frac{b^2x}{3c^2}-\frac{b^2\tan^{-1}(cx)}{3c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right.
\end{aligned}$$

Mathematica [A] time = 0.280131, size = 131, normalized size = 0.95

$$\frac{ib^2\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+a^2c^3x^3-abc^2x^2+ab\log\left(c^2x^2+1\right)-b\tan^{-1}(cx)\left(-2ac^3x^3+bc^2x^2+2b\log\left(1+e^{2i\tan^{-1}(\right.\right.}{3c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*c*x - a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(I + c^3*x^3)*ArcTan[c*x]^2 - b*
ArcTan[c*x]*(b + b*c^2*x^2 - 2*a*c^3*x^3 + 2*b*Log[1 + E^((2*I)*ArcTan[c*x]
)]) + a*b*Log[1 + c^2*x^2] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c
^3)
```

Maple [B] time = 0.014, size = 298, normalized size = 2.2

$$\frac{x^3a^2}{3}+\frac{b^2x^3\left(\arctan\left(cx\right)\right)^2}{3}-\frac{b^2\arctan\left(cx\right)x^2}{3c}+\frac{b^2\arctan\left(cx\right)\ln\left(c^2x^2+1\right)}{3c^3}+\frac{b^2x}{3c^2}-\frac{b^2\arctan\left(cx\right)}{3c^3}+\frac{\frac{i}{6}b^2\ln\left(cx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctan(c*x)^2-1/3/c*b^2*arctan(c*x)*x^2+1/3/c^3*b^2
*arctan(c*x)*ln(c^2*x^2+1)+1/3*b^2*x/c^2-1/3*b^2*arctan(c*x)/c^3+1/6*I/c^3*
b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/6*I/c^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+
1/6*I/c^3*b^2*ln(c^2*x^2+1)*ln(c*x-I)-1/6*I/c^3*b^2*ln(c^2*x^2+1)*ln(c*x+I)
+1/12*I/c^3*b^2*ln(c*x+I)^2+1/6*I/c^3*b^2*dilog(1/2*I*(c*x-I))-1/6*I/c^3*b^
2*dilog(-1/2*I*(c*x+I))-1/12*I/c^3*b^2*ln(c*x-I)^2+2/3*a*b*x^3*arctan(c*x)-
1/3*a*b*x^2/c+1/3/c^3*a*b*ln(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2x^3+\frac{1}{3}\left(2x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)ab+\frac{1}{48}\left(4x^3\arctan\left(cx\right)^2-x^3\log\left(c^2x^2+1\right)^2+48\int\frac{4c^2x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a*b + 1/48*(4*x^3*arctan(c*x)^2 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1/4
8*(4*c^2*x^4*log(c^2*x^2 + 1) - 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)*ar
ctan(c*x)^2 + 3*(c^2*x^4 + x^2)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^2\arctan\left(cx\right)^2+2\,abx^2\arctan\left(cx\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\,\mathrm{atan}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan (cx) + a)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*x^2, x)
```

3.17 $\int x \left(a+b \tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=76

$$\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{abx}{c}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2}-\frac{b^2x\tan^{-1}(cx)}{c}$$

```
[Out] -((a*b*x)/c) - (b^2*x*ArcTan[c*x])/c + (a + b*ArcTan[c*x])^2/(2*c^2) + (x^2
*(a + b*ArcTan[c*x])^2)/2 + (b^2*Log[1 + c^2*x^2])/(2*c^2)
```

Rubi [A] time = 0.106231, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4852, 4916, 4846, 260, 4884}

$$\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{abx}{c}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2}-\frac{b^2x\tan^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])^2,x]
```


```
[Out] -((a*b*x)/c) - (b^2*x*ArcTan[c*x])/c + (a + b*ArcTan[c*x])^2/(2*c^2) + (x^2
*(a + b*ArcTan[c*x])^2)/2 + (b^2*Log[1 + c^2*x^2])/(2*c^2)
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}(cx)\right)^2 dx = \frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-(bc)\int\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx$$
$$=\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{b\int\left(a+b\tan^{-1}(cx)\right)dx}{c}+\frac{b\int\frac{a+b\tan^{-1}(cx)}{1+c^2x^2}dx}{c}$$
$$=-\frac{abx}{c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{b^2\int\tan^{-1}(cx)\,dx}{c}$$
$$=-\frac{abx}{c}-\frac{b^2x\tan^{-1}(cx)}{c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2+b^2\int\frac{x}{1+c^2x^2}dx$$
$$=-\frac{abx}{c}-\frac{b^2x\tan^{-1}(cx)}{c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{b^2\log\left(1+c^2x^2\right)}{2c^2}$$

Mathematica [A] time = 0.0647758, size = 75, normalized size = 0.99

$$\frac{2b\tan^{-1}(cx)\left(ac^2x^2+a-bcx\right)+acx(acx-2b)+b^2\log\left(c^2x^2+1\right)+b^2\left(c^2x^2+1\right)\tan^{-1}(cx)^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*c*x*(-2*b + a*c*x) + 2*b*(a - b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
c^2*x^2)*ArcTan[c*x]^2 + b^2*Log[1 + c^2*x^2])/(2*c^2)
```

Maple [A] time = 0.012, size = 97, normalized size = 1.3

$$\frac{a^2x^2}{2}+\frac{x^2b^2\left(\arctan(cx)\right)^2}{2}+\frac{b^2\left(\arctan(cx)\right)^2}{2c^2}-\frac{b^2x\arctan(cx)}{c}+\frac{b^2\ln\left(c^2x^2+1\right)}{2c^2}+bx^2a\arctan(cx)+\frac{ab\arctan(cx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*x^2*b^2*arctan(c*x)^2+1/2/c^2*b^2*arctan(c*x)^2-b^2*x*arcta
n(c*x)/c+1/2*b^2*ln(c^2*x^2+1)/c^2+b*x^2*a*arctan(c*x)+1/c^2*a*b*arctan(c*x
)-a*b*x/c
```

Maxima [A] time = 1.53105, size = 140, normalized size = 1.84

$$\frac{1}{2}b^2x^2\arctan(cx)^2+\frac{1}{2}a^2x^2+\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)ab-\frac{1}{2}\left(2c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\arctan(cx)+\frac{\text{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctan(c*x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arc
tan(c*x)/c^3))*a*b - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arct
```

```
an(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2
```

Fricas [A] time = 2.44117, size = 189, normalized size = 2.49

$$\frac{a^2c^2x^2 - 2\,abcx + \left(b^2c^2x^2 + b^2\right)\arctan\left(cx\right)^2 + b^2\log\left(c^2x^2 + 1\right) + 2\left(abc^2x^2 - b^2cx + ab\right)\arctan\left(cx\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*c^2*x^2 - 2*a*b*c*x + (b^2*c^2*x^2 + b^2)*arctan(c*x)^2 + b^2*log(
c^2*x^2 + 1) + 2*(a*b*c^2*x^2 - b^2*c*x + a*b)*arctan(c*x))/c^2
```

Sympy [A] time = 0.934614, size = 107, normalized size = 1.41

$$\begin{cases} \frac{a^2x^2}{2} + abx^2\operatorname{atan}\left(cx\right) - \frac{abx}{c} + \frac{ab\operatorname{atan}\left(cx\right)}{c^2} + \frac{b^2x^2\operatorname{atan}^2\left(cx\right)}{2} - \frac{b^2x\operatorname{atan}\left(cx\right)}{c} + \frac{b^2\log\left(x^2 + \frac{1}{c^2}\right)}{2c^2} + \frac{b^2\operatorname{atan}^2\left(cx\right)}{2c^2} & \text{for}\ c \neq 0 \\ \frac{a^2x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*atan(c*x) - a*b*x/c + a*b*atan(c*x)/c**2
+ b**2*x**2*atan(c*x)**2/2 - b**2*x*atan(c*x)/c + b**2*log(x**2 + c**(-2))/
(2*c**2) + b**2*atan(c*x)**2/(2*c**2), Ne(c, 0)), (a**2*x**2/2, True))
```

Giac [A] time = 1.19749, size = 140, normalized size = 1.84

$$\frac{b^2c^2x^2\arctan\left(cx\right)^2 + 2\,abc^2x^2\arctan\left(cx\right) + a^2c^2x^2 - 2\,b^2cx\arctan\left(cx\right) - 2\,\pi ab\mathrm{sgn}\left(c\right)\mathrm{sgn}\left(x\right) - 2\,abcx + b^2\arctan}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2*x^2*arctan(c*x)^2 + 2*a*b*c^2*x^2*arctan(c*x) + a^2*c^2*x^2 -
2*b^2*c*x*arctan(c*x) - 2*pi*a*b*sgn(c)*sgn(x) - 2*a*b*c*x + b^2*arctan(c*x
)^2 + 2*a*b*arctan(c*x) + b^2*log(c^2*x^2 + 1))/c^2
```

3.18 $\int \left(a+b\tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=83

$$\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c}+\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}$$

```
[Out] (I*(a + b*ArcTan[c*x])^2)/c + x*(a + b*ArcTan[c*x])^2 + (2*b*(a + b*ArcTan[
c*x])*Log[2/(1 + I*c*x)])/c + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
```

Rubi [A] time = 0.0978475, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4846, 4920, 4854, 2402, 2315}

$$\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c}+\frac{2b\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^2)/c + x*(a + b*ArcTan[c*x])^2 + (2*b*(a + b*ArcTan[
c*x])*Log[2/(1 + I*c*x)])/c + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^{-1}(cx)\right)^2\,dx &= x\left(a+b\tan^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+(2b)\int \frac{a+b\tan^{-1}(cx)}{i-cx}\,dx\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+\frac{2b\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c}-\left(2b^2\right)\int -\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+\frac{2b\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c}+\frac{\left(2ib^2\right)\text{Su}}{}\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^2}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+\frac{2b\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{c}+\frac{ib^2\text{Li}_2\left(1\right.}{c}\end{aligned}$$

Mathematica [A] time = 0.0755589, size = 90, normalized size = 1.08

$$\frac{-ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+a\left(acx-b\log\left(c^2x^2+1\right)\right)+2b\tan^{-1}(cx)\left(acx+b\log\left(1+e^{2i\tan^{-1}(cx)}\right)\right)+b^2(cx-i)\,\text{t}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*(-I + c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(a*c*x + b*Log[1 + E^((2*I)
*ArcTan[c*x])]) + a*(a*c*x - b*Log[1 + c^2*x^2]) - I*b^2*PolyLog[2, -E^((2*
I)*ArcTan[c*x])])/c
```

Maple [A] time = 0.071, size = 128, normalized size = 1.5

$$xb^2\left(\arctan(cx)\right)^2-\frac{i\left(\arctan(cx)\right)^2b^2}{c}+2\,xab\arctan(cx)+2\,\frac{\arctan(cx)\,b^2}{c}\ln\left(\frac{(1+icx)^2}{c^2x^2+1}+1\right)-\frac{ib^2}{c}\text{polylog}\left(2,-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2,x)
```

```
[Out] x*b^2*arctan(c*x)^2-I/c*arctan(c*x)^2*b^2+2*x*a*b*arctan(c*x)+2/c*arctan(c*
x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*b^2-I/c*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)
)*b^2+a^2*x-1/c*a*b*ln(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\left(4\,x\arctan(cx)^2+192\,c^2\int \frac{x^2\arctan(cx)^2}{16\left(c^2x^2+1\right)}\,dx+16\,c^2\int \frac{x^2\log\left(c^2x^2+1\right)^2}{16\left(c^2x^2+1\right)}\,dx+64\,c^2\int \frac{x^2\log\left(c^2x^2+1\right)}{16\left(c^2x^2+1\right)}\,dx-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*x*arctan(c*x)^2 + 192*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2
 + 1), x) + 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
+ 64*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - x*log(c^2*
x^2 + 1)^2 + 4*arctan(c*x)^3/c - 128*c*integrate(1/16*x*arctan(c*x)/(c^2*x^
2 + 1), x) + 16*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x))*b^2 +
a^2*x + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2\arctan\left(cx\right)^2+2\,ab\arctan\left(cx\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{atan}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(cx\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2, x)
```

3.19 $\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=132

$$-ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,1\right.$$

```
[Out] 2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2
/(1 + I*c*x)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*PolyLog[3, -1
+ 2/(1 + I*c*x)])/2
```

Rubi [A] time = 0.249742, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4850, 4988, 4884, 4994, 6610}

$$-ib\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2
/(1 + I*c*x)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*PolyLog[3, -1
+ 2/(1 + I*c*x)])/2
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
```

```
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx = 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-(4bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)+(2bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx-(2bc)$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-ib\left(a+b\tan^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+icx}\right)+ib\left(a+b\right.$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-ib\left(a+b\tan^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+icx}\right)+ib\left(a+b\right.$$

Mathematica [A] time = 0.0717043, size = 144, normalized size = 1.09

$$\frac{1}{2}b\left(2i\text{PolyLog}\left(2,\frac{cx+i}{-cx+i}\right)\left(a+b\tan^{-1}(cx)\right)-2i\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right)\left(a+b\tan^{-1}(cx)\right)+b\left(\text{PolyLog}\left(3,\frac{cx+i}{-cx+i}\right)-\text{P}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^2*ArcTanh[(I + c*x)/(-I + c*x)] + (b*((2*I)*(a + b*Ar
cTan[c*x])*PolyLog[2, (I + c*x)/(I - c*x)] - (2*I)*(a + b*ArcTan[c*x])*Poly
Log[2, (I + c*x)/(-I + c*x)] + b*(PolyLog[3, (I + c*x)/(I - c*x)] - PolyLog
[3, (I + c*x)/(-I + c*x)])))/2
```

Maple [C] time = 0.394, size = 1128, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x,x)
```

```
[Out] -1/2*I*b^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2*Pi*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+a^2*ln(c*x)+b^2*ln(c*x)*arct
an(c*x)^2-b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^2*arctan(c*x)^2
*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^
2+1)^(1/2))-1/2*I*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(
```

```
(1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+I*a*b*ln(c*x)*ln(1+I*c*x)-I*a*b
*ln(c*x)*ln(1-I*c*x)+1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+2*a*b*ln(c*x)*arctan(c*x)+I*b^2*arct
an(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*I*b^2*arctan(c*x)*polylog(2,(
1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x
^2+1)^(1/2))+1/2*I*b^2*Pi*arctan(c*x)^2+I*a*b*dilog(1+I*c*x)-I*a*b*dilog(1-
I*c*x)-1/2*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*b^2*polylog(3,(1+I*c*x
)/(c^2*x^2+1)^(1/2))+2*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^
2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((
(1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I
*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csg
n(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \frac{1}{16} \int \frac{12\, b^2 \arctan(cx)^2 + b^2 \log\left(c^2x^2+1\right)^2 + 32\, ab \arctan(cx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^2 + 1)^2
+ 32*a*b*arctan(c*x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan(cx)^2 + 2\, ab \arctan(cx) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atan}(cx))^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/x, x)
```

3.20 $\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=82

$$-ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)-ic\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}+2bc\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)$$

```
[Out] (-I)*c*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/x + 2*b*c*(a + b*ArcTa
n[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rubi [A] time = 0.146934, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4852, 4924, 4868, 2447}

$$-ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)-ic\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}+2bc\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^2,x]
```


```
[Out] (-I)*c*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/x + 2*b*c*(a + b*ArcTa
n[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)]
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{x} + (2bc)\int \frac{a+b\tan^{-1}(cx)}{x\left(1+c^2x^2\right)}\,dx$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^2 - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x} + (2ibc)\int \frac{a+b\tan^{-1}(cx)}{x(i+cx)}\,dx$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^2 - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1-icx}\right) - \left(2b^2c\right.$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^2 - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tan^{-1}(cx)\right)\log\left(2-\frac{2}{1-icx}\right) - ib^2c\mathrm{L}$$

Mathematica [A] time = 0.14126, size = 102, normalized size = 1.24

$$\frac{-ib^2cx\mathrm{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right) - a\left(a+bcx\log\left(c^2x^2+1\right)-2bcx\log(cx)\right) + 2b\tan^{-1}(cx)\left(-a+bcx\log\left(1-e^{2i\tan^{-1}(cx)}\right)\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^2,x]
```

```
[Out] (b^2*(-1 - I*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-a + b*c*x*Log[1 - E^((2
*I)*ArcTan[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 + c^2*x^2]) - I*
b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x
```

Maple [B] time = 0.013, size = 323, normalized size = 3.9

$$-\frac{a^2}{x} - \frac{b^2\left(\arctan(cx)\right)^2}{x} - cb^2\arctan(cx)\ln\left(c^2x^2+1\right) + 2\,cb^2\ln(cx)\arctan(cx) + \frac{i}{2}cb^2\mathrm{dilog}\left(-\frac{i}{2}(cx+i)\right) + icb^2\mathrm{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2,x)
```

```
[Out] -a^2/x-b^2/x*arctan(c*x)^2-c*b^2*arctan(c*x)*ln(c^2*x^2+1)+2*c*b^2*ln(c*x)*
arctan(c*x)+1/2*I*c*b^2*dilog(-1/2*I*(c*x+I))+I*c*b^2*dilog(1+I*c*x)+1/2*I*
c*b^2*ln(c^2*x^2+1)*ln(c*x+I)-I*c*b^2*ln(c*x)*ln(1-I*c*x)-1/2*I*c*b^2*ln(c^
2*x^2+1)*ln(c*x-I)-1/2*I*c*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/4*I*c*b^2*ln(c
*x+I)^2-1/2*I*c*b^2*dilog(1/2*I*(c*x-I))+I*c*b^2*ln(c*x)*ln(1+I*c*x)+1/2*I*
c*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-I*c*b^2*dilog(1-I*c*x)+1/4*I*c*b^2*ln(c*
x-I)^2-2*a*b/x*arctan(c*x)-c*a*b*ln(c^2*x^2+1)+2*c*a*b*ln(c*x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan(cx)^2+2\,ab\arctan(cx)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\text{atan}(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/x^2, x)
```

3.21 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=79

$$-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}b^2c^2\log\left(c^2x^2+1\right)+b^2c^2\log(x)$$

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/x) - (c^2*(a + b*ArcTan[c*x])^2)/2 - (a + b*Arc
Tan[c*x])^2/(2*x^2) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^2])/2
```

Rubi [A] time = 0.127518, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4852, 4918, 266, 36, 29, 31, 4884}

$$-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}b^2c^2\log\left(c^2x^2+1\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^3,x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/x) - (c^2*(a + b*ArcTan[c*x])^2)/2 - (a + b*Arc
Tan[c*x])^2/(2*x^2) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^2])/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx &= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2\left(1+c^2x^2\right)}\,dx \\
&= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx-\left(bc^3\right)\int \frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx \\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\left(b^2c^2\right)\int \frac{1}{x\left(1+c^2\right.} \\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\left(b^2c^2\right)\text{Subst}\left(\int\right. \\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\left(b^2c^2\right)\text{Subst}\left(\int\right. \\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+b^2c^2\log(x)-\frac{1}{2}b^2c
\end{aligned}$$

Mathematica [A] time = 0.0649456, size = 90, normalized size = 1.14

$$-\frac{a^2+2b\tan^{-1}(cx)\left(ac^2x^2+a+bcx\right)+2abcx-2b^2c^2x^2\log(x)+b^2c^2x^2\log\left(c^2x^2+1\right)+b^2\left(c^2x^2+1\right)\tan^{-1}(cx)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^3,x]
```

```
[Out] -(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 + c^2*
x^2)*ArcTan[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b^2*c^2*x^2*Log[1 + c^2*x^2])/(
2*x^2)
```

Maple [A] time = 0.011, size = 110, normalized size = 1.4

$$-\frac{a^2}{2\,x^2}-\frac{b^2\left(\arctan(cx)\right)^2}{2\,x^2}-\frac{c^2b^2\left(\arctan(cx)\right)^2}{2}-\frac{cb^2\arctan(cx)}{x}-\frac{b^2c^2\ln\left(c^2x^2+1\right)}{2}+c^2b^2\ln(cx)-\frac{ab\arctan(cx)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2*b^2/x^2*arctan(c*x)^2-1/2*c^2*b^2*arctan(c*x)^2-c*b^2*arct
an(c*x)/x-1/2*b^2*c^2*ln(c^2*x^2+1)+c^2*b^2*ln(c*x)-a*b/x^2*arctan(c*x)-c^2
```

```
*a*b*arctan(c*x)-a*b*c/x
```

Maxima [A] time = 1.49703, size = 132, normalized size = 1.67

$$-\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)ab+\frac{1}{2}\left(\left(\arctan\left(cx\right)^2-\log\left(c^2x^2+1\right)+2\,\log\left(x\right)\right)c^2-2\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b + 1/2*((arctan(c*x)^2 - lo
g(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x))*b^2
 - 1/2*b^2*arctan(c*x)^2/x^2 - 1/2*a^2/x^2
```

Fricas [A] time = 2.54015, size = 221, normalized size = 2.8

$$-\frac{b^2c^2x^2\log\left(c^2x^2+1\right)-2\,b^2c^2x^2\log\left(x\right)+2\,abcx+\left(b^2c^2x^2+b^2\right)\arctan\left(cx\right)^2+a^2+2\left(abc^2x^2+b^2cx+ab\right)\arctan\left(cx\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2*x^2*log(c^2*x^2 + 1) - 2*b^2*c^2*x^2*log(x) + 2*a*b*c*x + (b^
2*c^2*x^2 + b^2)*arctan(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + b^2*c*x + a*b)*arct
an(c*x))/x^2
```

Sympy [A] time = 1.45158, size = 119, normalized size = 1.51

$$\begin{cases}-\frac{a^2}{2x^2}-abc^2\operatorname{atan}\left(cx\right)-\frac{abc}{x}-\frac{ab\operatorname{atan}\left(cx\right)}{x^2}+b^2c^2\log\left(x\right)-\frac{b^2c^2\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{b^2c^2\operatorname{atan}^2\left(cx\right)}{2}-\frac{b^2c\operatorname{atan}\left(cx\right)}{x}-\frac{b^2\operatorname{atan}^2\left(cx\right)}{2x^2} & \text{for}\ c \neq \\ -\frac{a^2}{2x^2} & \text{otherw}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*c**2*atan(c*x) - a*b*c/x - a*b*atan(c*x)/x*
*2 + b**2*c**2*log(x) - b**2*c**2*log(x**2 + c**(-2))/2 - b**2*c**2*atan(c*
x)**2/2 - b**2*c*atan(c*x)/x - b**2*atan(c*x)**2/(2*x**2), Ne(c, 0)), (-a**
2/(2*x**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(cx\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/x^3, x)
```

3.22 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=140

$$\frac{1}{3}ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{2}{3}bc^3\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{bc\left(a+b\tan^{-1}(\right.}{3x^2}$$

```
[Out] -(b^2*c^2)/(3*x) - (b^2*c^3*ArcTan[c*x])/3 - (b*c*(a + b*ArcTan[c*x]))/(3*x
^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/(3*x^3) - (2*
b*c^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*PolyLog
[2, -1 + 2/(1 - I*c*x)]
```

Rubi [A] time = 0.229991, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4852, 4918, 325, 203, 4924, 4868, 2447}

$$\frac{1}{3}ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{2}{3}bc^3\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{bc\left(a+b\tan^{-1}(\right.}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^4,x]
```

```
[Out] -(b^2*c^2)/(3*x) - (b^2*c^3*ArcTan[c*x])/3 - (b*c*(a + b*ArcTan[c*x]))/(3*x
^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/(3*x^3) - (2*
b*c^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*PolyLog
[2, -1 + 2/(1 - I*c*x)]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx &= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tan^{-1}(cx)}{x^3\left(1+c^2x^2\right)}\,dx\\
&= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx-\frac{1}{3}\left(2bc^3\right)\int \frac{a+b\tan^{-1}(cx)}{x\left(1+c^2x^2\right)}\,dx\\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(b^2c^2\right)\int \frac{}{x^2\left(1\right.}\\
&= -\frac{b^2c^2}{3x}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}-\frac{2}{3}bc^3\left(a+\right.\\
&= -\frac{b^2c^2}{3x}-\frac{1}{3}b^2c^3\tan^{-1}(cx)-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}\right.}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.364181, size = 153, normalized size = 1.09

$$-\frac{-ib^2c^3x^3\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+a^2+2abc^3x^3\log(cx)-abc^3x^3\log\left(c^2x^2+1\right)+b\tan^{-1}(cx)\left(2a+bc^3x^3+2bc^3x^3\right.}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^4,x]
```

```
[Out] -(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - I*c^3*x^3)*ArcTan[c*x]^2 + b*ArcTa
n[c*x]*(2*a + b*c*x + b*c^3*x^3 + 2*b*c^3*x^3*Log[1 - E^((2*I)*ArcTan[c*x])
]) + 2*a*b*c^3*x^3*Log[c*x] - a*b*c^3*x^3*Log[1 + c^2*x^2] - I*b^2*c^3*x^3*
PolyLog[2, E^((2*I)*ArcTan[c*x])])/(3*x^3)
```

Maple [B] time = 0.014, size = 399, normalized size = 2.9

$$-\frac{a^2}{3x^3}-\frac{b^2(\arctan(cx))^2}{3x^3}+\frac{c^3b^2\arctan(cx)\ln\left(c^2x^2+1\right)}{3}-\frac{cb^2\arctan(cx)}{3x^2}-\frac{2c^3b^2\ln(cx)\arctan(cx)}{3}-\frac{i}{6}c^3b^2\ln(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^4,x)
```

```
[Out] -1/3*a^2/x^3-1/3*b^2/x^3*arctan(c*x)^2+1/3*c^3*b^2*arctan(c*x)*ln(c^2*x^2+1
)-1/3*c*b^2*arctan(c*x)/x^2-2/3*c^3*b^2*ln(c*x)*arctan(c*x)-1/6*I*c^3*b^2*l
n(c*x-I)*ln(-1/2*I*(c*x+I))-1/12*I*c^3*b^2*ln(c*x-I)^2+1/3*I*c^3*b^2*dilog(
1-I*c*x)+1/6*I*c^3*b^2*ln(c^2*x^2+1)*ln(c*x-I)-1/3*I*c^3*b^2*ln(c*x)*ln(1+I
*c*x)+1/12*I*c^3*b^2*ln(c*x+I)^2-1/3*I*c^3*b^2*dilog(1+I*c*x)-1/6*I*c^3*b^2
*dilog(-1/2*I*(c*x+I))-1/3*b^2*c^3*arctan(c*x)-1/3*b^2*c^2/x+1/6*I*c^3*b^2*
dilog(1/2*I*(c*x-I))+1/6*I*c^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/6*I*c^3*b^
2*ln(c^2*x^2+1)*ln(c*x+I)+1/3*I*c^3*b^2*ln(c*x)*ln(1-I*c*x)-2/3*a*b/x^3*arc
tan(c*x)+1/3*c^3*a*b*ln(c^2*x^2+1)-1/3*c*a*b/x^2-2/3*c^3*a*b*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)ab+\frac{\frac{1}{4}\left(4x^3\int-\frac{12c^2x^2\log\left(c^2x^2+1\right)-56cx\arctan(cx)-108\left(c^2x^2+1\right)\arctan}{4\left(c^2x^6+x^4\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a
*b + 1/48*(48*x^3*integrate(-1/48*(4*c^2*x^2*log(c^2*x^2 + 1) - 8*c*x*arcta
n(c*x) - 36*(c^2*x^2 + 1)*arctan(c*x)^2 - 3*(c^2*x^2 + 1)*log(c^2*x^2 + 1)^
2)/(c^2*x^6 + x^4), x) - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x^3 - 1/
3*a^2/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan(cx)^2+2ab\arctan(cx)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{atan}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/x^4, x)
```

3.23 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=116

$$\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}-\frac{b^2c^2}{12x^2}+\frac{1}{3}b^2c^4\log\left(c^2x^2+1\right)$$

```
[Out] -(b^2*c^2)/(12*x^2) - (b*c*(a + b*ArcTan[c*x]))/(6*x^3) + (b*c^3*(a + b*Arc
Tan[c*x]))/(2*x) + (c^4*(a + b*ArcTan[c*x])^2)/4 - (a + b*ArcTan[c*x])^2/(4
*x^4) - (2*b^2*c^4*Log[x])/3 + (b^2*c^4*Log[1 + c^2*x^2])/3
```

Rubi [A] time = 0.218944, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4852, 4918, 266, 44, 36, 29, 31, 4884}

$$\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}-\frac{b^2c^2}{12x^2}+\frac{1}{3}b^2c^4\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^5,x]
```

```
[Out] -(b^2*c^2)/(12*x^2) - (b*c*(a + b*ArcTan[c*x]))/(6*x^3) + (b*c^3*(a + b*Arc
Tan[c*x]))/(2*x) + (c^4*(a + b*ArcTan[c*x])^2)/4 - (a + b*ArcTan[c*x])^2/(4
*x^4) - (2*b^2*c^4*Log[x])/3 + (b^2*c^4*Log[1 + c^2*x^2])/3
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tan^{-1}(cx)}{x^4\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx-\frac{1}{2}\left(bc^3\right)\int \frac{a+b\tan^{-1}(cx)}{x^2\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4x^4}+\frac{1}{6}\left(b^2c^2\right)\int \frac{1}{x^3\left(1+c^2x^2\right)}\,dx-\frac{1}{2}\left(bc^3\right)\int \frac{a}{}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx\right.}{4x^4}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx\right.}{4x^4}$$

$$= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\,\mathrm{t}\right.}{}$$

$$= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{6x^3}+\frac{bc^3\left(a+b\tan^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\,\mathrm{t}\right.}{}$$

Mathematica [A] time = 0.0875461, size = 128, normalized size = 1.1

$$\frac{-3a^2+6abc^3x^3+2b\tan^{-1}(cx)\left(3a\left(c^4x^4-1\right)+bcx\left(3c^2x^2-1\right)\right)-2abcx-b^2c^2x^2-8b^2c^4x^4\log(x)+4b^2c^4x^4\log\left(c^2x\right.}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^5,x]
```

```
[Out] (-3*a^2 - 2*a*b*c*x - b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(b*c*x*(-1 + 3*c^2*
x^2) + 3*a*(-1 + c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]^2
 - 8*b^2*c^4*x^4*Log[x] + 4*b^2*c^4*x^4*Log[1 + c^2*x^2])/(12*x^4)
```

Maple [A] time = 0.017, size = 147, normalized size = 1.3

$$-\frac{a^2}{4x^4}-\frac{b^2(\arctan(cx))^2}{4x^4}+\frac{c^4b^2(\arctan(cx))^2}{4}-\frac{cb^2\arctan(cx)}{6x^3}+\frac{b^2c^3\arctan(cx)}{2x}+\frac{b^2c^4\ln(c^2x^2+1)}{3}-\frac{b^2c^2}{12x^2}-\frac{2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-1/4*b^2/x^4*arctan(c*x)^2+1/4*c^4*b^2*arctan(c*x)^2-1/6*c*b^2*
arctan(c*x)/x^3+1/2*c^3*b^2*arctan(c*x)/x+1/3*b^2*c^4*ln(c^2*x^2+1)-1/12*b^
2*c^2/x^2-2/3*c^4*b^2*ln(c*x)-1/2*a*b/x^4*arctan(c*x)+1/2*c^4*a*b*arctan(c*
x)-1/6*a*b*c/x^3+1/2*c^3*a*b/x
```

Maxima [A] time = 1.53828, size = 205, normalized size = 1.77

$$\frac{1}{6}\left(\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)ab+\frac{1}{12}\left(2\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c\arctan(cx)-\frac{(3c^2x^2\operatorname{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b +
 1/12*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c*arctan(c*x) - (3*c^2*x
^2*arctan(c*x)^2 - 4*c^2*x^2*log(c^2*x^2 + 1) + 8*c^2*x^2*log(x) + 1)*c^2/x
^2)*b^2 - 1/4*b^2*arctan(c*x)^2/x^4 - 1/4*a^2/x^4
```

Fricas [A] time = 2.75023, size = 297, normalized size = 2.56

$$\frac{4b^2c^4x^4\log(c^2x^2+1)-8b^2c^4x^4\log(x)+6abc^3x^3-b^2c^2x^2-2abcx+3(b^2c^4x^4-b^2)\arctan(cx)^2-3a^2+2(3abc^4x^4}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^2*c^4*x^4*log(c^2*x^2 + 1) - 8*b^2*c^4*x^4*log(x) + 6*a*b*c^3*x^3
 - b^2*c^2*x^2 - 2*a*b*c*x + 3*(b^2*c^4*x^4 - b^2)*arctan(c*x)^2 - 3*a^2 +
2*(3*a*b*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x - 3*a*b)*arctan(c*x))/x^4
```

Sympy [A] time = 2.6149, size = 170, normalized size = 1.47

$$\begin{cases}-\frac{a^2}{4x^4}+\frac{abc^4\operatorname{atan}(cx)}{2}+\frac{abc^3}{2x}-\frac{abc}{6x^3}-\frac{ab\operatorname{atan}(cx)}{2x^4}-\frac{2b^2c^4\log(x)}{3}+\frac{b^2c^4\log\left(x^2+\frac{1}{c^2}\right)}{3}+\frac{b^2c^4\operatorname{atan}^2(cx)}{4}+\frac{b^2c^3\operatorname{atan}(cx)}{2x}-\frac{b^2c^2}{12x^2}-\frac{b^2c\operatorname{atan}(cx}{6x^3} \\ -\frac{a^2}{4x^4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**4*atan(c*x)/2 + a*b*c**3/(2*x) - a*b*c/(
6*x**3) - a*b*atan(c*x)/(2*x**4) - 2*b**2*c**4*log(x)/3 + b**2*c**4*log(x**
2 + c**(-2))/3 + b**2*c**4*atan(c*x)**2/4 + b**2*c**3*atan(c*x)/(2*x) - b**
2*c**2/(12*x**2) - b**2*c*atan(c*x)/(6*x**3) - b**2*atan(c*x)**2/(4*x**4),
Ne(c, 0)), (-a**2/(4*x**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/x^5, x)
```

3.24 $\int x^5 \left(a+b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=255

$$-\frac{23ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{30c^6}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}-\frac{23b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{15c^6}+$$

```
[Out] (19*b^3*x)/(60*c^5) - (b^3*x^3)/(60*c^3) - (19*b^3*ArcTan[c*x])/(60*c^6) -
(4*b^2*x^2*(a + b*ArcTan[c*x]))/(15*c^4) + (b^2*x^4*(a + b*ArcTan[c*x]))/(2
0*c^2) - (((23*I)/30)*b*(a + b*ArcTan[c*x])^2)/c^6 - (b*x*(a + b*ArcTan[c*x
])^2)/(2*c^5) + (b*x^3*(a + b*ArcTan[c*x])^2)/(6*c^3) - (b*x^5*(a + b*ArcTa
n[c*x])^2)/(10*c) + (a + b*ArcTan[c*x])^3/(6*c^6) + (x^6*(a + b*ArcTan[c*x]
)^3)/6 - (23*b^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^6) - (((23*I
)/30)*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^6
```

Rubi [A] time = 0.94766, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4852, 4916, 302, 203, 321, 4920, 4854, 2402, 2315, 4846, 4884}

$$-\frac{23ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{30c^6}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}-\frac{23b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{15c^6}+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (19*b^3*x)/(60*c^5) - (b^3*x^3)/(60*c^3) - (19*b^3*ArcTan[c*x])/(60*c^6) -
(4*b^2*x^2*(a + b*ArcTan[c*x]))/(15*c^4) + (b^2*x^4*(a + b*ArcTan[c*x]))/(2
0*c^2) - (((23*I)/30)*b*(a + b*ArcTan[c*x])^2)/c^6 - (b*x*(a + b*ArcTan[c*x
])^2)/(2*c^5) + (b*x^3*(a + b*ArcTan[c*x])^2)/(6*c^3) - (b*x^5*(a + b*ArcTa
n[c*x])^2)/(10*c) + (a + b*ArcTan[c*x])^3/(6*c^6) + (x^6*(a + b*ArcTan[c*x]
)^3)/6 - (23*b^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^6) - (((23*I
)/30)*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^6
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^5\left(a+b\tan^{-1}(cx)\right)^3dx=\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3-\frac{1}{2}(bc)\int\frac{x^6\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx$$

$$=\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3-\frac{b\int x^4\left(a+b\tan^{-1}(cx)\right)^2dx}{2c}+\frac{b\int\frac{x^4\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{2c}$$

$$=-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)^2}{10c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3+\frac{1}{5}b^2\int\frac{x^5\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx+\frac{b\int}{}$$

$$=\frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{6c^3}-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)^2}{10c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3-\frac{b\int\left(a+b\tan\right.}{2c^5}$$

$$=\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-\frac{bx\left(a+b\tan^{-1}(cx)\right)^2}{2c^5}+\frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{6c^3}-\frac{bx^5\left(a+b\tan^{-1}\right.}{10c}$$

$$=-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-\frac{23ib\left(a+b\tan^{-1}(cx)\right)^2}{30c^6}-\frac{bx\left(a+b\,\mathrm{t}\right.}{2}$$

$$=\frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-\frac{23ib\left(a+b\tan^{-1}(cx)\right)}{30c^6}$$

$$=\frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{19b^3\tan^{-1}(cx)}{60c^6}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-23il$$

$$=\frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{19b^3\tan^{-1}(cx)}{60c^6}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-23il$$

Mathematica [A] time = 0.764416, size = 291, normalized size = 1.14

$$\frac{46ib^3\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+b\tan^{-1}(cx)\left(30a^2\left(c^6x^6+1\right)-4abcx\left(3c^4x^4-5c^2x^2+15\right)+b^2\left(3c^4x^4-16c^2x^2-19\right)-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-19*a*b^2 - 30*a^2*b*c*x + 19*b^3*c*x - 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x^
3 - b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 - 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*b^
2*(b*(23*I - 15*c*x + 5*c^3*x^3 - 3*c^5*x^5) + 15*a*(1 + c^6*x^6))*ArcTan[c
*x]^2 + 10*b^3*(1 + c^6*x^6)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*(-19 - 16*c
^2*x^2 + 3*c^4*x^4) - 4*a*b*c*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 30*a^2*(1 +
c^6*x^6) - 92*b^2*Log[1 + E^((2*I)*ArcTan[c*x])]) + 46*a*b^2*Log[1 + c^2*x^
2] + (46*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(60*c^6)
```

Maple [B] time = 0.017, size = 528, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x))^3,x)
```

```
[Out] 1/20/c^2*b^3*arctan(c*x)*x^4-4/15/c^4*b^3*arctan(c*x)*x^2-1/10/c*b^3*arctan
(c*x)^2*x^5+1/6/c^3*b^3*arctan(c*x)^2*x^3+1/2*a^2*b*x^6*arctan(c*x)+1/2*a*b
```

```
^2*x^6*arctan(c*x)^2-23/60*I/c^6*b^3*dilog(-1/2*I*(c*x+I))-23/120*I/c^6*b^3
*ln(c*x-I)^2+23/120*I/c^6*b^3*ln(c*x+I)^2+23/60*I/c^6*b^3*dilog(1/2*I*(c*x-
I))+23/30/c^6*b^3*arctan(c*x)*ln(c^2*x^2+1)+23/30/c^6*a*b^2*ln(c^2*x^2+1)+1
/2/c^6*a^2*b*arctan(c*x)+1/20/c^2*a*b^2*x^4+1/6/c^3*a^2*b*x^3+1/2/c^6*a*b^2
*arctan(c*x)^2-1/2/c^5*b^3*arctan(c*x)^2*x-23/60*I/c^6*b^3*ln(c*x-I)*ln(-1/
2*I*(c*x+I))+1/3/c^3*a*b^2*x^3*arctan(c*x)-1/5/c*a*b^2*x^5*arctan(c*x)+23/6
0*I/c^6*b^3*ln(c*x+I)*ln(1/2*I*(c*x-I))+23/60*I/c^6*b^3*ln(c^2*x^2+1)*ln(c*
x-I)-23/60*I/c^6*b^3*ln(c^2*x^2+1)*ln(c*x+I)-4/15/c^4*x^2*a*b^2-1/2/c^5*x*a
^2*b-1/10/c*x^5*a^2*b-1/c^5*a*b^2*x*arctan(c*x)+19/60*b^3*x/c^5-1/60*b^3*x^
3/c^3-19/60*b^3*arctan(c*x)/c^6+1/6*b^3*x^6*arctan(c*x)^3+1/6/c^6*b^3*arcta
n(c*x)^3+1/6*x^6*a^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x^5\arctan(cx)^3 + 3\,ab^2x^5\arctan(cx)^2 + 3\,a^2bx^5\arctan(cx) + a^3x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctan(c*x)^3 + 3*a*b^2*x^5*arctan(c*x)^2 + 3*a^2*b*x^5*ar
ctan(c*x) + a^3*x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(a + b\,\mathrm{atan}(cx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**5*(a + b*atan(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx) + a)^3x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3*x^5, x)
```

3.25 $\int x^4 \left(a + b\tan^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=271

$$\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{5c^5}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{10c^5}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}-\frac{9ab^2x}{10c^4}+\frac{3bx^2(}{}$$

```
[Out] (-9*a*b^2*x)/(10*c^4) - (b^3*x^2)/(20*c^3) - (9*b^3*x*ArcTan[c*x])/(10*c^4)
 + (b^2*x^3*(a + b*ArcTan[c*x]))/(10*c^2) + (9*b*(a + b*ArcTan[c*x])^2)/(20
*c^5) + (3*b*x^2*(a + b*ArcTan[c*x])^2)/(10*c^3) - (3*b*x^4*(a + b*ArcTan[c
*x])^2)/(20*c) + ((I/5)*(a + b*ArcTan[c*x])^3)/c^5 + (x^5*(a + b*ArcTan[c*x
])^3)/5 + (3*b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(5*c^5) + (b^3*Log
[1 + c^2*x^2])/(2*c^5) + (((3*I)/5)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
2/(1 + I*c*x)])/c^5 + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(10*c^5)
```

Rubi [A] time = 0.755895, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4852, 4916, 266, 43, 4846, 260, 4884, 4920, 4854, 4994, 6610}

$$\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{5c^5}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{10c^5}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}-\frac{9ab^2x}{10c^4}+\frac{3bx^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x])^3,x]
```


```
[Out] (-9*a*b^2*x)/(10*c^4) - (b^3*x^2)/(20*c^3) - (9*b^3*x*ArcTan[c*x])/(10*c^4)
 + (b^2*x^3*(a + b*ArcTan[c*x]))/(10*c^2) + (9*b*(a + b*ArcTan[c*x])^2)/(20
*c^5) + (3*b*x^2*(a + b*ArcTan[c*x])^2)/(10*c^3) - (3*b*x^4*(a + b*ArcTan[c
*x])^2)/(20*c) + ((I/5)*(a + b*ArcTan[c*x])^3)/c^5 + (x^5*(a + b*ArcTan[c*x
])^3)/5 + (3*b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(5*c^5) + (b^3*Log
[1 + c^2*x^2])/(2*c^5) + (((3*I)/5)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
2/(1 + I*c*x)])/c^5 + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(10*c^5)
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^4\left(a+b\tan^{-1}(cx)\right)^3dx=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^3-\frac{1}{5}(3bc)\int\frac{x^5\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx$$

$$=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^3-\frac{(3b)\int x^3\left(a+b\tan^{-1}(cx)\right)^2dx}{5c}+\frac{(3b)\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{5c}$$

$$=-\frac{3bx^4\left(a+b\tan^{-1}(cx)\right)^2}{20c}+\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^3+\frac{1}{10}\left(3b^2\right)\int\frac{x^4\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}$$

$$=\frac{3bx^2\left(a+b\tan^{-1}(cx)\right)^2}{10c^3}-\frac{3bx^4\left(a+b\tan^{-1}(cx)\right)^2}{20c}+\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{5c^5}+\frac{1}{5}x^5\left(a+b\right.$$

$$=\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}+\frac{3bx^2\left(a+b\tan^{-1}(cx)\right)^2}{10c^3}-\frac{3bx^4\left(a+b\tan^{-1}(cx)\right)^2}{20c}+\frac{i\left(a+b\right.}{}$$

$$=-\frac{9ab^2x}{10c^4}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}+\frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{20c^5}+\frac{3bx^2\left(a+b\tan^{-1}(cx)\right)^2}{10c^3}-$$

$$=-\frac{9ab^2x}{10c^4}-\frac{9b^3x\tan^{-1}(cx)}{10c^4}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}+\frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{20c^5}+\frac{3bx^2\left(a\right.}{}$$

$$=-\frac{9ab^2x}{10c^4}-\frac{b^3x^2}{20c^3}-\frac{9b^3x\tan^{-1}(cx)}{10c^4}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}+\frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{20c^5}+$$

Mathematica [A] time = 0.8301, size = 396, normalized size = 1.46

$$-12ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\left(a+b\tan^{-1}(cx)\right)+6b^3\text{PolyLog}\left(3,-e^{2i\tan^{-1}(cx)}\right)-3a^2bc^4x^4+6a^2bc^2x^2-6a^2b\log$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-b^3 - 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 - b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 - 3*
a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 + 18*a*b^2*ArcTan[c*x] - 18*b^3*c*x*ArcTan[c*
x] + 12*a*b^2*c^2*x^2*ArcTan[c*x] + 2*b^3*c^3*x^3*ArcTan[c*x] - 6*a*b^2*c^4
*x^4*ArcTan[c*x] + 12*a^2*b*c^5*x^5*ArcTan[c*x] - (12*I)*a*b^2*ArcTan[c*x]^
2 + 9*b^3*ArcTan[c*x]^2 + 6*b^3*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^4*x^4*ArcTa
n[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTan[c*x]^2 - (4*I)*b^3*ArcTan[c*x]^3 + 4*b^3
*c^5*x^5*ArcTan[c*x]^3 + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])
] + 12*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*a^2*b*Log[1 + c
^2*x^2] + 10*b^3*Log[1 + c^2*x^2] - (12*I)*b^2*(a + b*ArcTan[c*x])*PolyLog[
2, -E^((2*I)*ArcTan[c*x])] + 6*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])])/(20*
c^5)
```

Maple [C] time = 3.677, size = 3053, normalized size = 11.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^3,x)
```

```
[Out] -3/80/c^2*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+
I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*x^3-3/10/c^5*a^2*b*
ln(c^2*x^2+1)+3/5/c^5*b^3*arctan(c*x)^2*ln(2)+I/c^5*b^3*arctan(c*x)+9/10/c^
5*a*b^2*arctan(c*x)+3/5*x^5*a^2*b*arctan(c*x)+3/5*x^5*a*b^2*arctan(c*x)^2-1
/5*I/c^5*b^3*arctan(c*x)^3+9/20/c^5*b^3*arctan(c*x)^2-1/c^5*b^3*ln((1+I*c*x
)^2/(c^2*x^2+1)+1)+3/10/c^5*b^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/5*x^5
*b^3*arctan(c*x)^3+9/160*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^4/(c^2
*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3*x^2-3/20*I/c^5*b^3*arctan(c*x)^2
*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2-3
/160/c^2*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x^3+3/80/c^2*b^3*arctan(c*x)^2*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^3+3/160
/c^2*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I*c
*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*x^3+9/160/c^4*b^3*arctan
(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)+1)^2)*x-9/80/c^4*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x-9/160/c^4*b^3*arctan(c*x)^2*
Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I
*(1+I*c*x)^2/(c^2*x^2+1)+I)*x+9/80/c^4*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^
2+1)+I)^2*x+1/5*x^5*a^3-9/10*a*b^2*x/c^4-9/10*b^3*x*arctan(c*x)/c^4-3/80*I/
c^5*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)+1)^2)^2+3/20*I/c^5*b^3*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x)
^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1
)+1)^2)^2+3/20*I/c^5*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((
1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))+3/10*I/c^5*b
^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))-9/160*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)+1)^2)^3*x^2-21/80*I/c^5*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x
^2+1)+I)*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2+
3/160*I/c^5*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+21/160*I/c^5*b^3*arctan(c*x)^2*Pi*csgn(I*
(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)
^2/(c^2*x^2+1)+I)+3/5/c^3*a*b^2*x^2*arctan(c*x)-3/5*I/c^5*b^3*arctan(c*x)*p
olylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/20*I/c^5*a*b^2*ln(c*x-I)^2+3/10*I/c^5*
a*b^2*dilog(-1/2*I*(c*x+I))-3/20*I/c^5*a*b^2*ln(c*x+I)^2-3/10*I/c^5*a*b^2*d
ilog(1/2*I*(c*x-I))-1/20*b^3*x^2/c^3-3/5/c^5*a*b^2*arctan(c*x)*ln(c^2*x^2+1
)-3/10/c*a*b^2*x^4*arctan(c*x)+9/80*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*x^2+9/160*I
/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I*c
*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*x^2-9/80*I/c^3*b^3*arcta
n(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^4/(c^2*x^2+1
)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*x^2-3/20*I/c^5*b^3*arctan(c*x)^2*Pi*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))-9/160*I/c^3*b^3*arct
an(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)+1)^2)*x^2+1/10*a*b^2*x^3/c^2-1/20/c^5*b^3-3/20/c*x^4*a^2*b+3/10/c^3
*b*a^2*x^2-3/10/c^5*b^3*arctan(c*x)^2*ln(c^2*x^2+1)+3/5/c^5*b^3*arctan(c*x)
^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-3/20/c*b^3*arctan(c*x)^2*x^4+3/10/c^3*b^
3*arctan(c*x)^2*x^2+1/10/c^2*b^3*arctan(c*x)*x^3+9/160/c^4*b^3*arctan(c*x)^
2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x-3/160/c^2*b^3*arctan(c*x)^2*
Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*x^3+3/160/c^2*b^3*arctan(c*x)^2*
Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3*x^3-9/
160/c^4*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)
^2/(c^2*x^2+1)+I)^3*x-3/20*I/c^5*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1))^3+21/160*I/c^5*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+
1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3-3/10*I/c^5*a*b^2*ln(c^2*x^2+1)*ln(c*x
-I)+3/10*I/c^5*a*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/10*I/c^5*a*b^2*ln(c^2*x
^2+1)*ln(c*x+I)-3/10*I/c^5*a*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+3/160*I/c^5*b^
```

```
3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-3/20*I/c^5*b^3*a
rctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^
2)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{40}b^3x^5\arctan(cx)^3 - \frac{3}{160}b^3x^5\arctan(cx)\log\left(c^2x^2+1\right)^2 + \frac{1}{5}a^3x^5 + \frac{3}{20}\left(4x^5\arctan(cx) - c\left(\frac{c^2x^4-2x^2}{c^4} + \frac{2\log}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/40*b^3*x^5*arctan(c*x)^3 - 3/160*b^3*x^5*arctan(c*x)*log(c^2*x^2 + 1)^2 +
 1/5*a^3*x^5 + 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c
^2*x^2 + 1)/c^6))*a^2*b + integrate(1/160*(12*b^3*c^2*x^6*arctan(c*x)*log(c
^2*x^2 + 1) + 140*(b^3*c^2*x^6 + b^3*x^4)*arctan(c*x)^3 + 12*(40*a*b^2*c^2*
x^6 - b^3*c*x^5 + 40*a*b^2*x^4)*arctan(c*x)^2 + 3*(b^3*c*x^5 + 5*(b^3*c^2*x
^6 + b^3*x^4)*arctan(c*x))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^4\arctan(cx)^3 + 3\,ab^2x^4\arctan(cx)^2 + 3\,a^2bx^4\arctan(cx) + a^3x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^4*arctan(c*x)^3 + 3*a*b^2*x^4*arctan(c*x)^2 + 3*a^2*b*x^4*ar
ctan(c*x) + a^3*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(a + b\,\mathrm{atan}(cx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**4*(a + b*atan(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx) + a)^3x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3*x^4, x)
```

3.26 $\int x^3 \left(a+b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=194

$$\frac{ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4}+\frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}+\frac{2b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4}+\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3}-\frac{(a+}{}$$

```
[Out] -(b^3*x)/(4*c^3) + (b^3*ArcTan[c*x])/(4*c^4) + (b^2*x^2*(a + b*ArcTan[c*x])
)/(4*c^2) + (I*b*(a + b*ArcTan[c*x])^2)/c^4 + (3*b*x*(a + b*ArcTan[c*x])^2)
/(4*c^3) - (b*x^3*(a + b*ArcTan[c*x])^2)/(4*c) - (a + b*ArcTan[c*x])^3/(4*c
^4) + (x^4*(a + b*ArcTan[c*x])^3)/4 + (2*b^2*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/c^4 + (I*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^4
```

Rubi [A] time = 0.545385, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4852, 4916, 321, 203, 4920, 4854, 2402, 2315, 4846, 4884}

$$\frac{ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4}+\frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}+\frac{2b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^4}+\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3}-\frac{(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] -(b^3*x)/(4*c^3) + (b^3*ArcTan[c*x])/(4*c^4) + (b^2*x^2*(a + b*ArcTan[c*x])
)/(4*c^2) + (I*b*(a + b*ArcTan[c*x])^2)/c^4 + (3*b*x*(a + b*ArcTan[c*x])^2)
/(4*c^3) - (b*x^3*(a + b*ArcTan[c*x])^2)/(4*c) - (a + b*ArcTan[c*x])^3/(4*c
^4) + (x^4*(a + b*ArcTan[c*x])^3)/4 + (2*b^2*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/c^4 + (I*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^4
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}(cx)\right)^3 dx = \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^3 - \frac{1}{4}(3bc)\int \frac{x^4\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx$$

$$= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^3 - \frac{(3b)\int x^2\left(a+b\tan^{-1}(cx)\right)^2 dx}{4c} + \frac{(3b)\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{4c}$$

$$= -\frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{4c} + \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^3 + \frac{1}{2}b^2\int \frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx + \frac{(3b)}{}$$

$$= \frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3} - \frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{4c} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{4c^4} + \frac{1}{4}x^4\left(a+b\tan^{-1}(\right.$$

$$= \frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3} - \frac{bx^3\left(a+b\tan^{-}\right.}{4c}$$

$$= -\frac{b^3x}{4c^3} + \frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3} - \frac{bx^3\left(a\right.}{}$$

$$= -\frac{b^3x}{4c^3} + \frac{b^3\tan^{-1}(cx)}{4c^4} + \frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tan^{-1}(\right.}{4c^3}$$

$$= -\frac{b^3x}{4c^3} + \frac{b^3\tan^{-1}(cx)}{4c^4} + \frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} + \frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tan^{-1}(\right.}{4c^3}$$

Mathematica [A] time = 0.523544, size = 225, normalized size = 1.16

$$\frac{-4ib^3\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right) + b\tan^{-1}(cx)\left(3a^2\left(c^4x^4-1\right) - 2abcx\left(c^2x^2-3\right) + b^2\left(c^2x^2+1\right) + 8b^2\log\left(1+e^{2i\tan^{-1}(cx}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2 + 3*a^2*b*c*x - b^3*c*x + a*b^2*c^2*x^2 - a^2*b*c^3*x^3 + a^3*c^4*x^
4 - b^2*(b*(4*I - 3*c*x + c^3*x^3) + a*(3 - 3*c^4*x^4))*ArcTan[c*x]^2 + b^3
*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(-2*a*b*c*x*(-3 + c^2*x^2) +
b^2*(1 + c^2*x^2) + 3*a^2*(-1 + c^4*x^4) + 8*b^2*Log[1 + E^((2*I)*ArcTan[c*
x])]) - 4*a*b^2*Log[1 + c^2*x^2] - (4*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*
x])])/(4*c^4)
```

Maple [B] time = 0.019, size = 445, normalized size = 2.3

$$\frac{ax^2b^2}{4c^2} + \frac{3a^2xb}{4c^3} - \frac{3ab^2(\arctan(cx))^2}{4c^4} + \frac{3x^4a^2b\arctan(cx)}{4} + \frac{3ab^2x^4(\arctan(cx))^2}{4} - \frac{\frac{i}{2}b^3\text{dilog}\left(\frac{i}{2}(cx-i)\right)}{c^4} + \frac{\frac{i}{2}b^3\text{dilo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^3,x)
```

```
[Out] 1/4/c^2*x^2*a*b^2+3/4/c^3*x*a^2*b-3/4/c^4*a*b^2*arctan(c*x)^2+3/4*x^4*a^2*b
*arctan(c*x)+3/4*a*b^2*x^4*arctan(c*x)^2-1/2*I/c^4*b^3*dilog(1/2*I*(c*x-I))
+1/2*I/c^4*b^3*dilog(-1/2*I*(c*x+I))+1/4*I/c^4*b^3*ln(c*x-I)^2-1/4*I/c^4*b^
3*ln(c*x+I)^2+3/4/c^3*b^3*arctan(c*x)^2*x+1/4/c^2*b^3*arctan(c*x)*x^2-1/c^4
*a*b^2*ln(c^2*x^2+1)-3/4/c^4*a^2*b*arctan(c*x)-1/c^4*b^3*arctan(c*x)*ln(c^2
```

```
*x^2+1)-1/4/c*b^3*arctan(c*x)^2*x^3-1/4/c*a^2*b*x^3-1/4*b^3*x/c^3+1/4*b^3*a
rctan(c*x)/c^4+1/4*b^3*x^4*arctan(c*x)^3-1/4/c^4*b^3*arctan(c*x)^3+1/2*I/c^
4*b^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*I/c^4*b^3*ln(c*x+I)*ln(1/2*I*(c*x-I)
)+3/2/c^3*a*b^2*x*arctan(c*x)-1/2/c*a*b^2*x^3*arctan(c*x)+1/2*I/c^4*b^3*ln(
c*x+I)*ln(c^2*x^2+1)-1/2*I/c^4*b^3*ln(c*x-I)*ln(c^2*x^2+1)+1/4*x^4*a^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^3\arctan\left(cx\right)^3 + 3\,ab^2x^3\arctan\left(cx\right)^2 + 3\,a^2bx^3\arctan\left(cx\right) + a^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c*x)^3 + 3*a*b^2*x^3*arctan(c*x)^2 + 3*a^2*b*x^3*ar
ctan(c*x) + a^3*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a + b\,\text{atan}\left(cx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(cx\right) + a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3*x^3, x)
```

3.27 $\int x^2 \left(a+b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=206

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3}-\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}+\frac{ab^2x}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{i\left(a+b\tan^{-1}\right.}{3c^3}$$

```
[Out] (a*b^2*x)/c^2 + (b^3*x*ArcTan[c*x])/c^2 - (b*(a + b*ArcTan[c*x])^2)/(2*c^3)
 - (b*x^2*(a + b*ArcTan[c*x])^2)/(2*c) - ((I/3)*(a + b*ArcTan[c*x])^3)/c^3
+ (x^3*(a + b*ArcTan[c*x])^3)/3 - (b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x
)])/c^3 - (b^3*Log[1 + c^2*x^2])/(2*c^3) - (I*b^2*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 + I*c*x)])/c^3 - (b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^3)
```

Rubi [A] time = 0.43386, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {4852, 4916, 4846, 260, 4884, 4920, 4854, 4994, 6610}

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3}-\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}+\frac{ab^2x}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{i\left(a+b\tan^{-1}\right.}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2*x)/c^2 + (b^3*x*ArcTan[c*x])/c^2 - (b*(a + b*ArcTan[c*x])^2)/(2*c^3)
 - (b*x^2*(a + b*ArcTan[c*x])^2)/(2*c) - ((I/3)*(a + b*ArcTan[c*x])^3)/c^3
+ (x^3*(a + b*ArcTan[c*x])^3)/3 - (b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x
)])/c^3 - (b^3*Log[1 + c^2*x^2])/(2*c^3) - (I*b^2*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 + I*c*x)])/c^3 - (b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^3)
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}(cx)\right)^3dx=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3-(bc)\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx$$

$$=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3-\frac{b\int x\left(a+b\tan^{-1}(cx)\right)^2dx}{c}+\frac{b\int\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{c}$$

$$=-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3+b^2\int\frac{x^2\left(a+\right.}{1}$$

$$=-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3-\frac{b\left(a+b\tan\right.}{}$$

$$=\frac{ab^2x}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+\right.$$

$$=\frac{ab^2x}{c^2}+\frac{b^3x\tan^{-1}(cx)}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}\right.}{3c^3}$$

$$=\frac{ab^2x}{c^2}+\frac{b^3x\tan^{-1}(cx)}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}\right.}{3c^3}$$

Mathematica [A] time = 0.565541, size = 269, normalized size = 1.31

$$6ab^2\left(i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+\left(c^3x^3+i\right)\tan^{-1}(cx)^2-\tan^{-1}(cx)\left(c^2x^2+2\log\left(1+e^{2i\tan^{-1}(cx)}\right)+1\right)+cx\right)+b^3\left(6i\,\text{ta}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTan[c*x] + 3*a^2*b*L
og[1 + c^2*x^2] + 6*a*b^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*
(1 + c^2*x^2 + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*A
rcTan[c*x])]) + b^3*(6*c*x*ArcTan[c*x] - 3*ArcTan[c*x]^2 - 3*c^2*x^2*ArcTan
[c*x]^2 + (2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[c*x]^3 - 6*ArcTan[c*x]^2*L
og[1 + E^((2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2] + (6*I)*ArcTan[c*x]*Poly
Log[2, -E^((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(6*
c^3)
```

Maple [C] time = 1.106, size = 2020, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^3,x)
```

```
[Out] -1/8*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*
(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)+1/4*I/c^3*b^3*arct
an(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^4/(c^2*x^2+
1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2-1/8*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/3*a
^3*x^3-1/2/c*b*a^2*x^2-1/2/c*b^3*arctan(c*x)^2*x^2+1/2/c^3*a^2*b*ln(c^2*x^2
+1)-1/c^3*b^3*arctan(c*x)^2*ln(2)-1/c^3*a*b^2*arctan(c*x)+1/2/c^3*b^3*arcta
n(c*x)^2*ln(c^2*x^2+1)-1/c^3*b^3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/
2))+a^2*b*x^3*arctan(c*x)+b^2*x^3*a*arctan(c*x)^2+1/3*I/c^3*b^3*arctan(c*x)
^3-I/c^3*b^3*arctan(c*x)+1/8/c^2*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1)+I)^2*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)
+I)*x+a*b^2*x/c^2+b^3*x*arctan(c*x)/c^2+1/4*I/c^3*b^3*arctan(c*x)^2*Pi*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+1/3*x^3*b^3*arctan(c*x
)^3-1/2/c^3*b^3*arctan(c*x)^2+1/c^3*b^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-1/2/c
^3*b^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-1/4/c^2*b^3*arctan(c*x)^2*Pi*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*
x)^2/(c^2*x^2+1)+I)^2*x-1/8/c^2*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*x+1/4/c^2*b^3*arctan(
c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)+1)^2)^2*x+1/4*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-1/2*I/c^3*b^3*arctan(c*x)^2*Pi
*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/4*
I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))^2-1/c*a*b^2*x^2*arctan(c*x)-1/4*I/c^3*a*b^2*ln(c*x-I)^2
-1/2*I/c^3*a*b^2*dilog(-1/2*I*(c*x+I))+1/4*I/c^3*a*b^2*ln(c*x+I)^2+1/2*I/c^
3*a*b^2*dilog(1/2*I*(c*x-I))-1/8*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x
)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3+I/c^3*b^3*arctan(c*x)*po
lylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/c^3*a*b^2*arctan(c*x)*ln(c^2*x^2+1)+1/8
/c^2*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/
```

```
(c^2*x^2+1)+I)^3*x-1/8/c^2*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)+1)^2)^3*x+1/2*I/c^3*a*b^2*ln(c^2*x^2+1)*ln(c*x-I)-1/2*I/c^3*a*b^2*ln(c
*x-I)*ln(-1/2*I*(c*x+I))-1/2*I/c^3*a*b^2*ln(c^2*x^2+1)*ln(c*x+I)+1/2*I/c^3*
a*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/8*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)+1)^2)^3+1/4*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*
x)^2/(c^2*x^2+1))^3+1/4*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3-1/4*I/c^3*b^3*arctan(c*x)^2*Pi*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)^2-1/4*I/c^3*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2
+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}\,b^3x^3\arctan\left(cx\right)^3-\frac{1}{32}\,b^3x^3\arctan\left(cx\right)\log\left(c^2x^2+1\right)^2+\frac{1}{3}\,a^3x^3+\frac{1}{2}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan(c*x)^3 - 1/32*b^3*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2 +
1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a^2*b + integrate(1/32*(4*b^3*c^2*x^4*arctan(c*x)*log(c^2*x^2 + 1) + 28*(b^
3*c^2*x^4 + b^3*x^2)*arctan(c*x)^3 + 4*(24*a*b^2*c^2*x^4 - b^3*c*x^3 + 24*a
*b^2*x^2)*arctan(c*x)^2 + (b^3*c*x^3 + 3*(b^3*c^2*x^4 + b^3*x^2)*arctan(c*x
))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x^2\arctan\left(cx\right)^3+3\,ab^2x^2\arctan\left(cx\right)^2+3\,a^2bx^2\arctan\left(cx\right)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctan(c*x)^3 + 3*a*b^2*x^2*arctan(c*x)^2 + 3*a^2*b*x^2*ar
ctan(c*x) + a^3*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\,\mathrm{atan}\left(cx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx) + a)^3 x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3*x^2, x)
```

3.28 $\int x\left(a+b\tan^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=131

$$-\frac{3ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2}-\frac{3b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2}-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2$$

```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c*x])^2)/c^2 - (3*b*x*(a + b*ArcTan[c*x])^2)/(2
*c) + (a + b*ArcTan[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTan[c*x])^3)/2 - (3*b^
2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 - (((3*I)/2)*b^3*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c^2
```

Rubi [A] time = 0.238823, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4852, 4916, 4846, 4920, 4854, 2402, 2315, 4884}

$$-\frac{3ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2}-\frac{3b^2\log\left(\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2}-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])^3,x]
```


```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c*x])^2)/c^2 - (3*b*x*(a + b*ArcTan[c*x])^2)/(2
*c) + (a + b*ArcTan[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTan[c*x])^3)/2 - (3*b^
2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 - (((3*I)/2)*b^3*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c^2
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}(cx)\right)^3\,dx = \frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{1}{2}(3bc)\int\frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx$$

$$=\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{(3b)\int\left(a+b\tan^{-1}(cx)\right)^2\,dx}{2c}+\frac{(3b)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{2c}$$

$$=-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3+\left(3b^2\right)\int\frac{x\left(a+b\right.}{1+}$$

$$=-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(\right.$$

$$=-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(\right.$$

$$=-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(\right.$$

$$=-\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(\right.$$

Mathematica [A] time = 0.283898, size = 152, normalized size = 1.16

$$\frac{3ib^3\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+a\left(acx(acx-3b)+3b^2\log\left(c^2x^2+1\right)\right)+3b^2\tan^{-1}(cx)^2\left(ac^2x^2+a+b(-cx+i)\right)+3b\,\text{ta}}{2c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^2 + b*(I - c*x))*ArcTan[c*x]^2 + b^3*(1 + c^2*x^2)*ArcT
an[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a - 2*b*c*x + a*c^2*x^2) - 2*b^2*Log[1 + E^
```

```
((2*I)*ArcTan[c*x])]) + a*(a*c*x*(-3*b + a*c*x) + 3*b^2*Log[1 + c^2*x^2]) +
 (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2)
```

Maple [B] time = 0.089, size = 352, normalized size = 2.7

$$\frac{x^2a^3}{2}+\frac{x^2b^3(\arctan(cx))^3}{2}+\frac{b^3(\arctan(cx))^3}{2c^2}-\frac{3b^3(\arctan(cx))^2x}{2c}+\frac{3b^3\arctan(cx)\ln\left(c^2x^2+1\right)}{2c^2}-\frac{\frac{3i}{4}b^3\mathrm{dilog}\left(\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^3,x)
```

```
[Out] 1/2*x^2*a^3+1/2*x^2*b^3*arctan(c*x)^3+1/2/c^2*b^3*arctan(c*x)^3-3/2/c*b^3*a
rctan(c*x)^2*x+3/2/c^2*b^3*arctan(c*x)*ln(c^2*x^2+1)-3/4*I/c^2*b^3*dilog(-1
/2*I*(c*x+I))-3/8*I/c^2*b^3*ln(c*x-I)^2-3/4*I/c^2*b^3*ln(c^2*x^2+1)*ln(c*x+
I)-3/4*I/c^2*b^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/8*I/c^2*b^3*ln(c*x+I)^2+3/4
*I/c^2*b^3*dilog(1/2*I*(c*x-I))+3/4*I/c^2*b^3*ln(c^2*x^2+1)*ln(c*x-I)+3/4*I
/c^2*b^3*ln(c*x+I)*ln(1/2*I*(c*x-I))+3/2*x^2*a*b^2*arctan(c*x)^2+3/2/c^2*a*
b^2*arctan(c*x)^2-3/c*a*b^2*x*arctan(c*x)+3/2/c^2*a*b^2*ln(c^2*x^2+1)+3/2*b
*a^2*x^2*arctan(c*x)-3/2/c*x*a^2*b+3/2/c^2*a^2*b*arctan(c*x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x\arctan(cx)^3+3\,ab^2x\arctan(cx)^2+3\,a^2bx\arctan(cx)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c*x)^3 + 3*a*b^2*x*arctan(c*x)^2 + 3*a^2*b*x*arctan(c
*x) + a^3*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\,\mathrm{atan}(cx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*atan(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan(cx) + a)^3 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3*x, x)
```

3.29 $\int \left(a+b\tan^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=119

$$\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{i\left(a+b\tan^{-1}(cx)\right)}{c}$$

```
[Out] (I*(a + b*ArcTan[c*x])^3)/c + x*(a + b*ArcTan[c*x])^3 + (3*b*(a + b*ArcTan[
c*x])^2*Log[2/(1 + I*c*x)])/c + ((3*I)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c)
```

Rubi [A] time = 0.209268, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4846, 4920, 4854, 4884, 4994, 6610}

$$\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{i\left(a+b\tan^{-1}(cx)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^3)/c + x*(a + b*ArcTan[c*x])^3 + (3*b*(a + b*ArcTan[
c*x])^2*Log[2/(1 + I*c*x)])/c + ((3*I)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c)
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
```

```
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^{-1}(cx)\right)^3\,dx &= x\left(a+b\tan^{-1}(cx)\right)^3-(3bc)\int\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+(3b)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{i-cx}\,dx\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{3b\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c}-\left(6b^2\right)\int\frac{(a\cdots}{\cdots}\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{3b\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c}+\frac{3ib^2\left(a+b\,\mathrm{t}\cdots\right.}{\cdots}\\ &= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{3b\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c}+\frac{3ib^2\left(a+b\,\mathrm{t}\cdots\right.}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.0921695, size = 192, normalized size = 1.61

$$\frac{3ab^2\left(-i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+cx\tan^{-1}(cx)^2-i\tan^{-1}(cx)^2+2\tan^{-1}(cx)\log\left(1+e^{2i\tan^{-1}(cx)}\right)\right)}{c}+\frac{b^3\left(-3i\tan^{-1}(cx\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTan[c*x] - (3*a^2*b*Log[1 + c^2*x^2])/(2*c) + (3*a*b^2
*((-I)*ArcTan[c*x]^2 + c*x*ArcTan[c*x]^2 + 2*ArcTan[c*x]*Log[1 + E^((2*I)*A
rcTan[c*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c + (b^3*((-I)*ArcTan
[c*x]^3 + c*x*ArcTan[c*x]^3 + 3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])
] - (3*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*PolyLog[3, -E
^((2*I)*ArcTan[c*x])])/2))/c
```

Maple [B] time = 0.134, size = 270, normalized size = 2.3

$$xa^3-\frac{ib^3(\arctan(cx))^3}{c}+b^3x(\arctan(cx))^3+3\frac{b^3(\arctan(cx))^2}{c}\ln\left(\frac{(1+icx)^2}{c^2x^2+1}+1\right)-\frac{3\,ib^3\arctan(cx)}{c}\mathrm{polylog}\left(2,-\frac{(1\cdots}{c\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3,x)
```

```
[Out] x*a^3-I/c*b^3*arctan(c*x)^3+b^3*x*arctan(c*x)^3+3/c*b^3*arctan(c*x)^2*ln((1
+I*c*x)^2/(c^2*x^2+1)+1)-3*I/c*b^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*
x^2+1))+3/2/c*b^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3*I/c*arctan(c*x)^2*a
```

```
*b^2+3*x*a*b^2*arctan(c*x)^2+6/c*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)*
a*b^2-3*I/c*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))*a*b^2+3*x*a^2*b*arctan(c*x)
-3/2/c*a^2*b*ln(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\,b^3 x\arctan\left(cx\right)^3-\frac{3}{32}\,b^3 x\arctan\left(cx\right)\log\left(c^2x^2+1\right)^2+\frac{7\,b^3\arctan\left(cx\right)^4}{32\,c}+28\,b^3c^2\int\frac{x^2\arctan\left(cx\right)^3}{32\left(c^2x^2+1\right)}\,dx+3\,b^3c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x*arctan(c*x)^3 - 3/32*b^3*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 7/32*
b^3*arctan(c*x)^4/c + 28*b^3*c^2*integrate(1/32*x^2*arctan(c*x)^3/(c^2*x^2
+ 1), x) + 3*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2
*x^2 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2 + 1)
, x) + 12*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2
+ 1), x) + a*b^2*arctan(c*x)^3/c - 12*b^3*c*integrate(1/32*x*arctan(c*x)^2/
(c^2*x^2 + 1), x) + 3*b^3*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^2 +
1), x) + a^3*x + 3*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x
^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3\arctan\left(cx\right)^3+3\,ab^2\arctan\left(cx\right)^2+3\,a^2b\arctan\left(cx\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
a^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{atan}\left(cx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(cx\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3, x)
```

3.30 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx$

Optimal. Leaf size=206

$$-\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{3}{2}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}ib\text{PolyLog}\left(2\right.$$

```
[Out] 2*(a + b*ArcTan[c*x])^3*ArcTanh[1 - 2/(1 + I*c*x)] - ((3*I)/2)*b*(a + b*Arc
Tan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + ((3*I)/2)*b*(a + b*ArcTan[c*x])
^2*PolyLog[2, -1 + 2/(1 + I*c*x)] - (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1
 - 2/(1 + I*c*x)])/2 + (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*
c*x)])/2 + ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)] - ((3*I)/4)*b^3*Poly
Log[4, -1 + 2/(1 + I*c*x)]
```

Rubi [A] time = 0.427019, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4850, 4988, 4884, 4994, 4998, 6610}

$$-\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{3}{2}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}ib\text{PolyLog}\left(2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x,x]
```


```
[Out] 2*(a + b*ArcTan[c*x])^3*ArcTanh[1 - 2/(1 + I*c*x)] - ((3*I)/2)*b*(a + b*Arc
Tan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + ((3*I)/2)*b*(a + b*ArcTan[c*x])
^2*PolyLog[2, -1 + 2/(1 + I*c*x)] - (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1
 - 2/(1 + I*c*x)])/2 + (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*
c*x)])/2 + ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)] - ((3*I)/4)*b^3*Poly
Log[4, -1 + 2/(1 + I*c*x)]
```


Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx = 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-(6bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)+(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx-(3bc)$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib\left(a+b\tan^{-1}(cx)\right)^2\mathrm{Li}_2\left(1-\frac{2}{1+icx}\right)+\frac{3}{2}ib\left(a\right.$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib\left(a+b\tan^{-1}(cx)\right)^2\mathrm{Li}_2\left(1-\frac{2}{1+icx}\right)+\frac{3}{2}ib\left(a\right.$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib\left(a+b\tan^{-1}(cx)\right)^2\mathrm{Li}_2\left(1-\frac{2}{1+icx}\right)+\frac{3}{2}ib\left(a\right.$$

Mathematica [A] time = 0.136384, size = 212, normalized size = 1.03

$$\frac{3}{4}ib\left(2\text{PolyLog}\left(2,\frac{cx+i}{-cx+i}\right)\left(a+b\tan^{-1}(cx)\right)^2-2\text{PolyLog}\left(2,\frac{cx+i}{cx-i}\right)\left(a+b\tan^{-1}(cx)\right)^2+b\left(-2i\text{PolyLog}\left(3,\frac{cx+i}{-cx+i}\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^3*ArcTanh[(I + c*x)/(-I + c*x)] + ((3*I)/4)*b*(2*(a +
 b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(I - c*x)] - 2*(a + b*ArcTan[c*x])^2
*PolyLog[2, (I + c*x)/(-I + c*x)] + b*((-2*I)*(a + b*ArcTan[c*x])*PolyLog[3
, (I + c*x)/(I - c*x)] + (2*I)*(a + b*ArcTan[c*x])*PolyLog[3, (I + c*x)/(-I
 + c*x)] + b*(-PolyLog[4, (I + c*x)/(I - c*x)] + PolyLog[4, (I + c*x)/(-I +
 c*x)])))
```

Maple [C] time = 0.301, size = 2309, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x,x)
```

```
[Out] -3/2*I*a^2*b*ln(c*x)*ln(1-I*c*x)+1/2*I*b^3*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)
-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^3+3/2*I*a^2*b*ln(c*x)*ln(1+I
*c*x)-6*I*a*b^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*a*b^
2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-6*I*a*b^2*arctan(c*x)*pol
ylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*a*b^2*Pi*arctan(c*x)^2-1/2*I*b^3
*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(
c*x)^3+1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^3*arctan(c*x)^3+6*a*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/
2*a*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+6*a*b^2*polylog(3,(1+I*c*x)/(c^
2*x^2+1)^(1/2))+b^3*ln(c*x)*arctan(c*x)^3-b^3*arctan(c*x)^3*ln((1+I*c*x)^2/
(c^2*x^2+1)-1)+b^3*arctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*b^3*ar
ctan(c*x)*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^3*arctan(c*x)^3*ln(1+(1+
I*c*x)/(c^2*x^2+1)^(1/2))+6*b^3*arctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1
)^(1/2))-3/2*b^3*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3/4*I*b^3*
polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))+6*I*b^3*polylog(4,(1+I*c*x)/(c^2*x^2+1)
^(1/2))+6*I*b^3*polylog(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*a*b^2*Pi*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+a^3*
ln(c*x)+1/2*I*b^3*Pi*arctan(c*x)^3-3*I*b^3*arctan(c*x)^2*polylog(2,(1+I*c*x
)/(c^2*x^2+1)^(1/2))-3*I*b^3*arctan(c*x)^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1)
^(1/2))+3/2*I*b^3*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3*a*b^2
*ln(c*x)*arctan(c*x)^2-3*a*b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+
3*a*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*a*b^2*arctan(c*x)
^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*a^2*b*ln(c*x)*arctan(c*x)+3/2*I*a^2*
b*dilog(1+I*c*x)-3/2*I*a^2*b*dilog(1-I*c*x)-1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)
-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^3+3/2*I*a*b^2*Pi*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3/2*I*
a*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*ar
ctan(c*x)^2+3/2*I*a*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-
1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))*arctan(c*x)^3-1/2*I*b^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+
1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*ar
ctan(c*x)^3-1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^3-3/2*I*a*b
^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*a*b^2*Pi*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-3/2*I*a*b^2*Pi*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(
c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^3*Pi*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^3-3/2
*I*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3\log(x)+\frac{1}{32}\int\frac{28\,b^3\arctan(cx)^3+3\,b^3\arctan(cx)\log\left(c^2x^2+1\right)^2+96\,ab^2\arctan(cx)^2+96\,a^2b\arctan(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x)^3 + 3*b^3*arctan(c*x)*log(c
^2*x^2 + 1)^2 + 96*a*b^2*arctan(c*x)^2 + 96*a^2*b*arctan(c*x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan(cx)^3 + 3\,ab^2 \arctan(cx)^2 + 3\,a^2 b \arctan(cx) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{atan}\,(cx))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/x, x)
```

3.31 $\int \frac{(a+b\tan^{-1}(cx))^3}{x^2}\,dx$

Optimal. Leaf size=116

$$-3ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{3}{2}b^3c\text{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)-ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{(a+b$$

```
[Out] (-I)*c*(a + b*ArcTan[c*x])^3 - (a + b*ArcTan[c*x])^3/x + 3*b*c*(a + b*ArcTa
n[c*x])^2*Log[2 - 2/(1 - I*c*x)] - (3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[
2, -1 + 2/(1 - I*c*x)] + (3*b^3*c*PolyLog[3, -1 + 2/(1 - I*c*x)])/2
```

Rubi [A] time = 0.267137, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4852, 4924, 4868, 4884, 4992, 6610}

$$-3ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{3}{2}b^3c\text{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)-ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{(a+b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^2,x]
```

```
[Out] (-I)*c*(a + b*ArcTan[c*x])^3 - (a + b*ArcTan[c*x])^3/x + 3*b*c*(a + b*ArcTa
n[c*x])^2*Log[2 - 2/(1 - I*c*x)] - (3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[
2, -1 + 2/(1 - I*c*x)] + (3*b^3*c*PolyLog[3, -1 + 2/(1 - I*c*x)])/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x} + (3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(1+c^2x^2\right)}\,dx$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^3 - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x} + (3ibc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(i+cx)}\,dx$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^3 - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x} + 3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1-icx}\right) - \left(6b^2\right.$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^3 - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x} + 3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1-icx}\right) - 3ib^2c$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^3 - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x} + 3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{2}{1-icx}\right) - 3ib^2c$$

Mathematica [A] time = 0.368906, size = 214, normalized size = 1.84

$$3ab^2c\left(-i\left(\tan^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)\right)-\frac{\tan^{-1}(cx)^2}{cx}+2\tan^{-1}(cx)\log\left(1-e^{2i\tan^{-1}(cx)}\right)\right)+b^3c\left(3i\tan^{-1}(cx)\text{P}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcTan[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1 +
c^2*x^2])/2 + 3*a*b^2*c*(-(ArcTan[c*x]^2/(c*x)) + 2*ArcTan[c*x]*Log[1 - E^(
(2*I)*ArcTan[c*x])] - I*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])
) + b^3*c*((-I/8)*Pi^3 + I*ArcTan[c*x]^3 - ArcTan[c*x]^3/(c*x) + 3*ArcTan[c
*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (3*I)*ArcTan[c*x]*PolyLog[2, E^((-2
*I)*ArcTan[c*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/2)
```

Maple [C] time = 0.341, size = 2159, normalized size = 18.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^2,x)
```

```
[Out] 3*c*b^3*ln(c*x)*arctan(c*x)^2+3*c*b^3*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2
+1)^(1/2))-3/4*I*c*b^3*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^
2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x
)^2/(c^2*x^2+1)+1)^2)+3/2*I*c*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*cs
gn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*
c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3*I*c*a*b^2*dilog(1+I*c*x)-3*I*c*a*b^2
*dilog(1-I*c*x)+3/2*I*c*b^3*Pi*arctan(c*x)^2+6*c*a*b^2*ln(c*x)*arctan(c*x)-
3*c*a*b^2*arctan(c*x)*ln(c^2*x^2+1)-6*I*c*b^3*arctan(c*x)*polylog(2,(1+I*c*
x)/(c^2*x^2+1)^(1/2))-6*I*c*b^3*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1
)^(1/2))+3/4*I*c*a*b^2*ln(c*x-I)^2+3/2*I*c*a*b^2*dilog(-1/2*I*(c*x+I))-3/4*
I*c*a*b^2*ln(c*x+I)^2-3/2*I*c*a*b^2*dilog(1/2*I*(c*x-I))+3/4*I*c*b^3*arctan
(c*x)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^
2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+3/2*I*c*b^3*arctan(c*x)^2*Pi*csgn(I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2+3/4*I*c*b^3*a
rctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)+1)^2)-3/2*I*c*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/
2*I*c*b^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+3/2*I*c*b^3*Pi*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+3*c*b^3*arctan(c
*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*c*b^3*arctan(c*x)^2*ln((1+I*c*x)^
2/(c^2*x^2+1)-1)-3/2*c*b^3*arctan(c*x)^2*ln(c^2*x^2+1)+3*c*b^3*arctan(c*x)^
2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+3*c*a^2*b*ln(c*x)-3/2*c*a^2*b*ln(c^2*x^2+
1)+3*c*b^3*arctan(c*x)^2*ln(2)-3*a^2*b/x*arctan(c*x)-3*a*b^2/x*arctan(c*x)^
2-I*c*b^3*arctan(c*x)^3-a^3/x-b^3/x*arctan(c*x)^3+6*c*b^3*polylog(3,(1+I*c*
x)/(c^2*x^2+1)^(1/2))+6*c*b^3*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/4*I
*c*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^
2+1)+1)^2)^3+3/4*I*c*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1
)^2)^3+3*I*c*a*b^2*ln(c*x)*ln(1+I*c*x)-3*I*c*a*b^2*ln(c*x)*ln(1-I*c*x)-3/2*
I*c*a*b^2*ln(c^2*x^2+1)*ln(c*x-I)+3/2*I*c*a*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I)
)-3/2*I*c*a*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+3/2*I*c*a*b^2*ln(c^2*x^2+1)*ln(
c*x+I)+3/2*I*c*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^3*arctan(c*x)^2+3/2*I*c*b^3*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/
((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-3/2*I*c*b^3*Pi*csgn(((1+I*c*x)
^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/4*I*c*b^3*
arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3+3/4*I*c*b^3*arctan(c*x)^
2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c
*x)^2/(c^2*x^2+1)+1)^2)^2-3/4*I*c*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))-3/2*I*c*b^3*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^
2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-3/2*I*c*b^3*arctan
(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)+1)^2)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\arctan(cx)^3+3\,ab^2\arctan(cx)^2+3\,a^2b\arctan(cx)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\mathrm{atan}(cx))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arctan(cx)+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/x^2, x)
```

3.32 $\int \frac{(a+b\tan^{-1}(cx))^3}{x^3}\,dx$

Optimal. Leaf size=133

$$-\frac{3}{2}ib^3c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+3b^2c^2\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2\left(a+b\right.$$

```
[Out] ((-3*I)/2)*b*c^2*(a + b*ArcTan[c*x])^2 - (3*b*c*(a + b*ArcTan[c*x])^2)/(2*x
) - (c^2*(a + b*ArcTan[c*x])^3)/2 - (a + b*ArcTan[c*x])^3/(2*x^2) + 3*b^2*c
^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - ((3*I)/2)*b^3*c^2*PolyLog[2
, -1 + 2/(1 - I*c*x)]
```

Rubi [A] time = 0.28473, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4852, 4918, 4924, 4868, 2447, 4884}

$$-\frac{3}{2}ib^3c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)+3b^2c^2\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}c^2\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^3,x]
```

```
[Out] ((-3*I)/2)*b*c^2*(a + b*ArcTan[c*x])^2 - (3*b*c*(a + b*ArcTan[c*x])^2)/(2*x
) - (c^2*(a + b*ArcTan[c*x])^3)/2 - (a + b*ArcTan[c*x])^3/(2*x^2) + 3*b^2*c
^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - ((3*I)/2)*b^3*c^2*PolyLog[2
, -1 + 2/(1 - I*c*x)]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
```

```
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^3}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx-\frac{1}{2}\left(3bc^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx$$

$$= -\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}+\left(3b^2c^2\right)\int \frac{a+b\tan}{x\left(1+\right.}$$

$$= -\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(\right.}{2x^2}$$

$$= -\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(\right.}{2x^2}$$

$$= -\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(\right.}{2x^2}$$

Mathematica [A] time = 0.322897, size = 176, normalized size = 1.32

$$-\frac{3ib^3c^2x^2\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+a\left(a(a+3bcx)-6b^2c^2x^2\log\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)\right)+3b^2\tan^{-1}(cx)^2\left(ac^2x^2+a+bcx(1+icx)\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^3,x]
```

```
[Out] -(3*b^2*(a + a*c^2*x^2 + b*c*x*(1 + I*c*x))*ArcTan[c*x]^2 + b^3*(1 + c^2*x^
2)*ArcTan[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a + 2*b*c*x + a*c^2*x^2) - 2*b^2*c^2
*x^2*Log[1 - E^((2*I)*ArcTan[c*x])]) + a*(a*(a + 3*b*c*x) - 6*b^2*c^2*x^2*L
og[(c*x)/Sqrt[1 + c^2*x^2]]) + (3*I)*b^3*c^2*x^2*PolyLog[2, E^((2*I)*ArcTan
[c*x])])/(2*x^2)
```

Maple [B] time = 0.043, size = 457, normalized size = 3.4

$$-\frac{3c^2ab^2\ln\left(c^2x^2+1\right)}{2}-\frac{3cb^3\left(\arctan(cx)\right)^2}{2x}-\frac{3a^2b\arctan(cx)}{2x^2}+\frac{3i}{8}c^2b^3\left(\ln(cx-i)\right)^2-\frac{3i}{4}c^2b^3\operatorname{dilog}\left(\frac{i}{2}(cx-i)\right)+\frac{3i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^3,x)
```

```
[Out] -3/2*c^2*a*b^2*ln(c^2*x^2+1)-3/2*c*b^3*arctan(c*x)^2/x-3/2*a^2*b/x^2*arctan
(c*x)+3/8*I*c^2*b^3*ln(c*x-I)^2-3/4*I*c^2*b^3*dilog(1/2*I*(c*x-I))+3/2*I*c^
2*b^3*dilog(1+I*c*x)-3/2*I*c^2*b^3*dilog(1-I*c*x)+3/4*I*c^2*b^3*dilog(-1/2*
I*(c*x+I))-3/2*a*b^2/x^2*arctan(c*x)^2-3/2*c^2*a*b^2*arctan(c*x)^2-3/2*c^2*
b^3*arctan(c*x)*ln(c^2*x^2+1)+3*c^2*b^3*ln(c*x)*arctan(c*x)-3/2*c^2*a^2*b*a
rctan(c*x)+3*c^2*a*b^2*ln(c*x)-3/2*c*a^2*b/x-3/8*I*c^2*b^3*ln(c*x+I)^2-1/2*
b^3/x^2*arctan(c*x)^3-1/2*c^2*b^3*arctan(c*x)^3-3*c*a*b^2*arctan(c*x)/x+3/4
*I*c^2*b^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/2*I*c^2*b^3*ln(c*x)*ln(1+I*c*x)-3
/4*I*c^2*b^3*ln(c*x+I)*ln(1/2*I*(c*x-I))-3/4*I*c^2*b^3*ln(c^2*x^2+1)*ln(c*x
-I)+3/4*I*c^2*b^3*ln(c^2*x^2+1)*ln(c*x+I)-3/2*I*c^2*b^3*ln(c*x)*ln(1-I*c*x)
-1/2*a^3/x^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan(cx)^3 + 3\,ab^2 \arctan(cx)^2 + 3\,a^2 b \arctan(cx) + a^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{atan}\,(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx)+a)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/x^3, x)
```

3.33 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal. Leaf size=213

$$ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^3c^3\text{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{3}ic^3$$

```
[Out] -((b^2*c^2*(a + b*ArcTan[c*x]))/x) - (b*c^3*(a + b*ArcTan[c*x])^2)/2 - (b*c
*(a + b*ArcTan[c*x])^2)/(2*x^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^3 - (a + b*
ArcTan[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 + c^2*x^2])/2 - b*
c^3*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + I*b^2*c^3*(a + b*ArcTan[
c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 - I*c*
x)])/2
```

Rubi [A] time = 0.476335, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4852, 4918, 266, 36, 29, 31, 4884, 4924, 4868, 4992, 6610}

$$ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}b^3c^3\text{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x}+\frac{1}{3}ic^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^4,x]
```


```
[Out] -((b^2*c^2*(a + b*ArcTan[c*x]))/x) - (b*c^3*(a + b*ArcTan[c*x])^2)/2 - (b*c
*(a + b*ArcTan[c*x])^2)/(2*x^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^3 - (a + b*
ArcTan[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 + c^2*x^2])/2 - b*
c^3*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + I*b^2*c^3*(a + b*ArcTan[
c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 - I*c*
x)])/2
```


Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4992

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - (2*I)/(I + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^4}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{3x^3} + (bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{3x^3} + (bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx - \left(bc^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^3 - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{3x^3} + \left(b^2c^2\right)\int \frac{a+b\mathrm{t}}{x^2\left(1\right.}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^3 - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{3x^3} - bc^3\left(a+b\tan^{-1}(\right.$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}bc^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+b\mathrm{t}\right.$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}bc^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+b\mathrm{t}\right.$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}bc^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+b\mathrm{t}\right.$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{x} - \frac{1}{2}bc^3\left(a+b\tan^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}ic^3\left(a+b\mathrm{t}\right.$$

Mathematica [A] time = 0.812281, size = 321, normalized size = 1.51

$$\frac{iab^2\left(c^3x^3\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+ic^2x^2+\left(c^3x^3+i\right)\tan^{-1}(cx)^2+icx\tan^{-1}(cx)\left(c^2x^2+2c^2x^2\log\left(1-e^{2i\tan^{-1}(cx)}\right)\right.\right.}{x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (a^2*b*c)/(2*x^2) - (a^2*b*ArcTan[c*x])/x^3 - a^2*b*c^3*Log[
x] + (a^2*b*c^3*Log[1 + c^2*x^2])/2 + (I*a*b^2*(I*c^2*x^2 + (I + c^3*x^3)*A
rcTan[c*x]^2 + I*c*x*ArcTan[c*x]*(1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E^((2*I)*
ArcTan[c*x])]) + c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x^3 + (b^3*c^3
*(I*Pi^3 - (24*ArcTan[c*x])/(c*x) - 12*ArcTan[c*x]^2 - (12*ArcTan[c*x]^2)/(
c^2*x^2) - (8*I)*ArcTan[c*x]^3 - (8*ArcTan[c*x]^3)/(c^3*x^3) - 24*ArcTan[c*
x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + 24*Log[(c*x)/Sqrt[1 + c^2*x^2]] - (2
4*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, E^((-2*
I)*ArcTan[c*x])]))/24
```

Maple [C] time = 1.735, size = 5974, normalized size = 28.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan(cx)^3 + 3\,ab^2 \arctan(cx)^2 + 3\,a^2 b \arctan(cx) + a^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atan}(cx))^3}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^3}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/x^4, x)
```

3.34 $\int \frac{(a+b\tan^{-1}(cx))^3}{x^5}\,dx$

Optimal. Leaf size=198

$$ib^3c^4\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}-2b^2c^4\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}c^4\left(a+b\tan\right.$$

```
[Out] -(b^3*c^3)/(4*x) - (b^3*c^4*ArcTan[c*x])/4 - (b^2*c^2*(a + b*ArcTan[c*x]))/
(4*x^2) + I*b*c^4*(a + b*ArcTan[c*x])^2 - (b*c*(a + b*ArcTan[c*x])^2)/(4*x^
3) + (3*b*c^3*(a + b*ArcTan[c*x])^2)/(4*x) + (c^4*(a + b*ArcTan[c*x])^3)/4
- (a + b*ArcTan[c*x])^3/(4*x^4) - 2*b^2*c^4*(a + b*ArcTan[c*x])*Log[2 - 2/(
1 - I*c*x)] + I*b^3*c^4*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rubi [A] time = 0.600576, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4852, 4918, 325, 203, 4924, 4868, 2447, 4884}

$$ib^3c^4\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}-2b^2c^4\log\left(2-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)+\frac{1}{4}c^4\left(a+b\tan\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^5,x]
```

```
[Out] -(b^3*c^3)/(4*x) - (b^3*c^4*ArcTan[c*x])/4 - (b^2*c^2*(a + b*ArcTan[c*x]))/
(4*x^2) + I*b*c^4*(a + b*ArcTan[c*x])^2 - (b*c*(a + b*ArcTan[c*x])^2)/(4*x^
3) + (3*b*c^3*(a + b*ArcTan[c*x])^2)/(4*x) + (c^4*(a + b*ArcTan[c*x])^3)/4
- (a + b*ArcTan[c*x])^3/(4*x^4) - 2*b^2*c^4*(a + b*ArcTan[c*x])*Log[2 - 2/(
1 - I*c*x)] + I*b^3*c^4*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^5}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx-\frac{1}{4}\left(3bc^3\right)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{4x^3}-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{4x^4}+\frac{1}{2}\left(b^2c^2\right)\int \frac{a+b\tan^{-1}(cx)}{x^3\left(1+c^2x^2\right)}\,dx-\frac{1}{4}\left(3bc^3\right)\int$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{4x^3}+\frac{3bc^3\left(a+b\tan^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx\right.}{4x^4}$$

$$= -\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}+ibc^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{4x^3}+\frac{3bc^3\left(a+b\tan^{-1}\right.}{4x}$$

$$= -\frac{b^3c^3}{4x}-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}+ibc^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bc\left(a+b\tan^{-1}(cx)\right)^2}{4x^3}+\frac{3bc^3\left(a+\right.}{}$$

$$= -\frac{b^3c^3}{4x}-\frac{1}{4}b^3c^4\tan^{-1}(cx)-\frac{b^2c^2\left(a+b\tan^{-1}(cx)\right)}{4x^2}+ibc^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bc\left(a+b\tan^{-1}\right.}{4x^3}$$

Mathematica [A] time = 0.655673, size = 265, normalized size = 1.34

$$-\frac{-4ib^3c^4x^4\text{PolyLog}\left(2,e^{2i\tan^{-1}(cx)}\right)+b\tan^{-1}(cx)\left(a^2\left(3-3c^4x^4\right)+ab\left(2cx-6c^3x^3\right)+b^2c^2x^2\left(c^2x^2+1\right)+8b^2c^4x^4\log\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^5,x]
```

```
[Out] -(a^3 + a^2*b*c*x + a*b^2*c^2*x^2 - 3*a^2*b*c^3*x^3 + b^3*c^3*x^3 + a*b^2*c
^4*x^4 + b^2*(b*c*x*(1 - 3*c^2*x^2 - (4*I)*c^3*x^3) + a*(3 - 3*c^4*x^4))*Ar
cTan[c*x]^2 - b^3*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*c^2*x^2
*(1 + c^2*x^2) + a*b*(2*c*x - 6*c^3*x^3) + a^2*(3 - 3*c^4*x^4) + 8*b^2*c^4*
x^4*Log[1 - E^((2*I)*ArcTan[c*x])]) + 8*a*b^2*c^4*x^4*Log[(c*x)/Sqrt[1 + c^
2*x^2]] - (4*I)*b^3*c^4*x^4*PolyLog[2, E^((2*I)*ArcTan[c*x])])/(4*x^4)
```

Maple [B] time = 0.1, size = 550, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^5,x)
```

```
[Out] 1/2*I*c^4*b^3*ln(c^2*x^2+1)*ln(c*x-I)-1/4*b^3*c^3/x-2*c^4*b^3*ln(c*x)*arcta
n(c*x)+c^4*b^3*arctan(c*x)*ln(c^2*x^2+1)-2*c^4*a*b^2*ln(c*x)+I*c^4*b^3*dilo
g(1-I*c*x)+c^4*a*b^2*ln(c^2*x^2+1)-3/4*a*b^2/x^4*arctan(c*x)^2-3/4*a^2*b/x^
4*arctan(c*x)-1/2*I*c^4*b^3*dilog(-1/2*I*(c*x+I))-1/4*I*c^4*b^3*ln(c*x-I)^2
+1/4*I*c^4*b^3*ln(c*x+I)^2-I*c^4*b^3*dilog(1+I*c*x)+1/2*I*c^4*b^3*dilog(1/2
*I*(c*x-I))-1/4*c^2*a*b^2/x^2-1/4*c*b^3*arctan(c*x)^2/x^3+3/4*c^3*b^3*arcta
n(c*x)^2/x-1/4*c^2*b^3*arctan(c*x)/x^2+3/4*c^4*a^2*b*arctan(c*x)+3/4*c^4*a*
b^2*arctan(c*x)^2-1/4*c*a^2*b/x^3+3/4*c^3*a^2*b/x-1/2*I*c^4*b^3*ln(c^2*x^2+
1)*ln(c*x+I)-I*c^4*b^3*ln(c*x)*ln(1+I*c*x)+I*c^4*b^3*ln(c*x)*ln(1-I*c*x)-1/
2*c*a*b^2*arctan(c*x)/x^3+3/2*c^3*a*b^2*arctan(c*x)/x-1/2*I*c^4*b^3*ln(c*x-
I)*ln(-1/2*I*(c*x+I))+1/2*I*c^4*b^3*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/4*c^4*b^3
*arctan(c*x)^3-1/4*b^3/x^4*arctan(c*x)^3-1/4*b^3*c^4*arctan(c*x)-1/4*a^3/x^
4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\arctan(cx)^3+3ab^2\arctan(cx)^2+3a^2b\arctan(cx)+a^3}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/x^5, x)
```

3.35 $\int \frac{x}{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=10

$$\text{Unintegrable}\left(\frac{x}{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x/ArcTan[a*x], x]
```

Rubi [A] time = 0.0084395, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/ArcTan[a*x],x]
```

```
[Out] Defer[Int][x/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x}{\tan^{-1}(ax)}\,dx = \int \frac{x}{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.610712, size = 0, normalized size = 0.

$$\int \frac{x}{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/ArcTan[a*x],x]
```

```
[Out] Integrate[x/ArcTan[a*x], x]
```

Maple [A] time = 0.189, size = 0, normalized size = 0.

$$\int \frac{x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x),x)
```

```
[Out] int(x/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x),x)
```

```
[Out] Integral(x/atan(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/arctan(a*x), x)
```

3.36 $\int \frac{1}{\tan^{-1}(ax)} dx$

Optimal. Leaf size=8

$$\text{Unintegrable}\left(\frac{1}{\tan^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(-1), x]
```

Rubi [A] time = 0.0032902, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(-1),x]

[Out] Defer[Int][ArcTan[a*x]^(-1), x]
```

Rubi steps

$$\int \frac{1}{\tan^{-1}(ax)} dx = \int \frac{1}{\tan^{-1}(ax)} dx$$

Mathematica [A] time = 0.0109886, size = 0, normalized size = 0.

$$\int \frac{1}{\tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(-1),x]

[Out] Integrate[ArcTan[a*x]^(-1), x]
```

Maple [A] time = 0.148, size = 0, normalized size = 0.

$$\int (\arctan(ax))^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x),x)

[Out] int(1/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/arctan(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/arctan(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x),x)
```

```
[Out] Integral(1/atan(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/arctan(a*x), x)
```

3.37 $\int \frac{1}{x \tan^{-1}(ax)} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x \tan^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcTan[a*x]), x]
```

Rubi [A] time = 0.0145322, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x \tan^{-1}(ax)} dx = \int \frac{1}{x \tan^{-1}(ax)} dx$$

Mathematica [A] time = 0.425471, size = 0, normalized size = 0.

$$\int \frac{1}{x \tan^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x*ArcTan[a*x]), x]
```

Maple [A] time = 0.12, size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x),x)
```

```
[Out] int(1/x/arctan(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arctan(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \arctan(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/(x*arctan(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x),x)
```

```
[Out] Integral(1/(x*atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arctan(a*x)), x)
```

3.38 $\int \frac{x}{\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=10

$$\text{Unintegrable}\left(\frac{x}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[x/ArcTan[a*x]^2, x]
```

Rubi [A] time = 0.0075483, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][x/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x}{\tan^{-1}(ax)^2}\,dx = \int \frac{x}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.590724, size = 0, normalized size = 0.

$$\int \frac{x}{\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[x/ArcTan[a*x]^2, x]
```

Maple [A] time = 0.368, size = 0, normalized size = 0.

$$\int \frac{x}{(\arctan(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^2,x)
```

```
[Out] int(x/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2x^3 - \arctan(ax)\int \frac{3a^2x^2+1}{\arctan(ax)}\,dx + x}{a\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^3 - arctan(a*x)*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) + x)/(a*a
rctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\arctan(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arctan(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**2,x)
```

```
[Out] Integral(x/atan(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/arctan(a*x)^2, x)
```

3.39 $\int \frac{1}{\tan^{-1}(ax)^2} dx$

Optimal. Leaf size=8

$$\text{Unintegrable}\left(\frac{1}{\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(-2), x]
```

Rubi [A] time = 0.0032723, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(-2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(-2), x]
```

Rubi steps

$$\int \frac{1}{\tan^{-1}(ax)^2} dx = \int \frac{1}{\tan^{-1}(ax)^2} dx$$

Mathematica [A] time = 0.880677, size = 0, normalized size = 0.

$$\int \frac{1}{\tan^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(-2),x]
```

```
[Out] Integrate[ArcTan[a*x]^(-2), x]
```

Maple [A] time = 0.149, size = 0, normalized size = 0.

$$\int (\arctan(ax))^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^2,x)
```

```
[Out] int(1/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2x^2 - 2\,a^2\arctan(ax)\int\frac{x}{\arctan(ax)}\,dx + 1}{a\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 - 2*a^2*arctan(a*x)*integrate(x/arctan(a*x), x) + 1)/(a*arctan(a*
x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\arctan(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^(-2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**2,x)
```

```
[Out] Integral(atan(a*x)**(-2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(-2), x)
```

3.40 $\int \frac{1}{x\tan^{-1}(ax)^2}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{x\tan^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[1/(x*ArcTan[a*x]^2), x]
```

Rubi [A] time = 0.0130932, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x\tan^{-1}(ax)^2}\,dx=\int \frac{1}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.01026, size = 0, normalized size = 0.

$$\int \frac{1}{x\tan^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*ArcTan[a*x]^2), x]
```

Maple [A] time = 0.135, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\arctan(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^2,x)
```

```
[Out] int(1/x/arctan(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2x^2 - x\arctan(ax)\int \frac{(ax+1)(ax-1)}{x^2\arctan(ax)}\,dx + 1}{ax\arctan(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 - x*arctan(a*x)*integrate((a^2*x^2 - 1)/(x^2*arctan(a*x)), x) + 1
)/(a*x*arctan(a*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\arctan(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arctan(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x*atan(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\arctan(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arctan(a*x)^2), x)
```

3.41 $\int x\sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(x\sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x*Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0073851, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x\sqrt{\tan^{-1}(ax)}\,dx = \int x\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.95853, size = 0, normalized size = 0.

$$\int x\sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x*Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.246, size = 0, normalized size = 0.

$$\int x\sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(arctan(a*x)), x)
```

3.42 $\int \sqrt{\tan^{-1}(ax)}\,dx$

Optimal. Leaf size=10

$$\text{Unintegrable}\left(\sqrt{\tan^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.003199, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \sqrt{\tan^{-1}(ax)}\,dx = \int \sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.7412, size = 0, normalized size = 0.

$$\int \sqrt{\tan^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.131, size = 0, normalized size = 0.

$$\int \sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x)), x)
```

3.43 $\int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\sqrt{\tan^{-1}(ax)}}{x},x\right)$$

```
[Out] Unintegrable[Sqrt[ArcTan[a*x]]/x, x]
```

Rubi [A] time = 0.0131621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/x, x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx$$

Mathematica [A] time = 1.24583, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/x,x]
```

```
[Out] Integrate[Sqrt[ArcTan[a*x]]/x, x]
```

Maple [A] time = 0.26, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{\arctan (ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x,x)
```

```
[Out] int(arctan(a*x)^(1/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atan}{\left(ax \right)}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(atan(a*x))/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctan(a*x))/x, x)
```

3.44 $\int x \tan^{-1}(ax)^{3/2} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(x \tan^{-1}(ax)^{3/2}, x\right)$$

```
[Out] Unintegrable[x*ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0071999, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x \tan^{-1}(ax)^{3/2} \, dx = \int x \tan^{-1}(ax)^{3/2} \, dx$$

Mathematica [A] time = 0.948413, size = 0, normalized size = 0.

$$\int x \tan^{-1}(ax)^{3/2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x*ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.206, size = 0, normalized size = 0.

$$\int x \left(\arctan(ax)\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{atan}^{\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2),x)
```

```
[Out] Integral(x*atan(a*x)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x \arctan(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(a*x)^(3/2), x)
```

3.45 $\int \tan^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=10

$$\text{Unintegrable}\left(\tan^{-1}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0030674, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \tan^{-1}(ax)^{3/2}\,dx = \int \tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A] time = 2.14918, size = 0, normalized size = 0.

$$\int \tan^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.135, size = 0, normalized size = 0.

$$\int (\arctan(ax))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atan}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \arctan(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2), x)
```

3.46 $\int \frac{\tan^{-1}(ax)^{3/2}}{x} \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\tan^{-1}(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(3/2)/x, x]
```

Rubi [A] time = 0.0125215, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x} \, dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x} \, dx$$

Mathematica [A] time = 1.06873, size = 0, normalized size = 0.

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/x,x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2)/x, x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan(ax))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x,x)
```

```
[Out] int(arctan(a*x)^(3/2)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(3/2)/x, x)
```

3.47 $\int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{x}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[x/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0074883, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx = \int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A] time = 1.23157, size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.225, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt{\arctan(ax)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(arctan(a*x)), x)
```

3.48 $\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=10

$$\text{Unintegrable}\left(\frac{1}{\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/Sqrt[ArcTan[a*x]], x]
```

Rubi [A] time = 0.0031163, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][1/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx=\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.0195609, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[1/Sqrt[ArcTan[a*x]], x]
```

Maple [A] time = 0.132, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(atan(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(arctan(a*x)), x)
```

3.49 $\int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{\tan^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi [A] time = 0.0138701, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.12954, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A] time = 0.223, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(atan(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(arctan(a*x))), x)
```

3.50 $\int \frac{x}{\tan^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{x}{\tan^{-1}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable[x/ArcTan[a*x]^(3/2), x]
```

Rubi [A] time = 0.0073448, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\tan^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[x/ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x}{\tan^{-1}(ax)^{3/2}} dx = \int \frac{x}{\tan^{-1}(ax)^{3/2}} dx$$

Mathematica [A] time = 1.4769, size = 0, normalized size = 0.

$$\int \frac{x}{\tan^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[x/ArcTan[a*x]^(3/2), x]
```

Maple [A] time = 0.239, size = 0, normalized size = 0.

$$\int x (\arctan(ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(3/2),x)
```

```
[Out] int(x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x/atan(a*x)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\arctan(ax)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/arctan(a*x)^(3/2), x)
```

3.51 $\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=10

$$\text{Unintegrable}\left(\frac{1}{\tan^{-1}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable[ArcTan[a*x]^(-3/2), x]
```

Rubi [A] time = 0.0030854, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTan[a*x]^(-3/2),x]

[Out] Defer[Int][ArcTan[a*x]^(-3/2), x]
```

Rubi steps

$$\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A] time = 2.53052, size = 0, normalized size = 0.

$$\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(-3/2),x]

[Out] Integrate[ArcTan[a*x]^(-3/2), x]
```

Maple [A] time = 0.135, size = 0, normalized size = 0.

$$\int (\arctan(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(3/2),x)

[Out] int(1/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(-3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x)^(-3/2), x)
```

3.52 $\int \frac{1}{x \tan^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \tan^{-1}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi [A] time = 0.0128075, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \tan^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x \tan^{-1}(ax)^{3/2}} dx = \int \frac{1}{x \tan^{-1}(ax)^{3/2}} dx$$

Mathematica [A] time = 2.72044, size = 0, normalized size = 0.

$$\int \frac{1}{x \tan^{-1}(ax)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcTan[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A] time = 0.183, size = 0, normalized size = 0.

$$\int \frac{1}{x} (\arctan (ax))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*atan(a*x)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \arctan(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arctan(a*x)^(3/2)), x)
```

3.53 $\int \sqrt{x}\tan^{-1}(x)\,dx$

Optimal. Leaf size=117

$$\frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{4\sqrt{x}}{3}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.$$

```
[Out] (-4*Sqrt[x])/3 - (Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]])/3 + (Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[x]])/3 + (2*x^(3/2)*ArcTan[x])/3 - Log[1 - Sqrt[2]*Sqrt[x]
 + x]/(3*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]/(3*Sqrt[2])
```

Rubi [A] time = 0.0674999, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {4852, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{4\sqrt{x}}{3}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTan[x],x]
```

```
[Out] (-4*Sqrt[x])/3 - (Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]])/3 + (Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[x]])/3 + (2*x^(3/2)*ArcTan[x])/3 - Log[1 - Sqrt[2]*Sqrt[x]
 + x]/(3*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]/(3*Sqrt[2])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\tan^{-1}(x)\,dx &= \frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{2}{3}\int \frac{x^{3/2}}{1+x^2}\,dx\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)+\frac{2}{3}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)+\frac{2}{3}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{2}{3}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\right.\\
&= -\frac{4\sqrt{x}}{3}+\frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}}+\frac{1}{3}\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.\\
&= -\frac{4\sqrt{x}}{3}-\frac{1}{3}\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)+\frac{2}{3}x^{3/2}\tan^{-1}(x)-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0250164, size = 108, normalized size = 0.92

$$\frac{1}{6}\left(4x^{3/2}\tan^{-1}(x)-8\sqrt{x}-\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+2\sqrt{2}\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTan[x],x]
```

```
[Out] (-8*Sqrt[x] - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*Sqrt[x]] + 4*x^(3/2)*ArcTan[x] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] +
x] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/6
```

Maple [A] time = 0.023, size = 74, normalized size = 0.6

$$\frac{2\arctan(x)}{3}x^{\frac{3}{2}}-\frac{4}{3}\sqrt{x}+\frac{\sqrt{2}}{3}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{3}\arctan\left(-1+\sqrt{2}\sqrt{x}\right)+\frac{\sqrt{2}}{6}\ln\left(\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)*arctan(x)-4/3*x^(1/2)+1/3*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+1/3
*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/6*2^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1
+x-2^(1/2)*x^(1/2)))
```

Maxima [A] time = 1.48464, size = 116, normalized size = 0.99

$$\frac{2}{3}x^{\frac{3}{2}}\arctan(x)+\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{3}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{6}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x)
)) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)*l
og(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 4
/3*sqrt(x)
```

Fricas [A] time = 2.79439, size = 404, normalized size = 3.45

$$\frac{2}{3}(x\arctan(x)-2)\sqrt{x}-\frac{2}{3}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x*arctan(x) - 2)*sqrt(x) - 2/3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqr
t(x) + x + 1) - sqrt(2)*sqrt(x) - 1) - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(
-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 1/6*sqrt(2)*log(4*sq
```

```
rt(2)*sqrt(x) + 4*x + 4) - 1/6*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A] time = 8.1696, size = 104, normalized size = 0.89

$$\frac{2x^{\frac{3}{2}}\operatorname{atan}(x)}{3}-\frac{4\sqrt{x}}{3}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)}{6}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{3}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*x**(1/2),x)
```

```
[Out] 2*x**(3/2)*atan(x)/3 - 4*sqrt(x)/3 - sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)/
6 + sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1)/6 + sqrt(2)*atan(sqrt(2)*sqrt(x) -
 1)/3 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/3
```

Giac [A] time = 1.21178, size = 116, normalized size = 0.99

$$\frac{2}{3}x^{\frac{3}{2}}\arctan(x)+\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{3}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{6}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x)
)) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)*l
og(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 4
/3*sqrt(x)
```

3.54 $\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((dx)^m \left(a+b\tan^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

Rubi [A] time = 0.0216028, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^3,x]

[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx$$

Mathematica [A] time = 3.73086, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3,x]

[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

Maple [A] time = 1.937, size = 0, normalized size = 0.

$$\int (dx)^m (a+b\arctan(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x))^3,x)

[Out] int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^3\arctan\left(cx\right)^3+3\,ab^2\arctan\left(cx\right)^2+3\,a^2b\arctan\left(cx\right)+a^3\right)\left(dx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)*(d*x)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\left(dx\right)^m\left(a+b\,\mathrm{atan}\left(cx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x))**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(cx\right)+a\right)^3\left(dx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3*(d*x)^m, x)
```

3.55 $\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((dx)^m \left(a+b\tan^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

Rubi [A] time = 0.0212714, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^2,x]

[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 2.43325, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tan^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2,x]

[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

Maple [A] time = 1.633, size = 0, normalized size = 0.

$$\int (dx)^m (a+b\arctan(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x))^2,x)

[Out] int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \arctan(cx)^2 + 2\,ab\arctan(cx) + a^2\right)(dx)^m\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)*(d*x)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \,(a + b\,\text{atan}\,(cx))^2\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\arctan(cx) + a)^2\,(dx)^m\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2*(d*x)^m, x)
```

3.56 $\int (dx)^m \left(a+b\tan^{-1}(cx)\right) dx$

Optimal. Leaf size=73

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}(cx)\right)}{d(m+1)}-\frac{bc(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{d^2(m+1)(m+2)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyperg
eometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d^2*(1 + m)*(2 + m))
```

Rubi [A] time = 0.0311376, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4852, 364}

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}(cx)\right)}{d(m+1)}-\frac{bc(dx)^{m+2}{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{d^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyperg
eometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d^2*(1 + m)*(2 + m))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tan^{-1}(cx)\right)dx &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}(cx)\right)}{d(1+m)}-\frac{(bc)\int\frac{(dx)^{1+m}}{1+c^2x^2}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}(cx)\right)}{d(1+m)}-\frac{bc(dx)^{2+m}{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{d^2(1+m)(2+m)}\end{aligned}$$

Mathematica [A] time = 0.0313844, size = 60, normalized size = 0.82

$$-\frac{x(dx)^m\left(bcx\text{Hypergeometric2F1}\left(1,\frac{m}{2}+1,\frac{m}{2}+2,-c^2x^2\right)-(m+2)\left(a+b\tan^{-1}(cx)\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((x*(d*x)^m*(-((2 + m)*(a + b*ArcTan[c*x])) + b*c*x*Hypergeometric2F1[1, 1
 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)))
```

Maple [F] time = 1.434, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \arctan(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \arctan(cx) + a)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{atan}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan (cx) + a) (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)*(d*x)^m, x)
```

3.57 $\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\tan^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTan[c*x]), x]
```

Rubi [A] time = 0.0244613, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.253378, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x]), x]
```

Maple [A] time = 0.824, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x)),x)
```

```
[Out] int((d*x)^m/(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\arctan{(cx)} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{b\arctan{(cx)} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctan(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b\,\mathrm{atan}{(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x)),x)
```

```
[Out] Integral((d*x)**m/(a + b*atan(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\arctan{(cx)} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x) + a), x)
```

3.58 $\int \left(a+b\tan^{-1}(cx)\right)^p\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\left(a+b\tan^{-1}(cx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*ArcTan[c*x])^p, x]
```

Rubi [A] time = 0.0049583, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\tan^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Defer[Int][(a + b*ArcTan[c*x])^p, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(cx)\right)^p\,dx=\int \left(a+b\tan^{-1}(cx)\right)^p\,dx$$

Mathematica [A] time = 0.437439, size = 0, normalized size = 0.

$$\int \left(a+b\tan^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^p, x]
```

Maple [A] time = 0.41, size = 0, normalized size = 0.

$$\int (a+b\arctan(cx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^p,x)
```

```
[Out] int((a+b*arctan(c*x))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan (cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*arctan(c*x) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \arctan (cx) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)^p, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan} (cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**p,x)
```

```
[Out] Integral((a + b*atan(c*x))**p, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \arctan (cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^p, x)
```

3.59 $\int (dx)^m \left(a + b\tan^{-1}(cx)\right)^p \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((dx)^m \left(a + b\tan^{-1}(cx)\right)^p, x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTan[c*x])^p, x]
```

Rubi [A] time = 0.0213625, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a + b\tan^{-1}(cx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^p, x]
```

Rubi steps

$$\int (dx)^m \left(a + b\tan^{-1}(cx)\right)^p \, dx = \int (dx)^m \left(a + b\tan^{-1}(cx)\right)^p \, dx$$

Mathematica [A] time = 0.327939, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b\tan^{-1}(cx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p, x]
```

Maple [A] time = 0.986, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b\arctan(cx)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \arctan (cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \arctan (cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

3.60 $\int x^7 \left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=54

$$\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{bx^2}{8c^3}-\frac{b\tan^{-1}\left(cx^2\right)}{8c^4}-\frac{bx^6}{24c}$$

```
[Out] (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*ArcTan[c*x^2])/(8*c^4) + (x^8*(a + b*
ArcTan[c*x^2]))/8
```

Rubi [A] time = 0.0351121, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 302, 203}

$$\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{bx^2}{8c^3}-\frac{b\tan^{-1}\left(cx^2\right)}{8c^4}-\frac{bx^6}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*ArcTan[c*x^2])/(8*c^4) + (x^8*(a + b*
ArcTan[c*x^2]))/8
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+b\tan^{-1}\left(cx^2\right)\right)dx &= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\int\frac{x^9}{1+c^2x^4}\,dx\\ &= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\,\mathrm{Subst}\left(\int\frac{x^4}{1+c^2x^2}\,dx,x,x^2\right)\\ &= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\,\mathrm{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)dx,x,x^2\right)\\ &= \frac{bx^2}{8c^3}-\frac{bx^6}{24c}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{8c^3}\\ &= \frac{bx^2}{8c^3}-\frac{bx^6}{24c}-\frac{b\tan^{-1}\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0080472, size = 59, normalized size = 1.09

$$\frac{ax^8}{8}+\frac{bx^2}{8c^3}-\frac{b\tan^{-1}\left(cx^2\right)}{8c^4}-\frac{bx^6}{24c}+\frac{1}{8}bx^8\tan^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) - (b*x^6)/(24*c) + (a*x^8)/8 - (b*ArcTan[c*x^2])/(8*c^4) +
(b*x^8*ArcTan[c*x^2])/8
```

Maple [A] time = 0.024, size = 50, normalized size = 0.9

$$\frac{x^8a}{8}+\frac{bx^8\arctan\left(cx^2\right)}{8}-\frac{bx^6}{24\,c}+\frac{bx^2}{8\,c^3}-\frac{b\arctan\left(cx^2\right)}{8\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctan(c*x^2)-1/24*b*x^6/c+1/8*b*x^2/c^3-1/8*b*arctan(c
*x^2)/c^4
```

Maxima [A] time = 1.45766, size = 73, normalized size = 1.35

$$\frac{1}{8}\,ax^8+\frac{1}{24}\left(3\,x^8\arctan\left(cx^2\right)-c\left(\frac{c^2x^6-3\,x^2}{c^4}+\frac{3\,\arctan\left(cx^2\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/24*(3*x^8*arctan(c*x^2) - c*((c^2*x^6 - 3*x^2)/c^4 + 3*arctan
(c*x^2)/c^5))*b
```

Fricas [A] time = 2.62789, size = 111, normalized size = 2.06

$$\frac{3\,ac^4x^8 - bc^3x^6 + 3\,bcx^2 + 3\left(bc^4x^8 - b\right)\arctan\left(cx^2\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*a*c^4*x^8 - b*c^3*x^6 + 3*b*c*x^2 + 3*(b*c^4*x^8 - b)*arctan(c*x^2)
)/c^4
```

Sympy [A] time = 128.876, size = 58, normalized size = 1.07

$$\begin{cases} \frac{ax^8}{8} + \frac{bx^8\operatorname{atan}\left(cx^2\right)}{8} - \frac{bx^6}{24c} + \frac{bx^2}{8c^3} - \frac{b\operatorname{atan}\left(cx^2\right)}{8c^4} & \text{for}\ c \neq 0 \\ \frac{ax^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atan(c*x**2)/8 - b*x**6/(24*c) + b*x**2/(8*c**
3) - b*atan(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

Giac [A] time = 1.09653, size = 81, normalized size = 1.5

$$\frac{3\,acx^8 + \left(3\,cx^8\arctan\left(cx^2\right) - \frac{3\,\arctan\left(cx^2\right)}{c^3} - \frac{c^9x^6 - 3\,c^7x^2}{c^9}\right)b}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/24*(3*a*c*x^8 + (3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x^6 -
 3*c^7*x^2)/c^9)*b)/c
```

3.61 $\int x^5 \left(a+b\tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=47

$$\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(c^2x^4+1\right)}{12c^3}-\frac{bx^4}{12c}$$

```
[Out] -(b*x^4)/(12*c) + (x^6*(a + b*ArcTan[c*x^2]))/6 + (b*Log[1 + c^2*x^4])/(12*
c^3)
```

Rubi [A] time = 0.0308981, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5033, 266, 43}

$$\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(c^2x^4+1\right)}{12c^3}-\frac{bx^4}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -(b*x^4)/(12*c) + (x^6*(a + b*ArcTan[c*x^2]))/6 + (b*Log[1 + c^2*x^4])/(12*
c^3)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)dx &= \frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{3}(bc)\int\frac{x^7}{1+c^2x^4}\,dx\\
&= \frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc)\operatorname{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^4\right)\\
&= \frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc)\operatorname{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)dx,x,x^4\right)\\
&= -\frac{bx^4}{12c}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1+c^2x^4\right)}{12c^3}
\end{aligned}$$

Mathematica [A] time = 0.0132921, size = 52, normalized size = 1.11

$$\frac{ax^6}{6}+\frac{b\log\left(c^2x^4+1\right)}{12c^3}-\frac{bx^4}{12c}+\frac{1}{6}bx^6\tan^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -(b*x^4)/(12*c) + (a*x^6)/6 + (b*x^6*ArcTan[c*x^2])/6 + (b*Log[1 + c^2*x^4]
)/(12*c^3)
```

Maple [A] time = 0.024, size = 45, normalized size = 1.

$$\frac{x^6a}{6}+\frac{bx^6\arctan\left(cx^2\right)}{6}-\frac{bx^4}{12\,c}+\frac{b\ln\left(c^2x^4+1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctan(c*x^2)-1/12*b*x^4/c+1/12*b*ln(c^2*x^4+1)/c^3
```

Maxima [A] time = 0.983944, size = 65, normalized size = 1.38

$$\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\arctan\left(cx^2\right)-\left(\frac{x^4}{c^2}-\frac{\log\left(c^2x^4+1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*c)
*b
```

Fricas [A] time = 2.58199, size = 115, normalized size = 2.45

$$\frac{2\,bc^3x^6\arctan\left(cx^2\right)+2\,ac^3x^6-bc^2x^4+b\log\left(c^2x^4+1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b*c^3*x^6*arctan(c*x^2) + 2*a*c^3*x^6 - b*c^2*x^4 + b*log(c^2*x^4 +
 1))/c^3
```

Sympy [A] time = 76.1698, size = 80, normalized size = 1.7

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{atan}(cx^2)}{6} - \frac{bx^4}{12c} + \frac{b\log\left(x^2 + i\sqrt{\frac{1}{c^2}}\right)}{6c^3} + \frac{ib\operatorname{atan}(cx^2)}{6c^8\left(\frac{1}{c^2}\right)^{\frac{5}{2}}} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x**2)/6 - b*x**4/(12*c) + b*log(x**2 +
I*sqrt(c**(-2)))/(6*c**3) + I*b*atan(c*x**2)/(6*c**8*(c**(-2))**(5/2)), Ne(
c, 0)), (a*x**6/6, True))
```

Giac [A] time = 1.12515, size = 63, normalized size = 1.34

$$\frac{2\,acx^6 + \left(2\,cx^6 \arctan\left(cx^2\right) - x^4 + \frac{\log\left(c^2x^4+1\right)}{c^2}\right)b}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*(2*a*c*x^6 + (2*c*x^6*arctan(c*x^2) - x^4 + log(c^2*x^4 + 1)/c^2)*b)/c
```

3.62 $\int x^3 \left(a + b\tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=43

$$\frac{1}{4}x^4\left(a + b\tan^{-1}\left(cx^2\right)\right) + \frac{b\tan^{-1}\left(cx^2\right)}{4c^2} - \frac{bx^2}{4c}$$

```
[Out] -(b*x^2)/(4*c) + (b*ArcTan[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTan[c*x^2]))/4
```

Rubi [A] time = 0.026978, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 321, 203}

$$\frac{1}{4}x^4\left(a + b\tan^{-1}\left(cx^2\right)\right) + \frac{b\tan^{-1}\left(cx^2\right)}{4c^2} - \frac{bx^2}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -(b*x^2)/(4*c) + (b*ArcTan[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTan[c*x^2]))/4
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{2}(bc)\int\frac{x^5}{1+c^2x^4}\,dx\\ &= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\operatorname{Subst}\left(\int\frac{x^2}{1+c^2x^2}\,dx,x,x^2\right)\\ &= -\frac{bx^2}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\operatorname{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{4c}\\ &= -\frac{bx^2}{4c}+\frac{b\tan^{-1}\left(cx^2\right)}{4c^2}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0053972, size = 48, normalized size = 1.12

$$\frac{ax^4}{4}+\frac{b\tan^{-1}\left(cx^2\right)}{4c^2}-\frac{bx^2}{4c}+\frac{1}{4}bx^4\tan^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -(b*x^2)/(4*c) + (a*x^4)/4 + (b*ArcTan[c*x^2])/(4*c^2) + (b*x^4*ArcTan[c*x^
2])/4
```

Maple [A] time = 0.025, size = 41, normalized size = 1.

$$\frac{x^4a}{4}+\frac{bx^4\arctan\left(cx^2\right)}{4}-\frac{bx^2}{4c}+\frac{b\arctan\left(cx^2\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctan(c*x^2)-1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2
```

Maxima [A] time = 1.51948, size = 58, normalized size = 1.35

$$\frac{1}{4}ax^4+\frac{1}{4}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/4*(x^4*arctan(c*x^2) - c*(x^2/c^2 - arctan(c*x^2)/c^3))*b
```

Fricas [A] time = 2.60519, size = 85, normalized size = 1.98

$$\frac{ac^2x^4-bcx^2+\left(bc^2x^4+b\right)\arctan\left(cx^2\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*c^2*x^4 - b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2))/c^2
```

Sympy [A] time = 40.6319, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atan}\left(cx^2\right)}{4} - \frac{bx^2}{4c} + \frac{b \operatorname{atan}\left(cx^2\right)}{4c^2} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x**2)/4 - b*x**2/(4*c) + b*atan(c*x**2)
/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

Giac [A] time = 1.15399, size = 58, normalized size = 1.35

$$\frac{acx^4 + \frac{\left(c^2x^4 \arctan\left(cx^2\right) - cx^2 + \arctan\left(cx^2\right)\right)b}{c}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/4*(a*c*x^4 + (c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*b/c)/c
```

3.63 $\int x \left(a+b \tan^{-1}\left(c x^2\right)\right) \, dx$

Optimal. Leaf size=36

$$\frac{1}{2} x^2 \left(a+b \tan^{-1}\left(c x^2\right)\right)-\frac{b \log \left(c^2 x^4+1\right)}{4 c}$$

```
[Out] (x^2*(a + b*ArcTan[c*x^2]))/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Rubi [A] time = 0.01395, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 260}

$$\frac{1}{2} x^2 \left(a+b \tan^{-1}\left(c x^2\right)\right)-\frac{b \log \left(c^2 x^4+1\right)}{4 c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (x^2*(a + b*ArcTan[c*x^2]))/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int x \left(a+b \tan^{-1}\left(c x^2\right)\right) \, dx &= \frac{1}{2} x^2 \left(a+b \tan^{-1}\left(c x^2\right)\right)-(b c) \int \frac{x^3}{1+c^2 x^4} \, dx\\ &= \frac{1}{2} x^2 \left(a+b \tan^{-1}\left(c x^2\right)\right)-\frac{b \log \left(1+c^2 x^4\right)}{4 c} \end{aligned}$$

Mathematica [A] time = 0.0064483, size = 41, normalized size = 1.14

$$\frac{a x^2}{2}-\frac{b \log \left(c^2 x^4+1\right)}{4 c}+\frac{1}{2} b x^2 \tan^{-1}\left(c x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTan[c*x^2])/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Maple [A] time = 0.019, size = 36, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^2 \arctan(cx^2)}{2} - \frac{b \ln(c^2x^4 + 1)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctan(c*x^2)-1/4*b*ln(c^2*x^4+1)/c
```

Maxima [A] time = 1.02717, size = 51, normalized size = 1.42

$$\frac{1}{2}ax^2 + \frac{(2\,cx^2 \arctan(cx^2) - \log(c^2x^4 + 1))b}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b/c
```

Fricas [A] time = 2.93178, size = 89, normalized size = 2.47

$$\frac{2\,bcx^2 \arctan(cx^2) + 2\,acx^2 - b \log(c^2x^4 + 1)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^2*arctan(c*x^2) + 2*a*c*x^2 - b*log(c^2*x^4 + 1))/c
```

Sympy [A] time = 20.8985, size = 70, normalized size = 1.94

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{atan}(cx^2)}{2} - \frac{b \log\left(x^2 + i\sqrt{\frac{1}{c^2}}\right)}{2c} - \frac{ib \operatorname{atan}(cx^2)}{2c^4\left(\frac{1}{c^2}\right)^{\frac{3}{2}}} & \text{for}\ c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x**2)/2 - b*log(x**2 + I*sqrt(c**(-2)))
/(2*c) - I*b*atan(c*x**2)/(2*c**4*(c**(-2))**(3/2)), Ne(c, 0)), (a*x**2/2,
True))
```

Giac [A] time = 1.15435, size = 54, normalized size = 1.5

$$\frac{2\,acx^2 + \left(2\,cx^2 \arctan\left(cx^2\right) - \log\left(c^2x^4 + 1\right)\right)b}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*c*x^2 + (2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b)/c
```

3.64 $\int \frac{a+b\tan^{-1}(cx^2)}{x}\,dx$

Optimal. Leaf size=39

$$\frac{1}{4}ib\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{4}ib\text{PolyLog}\left(2,icx^2\right)+a\log(x)$$

```
[Out] a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

Rubi [A] time = 0.0454425, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5031, 4848, 2391}

$$\frac{1}{4}ib\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{4}ib\text{PolyLog}\left(2,icx^2\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x,x]
```

```
[Out] a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{a+b\tan^{-1}(cx)}{x}\,dx,x,x^2\right)\\
&= a\log(x)+\frac{1}{4}(ib)\,\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,x^2\right)-\frac{1}{4}(ib)\,\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx,x,x^2\right)\\
&= a\log(x)+\frac{1}{4}ib\text{Li}_2\left(-icx^2\right)-\frac{1}{4}ib\text{Li}_2\left(icx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0047769, size = 39, normalized size = 1.

$$\frac{1}{4}ib\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{4}ib\text{PolyLog}\left(2,icx^2\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x,x]
```

```
[Out] a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

Maple [C] time = 0.082, size = 63, normalized size = 1.6

$$a\ln(x)+b\ln(x)\arctan\left(cx^2\right)-\frac{b}{2c}\sum_{\_R1=\mathrm{RootOf}\left(c^2\_Z^4+1\right)}\frac{1}{\_R1^2}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x,x)
```

```
[Out] a*ln(x)+b*ln(x)*arctan(c*x^2)-1/2*b/c*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+di
log((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\arctan\left(cx^2\right)}{x}\,dx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x^2)/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\arctan\left(cx^2\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^2) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{atan}\left(cx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan\left(cx^2\right) + a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)/x, x)
```

3.65 $\int \frac{a+b\tan^{-1}(cx^2)}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2}-\frac{1}{4}bc\log\left(c^2x^4+1\right)+bc\log(x)$$

```
[Out] -(a + b*ArcTan[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 + c^2*x^4])/4
```

Rubi [A] time = 0.0236941, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5033, 266, 36, 29, 31}

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2}-\frac{1}{4}bc\log\left(c^2x^4+1\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^3,x]
```

```
[Out] -(a + b*ArcTan[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 + c^2*x^4])/4
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^3}\,dx = -\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2}+(bc)\int \frac{1}{x\left(1+c^2x^4\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2}+\frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx,x,x^4\right)$$
$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2}+\frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)-\frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx,x,x^4\right)$$
$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)$$

Mathematica [A] time = 0.0062847, size = 44, normalized size = 1.13

$$-\frac{a}{2x^2}-\frac{1}{4}bc\log\left(c^2x^4+1\right)-\frac{b\tan^{-1}\left(cx^2\right)}{2x^2}+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*ArcTan[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 + c^2*x^4]
)/4
```

Maple [A] time = 0.027, size = 39, normalized size = 1.

$$-\frac{a}{2\,x^2}-\frac{b\arctan\left(cx^2\right)}{2\,x^2}-\frac{bc\ln\left(c^2x^4+1\right)}{4}+bc\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c*x^2)-1/4*b*c*ln(c^2*x^4+1)+b*c*ln(x)
```

Maxima [A] time = 0.998128, size = 55, normalized size = 1.41

$$-\frac{1}{4}\left(c\left(\log\left(c^2x^4+1\right)-\log\left(x^4\right)\right)+\frac{2\arctan\left(cx^2\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*b - 1/2*a/x^2
```

Fricas [A] time = 2.74874, size = 111, normalized size = 2.85

$$-\frac{bcx^2\log\left(c^2x^4+1\right)-4\,bcx^2\log(x)+2\,b\arctan\left(cx^2\right)+2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2*log(c^2*x^4 + 1) - 4*b*c*x^2*log(x) + 2*b*arctan(c*x^2) + 2*a
)/x^2
```

Sympy [A] time = 45.4078, size = 593, normalized size = 15.21

$$\begin{cases} -\frac{a}{2x^2} \\ -\frac{a-\infty ib}{2x^2} \\ -\frac{a+\infty ib}{2x^2} \\ -\frac{iac^2x^4\sqrt{\frac{1}{c^2}}}{2ic^2x^6\sqrt{\frac{1}{c^2}}+2ix^2\sqrt{\frac{1}{c^2}}} - \frac{ia\sqrt{\frac{1}{c^2}}}{2ic^2x^6\sqrt{\frac{1}{c^2}}+2ix^2\sqrt{\frac{1}{c^2}}} + \frac{2ibc^3x^6\sqrt{\frac{1}{c^2}}\log(x)}{2ic^2x^6\sqrt{\frac{1}{c^2}}+2ix^2\sqrt{\frac{1}{c^2}}} - \frac{ibc^3x^6\sqrt{\frac{1}{c^2}}\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{2ic^2x^6\sqrt{\frac{1}{c^2}}+2ix^2\sqrt{\frac{1}{c^2}}} + \frac{bc^2x^6\operatorname{atan}(cx^2)}{2ic^2x^6\sqrt{\frac{1}{c^2}}+2ix^2\sqrt{\frac{1}{c^2}}} - \frac{ibc^2x^4\sqrt{\frac{1}{c^2}}\operatorname{ata}}{2ic^2x^6\sqrt{\frac{1}{c^2}}+2i} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2), Eq(c, 0)), (-(a - oo*I*b)/(2*x**2), Eq(c, -I/x**2))
, (-(a + oo*I*b)/(2*x**2), Eq(c, I/x**2)), (-I*a*c**2*x**4*sqrt(c**(-2))/(2
*I*c**2*x**6*sqrt(c**(-2)) + 2*I*x**2*sqrt(c**(-2))) - I*a*sqrt(c**(-2))/(2
*I*c**2*x**6*sqrt(c**(-2)) + 2*I*x**2*sqrt(c**(-2))) + 2*I*b*c**3*x**6*sqrt
(c**(-2))*log(x)/(2*I*c**2*x**6*sqrt(c**(-2)) + 2*I*x**2*sqrt(c**(-2))) - I
*b*c**3*x**6*sqrt(c**(-2))*log(x**2 + I*sqrt(c**(-2)))/(2*I*c**2*x**6*sqrt(
c**(-2)) + 2*I*x**2*sqrt(c**(-2))) + b*c**2*x**6*atan(c*x**2)/(2*I*c**2*x**
6*sqrt(c**(-2)) + 2*I*x**2*sqrt(c**(-2))) - I*b*c**2*x**4*sqrt(c**(-2))*ata
n(c*x**2)/(2*I*c**2*x**6*sqrt(c**(-2)) + 2*I*x**2*sqrt(c**(-2))) + 2*I*b*c*
x**2*sqrt(c**(-2))*log(x)/(2*I*c**2*x**6*sqrt(c**(-2)) + 2*I*x**2*sqrt(c**(
-2))) - I*b*c*x**2*sqrt(c**(-2))*log(x**2 + I*sqrt(c**(-2)))/(2*I*c**2*x**6
*sqrt(c**(-2)) + 2*I*x**2*sqrt(c**(-2))) + b*x**2*atan(c*x**2)/(2*I*c**2*x*
*6*sqrt(c**(-2)) + 2*I*x**2*sqrt(c**(-2))) - I*b*sqrt(c**(-2))*atan(c*x**2)
/(2*I*c**2*x**6*sqrt(c**(-2)) + 2*I*x**2*sqrt(c**(-2))), True))
```

Giac [A] time = 1.22596, size = 81, normalized size = 2.08

$$-\frac{bc^3x^2\log\left(c^2x^4+1\right) - 2\,bc^3x^2\log\left(cx^2\right) + 2\,bc^2\arctan\left(cx^2\right) + 2\,ac^2}{4\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*c^3*x^2*log(c^2*x^4 + 1) - 2*b*c^3*x^2*log(c*x^2) + 2*b*c^2*arctan(
c*x^2) + 2*a*c^2)/(c^2*x^2)
```

3.66 $\int \frac{a+b\tan^{-1}(cx^2)}{x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{a+b\tan^{-1}(cx^2)}{4x^4}-\frac{1}{4}bc^2\tan^{-1}(cx^2)-\frac{bc}{4x^2}$$

```
[Out] -(b*c)/(4*x^2) - (b*c^2*ArcTan[c*x^2])/4 - (a + b*ArcTan[c*x^2])/(4*x^4)
```

Rubi [A] time = 0.0247925, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 325, 203}

$$-\frac{a+b\tan^{-1}(cx^2)}{4x^4}-\frac{1}{4}bc^2\tan^{-1}(cx^2)-\frac{bc}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^5,x]
```

```
[Out] -(b*c)/(4*x^2) - (b*c^2*ArcTan[c*x^2])/4 - (a + b*ArcTan[c*x^2])/(4*x^4)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^5}\,dx &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{2}(bc)\int \frac{1}{x^3\left(1+c^2x^4\right)}\,dx \\
&= -\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx,x,x^2\right) \\
&= -\frac{bc}{4x^2}-\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}-\frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx,x,x^2\right) \\
&= -\frac{bc}{4x^2}-\frac{1}{4}bc^2\tan^{-1}\left(cx^2\right)-\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}
\end{aligned}
$$

Mathematica [C] time = 0.0067644, size = 48, normalized size = 1.17

$$
-\frac{bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^4\right)}{4x^2}-\frac{a}{4x^4}-\frac{b\tan^{-1}\left(cx^2\right)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^5,x]
```

```
[Out] -a/(4*x^4) - (b*ArcTan[c*x^2])/(4*x^4) - (b*c*Hypergeometric2F1[-1/2, 1, 1/
2, -(c^2*x^4)])/(4*x^2)
```

Maple [A] time = 0.024, size = 39, normalized size = 1.

$$
-\frac{a}{4\,x^4}-\frac{b\arctan\left(cx^2\right)}{4\,x^4}-\frac{bc}{4\,x^2}-\frac{bc^2\arctan\left(cx^2\right)}{4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctan(c*x^2)-1/4*b*c/x^2-1/4*b*c^2*arctan(c*x^2)
```

Maxima [A] time = 1.50057, size = 47, normalized size = 1.15

$$
-\frac{1}{4}\left(\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c+\frac{\arctan\left(cx^2\right)}{x^4}\right)b-\frac{a}{4\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*b - 1/4*a/x^4
```

Fricas [A] time = 2.68126, size = 76, normalized size = 1.85

$$
-\frac{bcx^2+\left(bc^2x^4+b\right)\arctan\left(cx^2\right)+a}{4\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2) + a)/x^4
```

Sympy [A] time = 36.2676, size = 42, normalized size = 1.02

$$-\frac{a}{4x^4}-\frac{bc^2\operatorname{atan}\left(cx^2\right)}{4}-\frac{bc}{4x^2}-\frac{b\operatorname{atan}\left(cx^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**5,x)
```

```
[Out] -a/(4*x**4) - b*c**2*atan(c*x**2)/4 - b*c/(4*x**2) - b*atan(c*x**2)/(4*x**4
)
```

Giac [B] time = 1.20103, size = 100, normalized size = 2.44

$$\frac{bc^5ix^4\log\left(cix^2+1\right)-bc^5ix^4\log\left(-cix^2+1\right)-2\,bc^4x^2-2\,bc^3\arctan\left(cx^2\right)-2\,ac^3}{8\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(b*c^5*i*x^4*log(c*i*x^2 + 1) - b*c^5*i*x^4*log(-c*i*x^2 + 1) - 2*b*c^4
*x^2 - 2*b*c^3*arctan(c*x^2) - 2*a*c^3)/(c^3*x^4)
```

3.67 $\int \frac{a+b\tan^{-1}(cx^2)}{x^7}\,dx$

Optimal. Leaf size=55

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{12}bc^3\log\left(c^2x^4+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{12x^4}$$

```
[Out] -(b*c)/(12*x^4) - (a + b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^4])/12
```

Rubi [A] time = 0.0322249, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5033, 266, 44}

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{12}bc^3\log\left(c^2x^4+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^7,x]
```

```
[Out] -(b*c)/(12*x^4) - (a + b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^4])/12
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^7}\,dx = -\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6} + \frac{1}{3}(bc)\int \frac{1}{x^5\left(1+c^2x^4\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6} + \frac{1}{12}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x\right)}\,dx, x, x^4\right)$$
$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6} + \frac{1}{12}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx, x, x^4\right)$$
$$= -\frac{bc}{12x^4} - \frac{a+b\tan^{-1}\left(cx^2\right)}{6x^6} - \frac{1}{3}bc^3\log(x) + \frac{1}{12}bc^3\log\left(1+c^2x^4\right)$$

Mathematica [A] time = 0.0137974, size = 60, normalized size = 1.09

$$-\frac{a}{6x^6} + \frac{1}{12}bc^3\log\left(c^2x^4+1\right) - \frac{1}{3}bc^3\log(x) - \frac{bc}{12x^4} - \frac{b\tan^{-1}\left(cx^2\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^7,x]
```

```
[Out] -a/(6*x^6) - (b*c)/(12*x^4) - (b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3
+ (b*c^3*Log[1 + c^2*x^4])/12
```

Maple [A] time = 0.029, size = 51, normalized size = 0.9

$$-\frac{a}{6\,x^6} - \frac{b\arctan\left(cx^2\right)}{6\,x^6} + \frac{bc^3\ln\left(c^2x^4+1\right)}{12} - \frac{bc}{12\,x^4} - \frac{bc^3\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctan(c*x^2)+1/12*b*c^3*ln(c^2*x^4+1)-1/12*b*c/x^4-1/
3*b*c^3*ln(x)
```

Maxima [A] time = 1.00167, size = 72, normalized size = 1.31

$$\frac{1}{12}\left(\left(c^2\log\left(c^2x^4+1\right) - c^2\log\left(x^4\right) - \frac{1}{x^4}\right)c - \frac{2\arctan\left(cx^2\right)}{x^6}\right)b - \frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*((c^2*log(c^2*x^4 + 1) - c^2*log(x^4) - 1/x^4)*c - 2*arctan(c*x^2)/x^6
)*b - 1/6*a/x^6
```

Fricas [A] time = 2.82218, size = 130, normalized size = 2.36

$$\frac{bc^3x^6\log\left(c^2x^4+1\right)-4\,bc^3x^6\log\left(x\right)-bcx^2-2\,b\arctan\left(cx^2\right)-2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*x^6*log(c^2*x^4 + 1) - 4*b*c^3*x^6*log(x) - b*c*x^2 - 2*b*arcta
n(c*x^2) - 2*a)/x^6
```

Sympy [A] time = 140.139, size = 784, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**7,x)
```

```
[Out] Piecewise(((-(a - oo*I*b)/(6*x**6), Eq(c, -I/x**2)), (-(a + oo*I*b)/(6*x**6)
, Eq(c, I/x**2)), (-a/(6*x**6), Eq(c, 0)), (2*I*a*c**6*x**4*(c**(-2))**(7/2
)/(-12*I*c**6*x**10*(c**(-2))**(7/2) - 12*I*c**4*x**6*(c**(-2))**(7/2)) + 2
*I*a*c**4*(c**(-2))**(7/2)/(-12*I*c**6*x**10*(c**(-2))**(7/2) - 12*I*c**4*x
**6*(c**(-2))**(7/2)) + 4*I*b*c**9*x**10*(c**(-2))**(7/2)*log(x)/(-12*I*c**
6*x**10*(c**(-2))**(7/2) - 12*I*c**4*x**6*(c**(-2))**(7/2)) - 2*I*b*c**9*x*
*10*(c**(-2))**(7/2)*log(x**2 + I*sqrt(c**(-2)))/(-12*I*c**6*x**10*(c**(-2)
)**(7/2) - 12*I*c**4*x**6*(c**(-2))**(7/2)) - I*b*c**9*x**10*(c**(-2))**(7/
2)/(-12*I*c**6*x**10*(c**(-2))**(7/2) - 12*I*c**4*x**6*(c**(-2))**(7/2)) +
4*I*b*c**7*x**6*(c**(-2))**(7/2)*log(x)/(-12*I*c**6*x**10*(c**(-2))**(7/2)
- 12*I*c**4*x**6*(c**(-2))**(7/2)) - 2*I*b*c**7*x**6*(c**(-2))**(7/2)*log(x
**2 + I*sqrt(c**(-2)))/(-12*I*c**6*x**10*(c**(-2))**(7/2) - 12*I*c**4*x**6*
(c**(-2))**(7/2)) + 2*I*b*c**6*x**4*(c**(-2))**(7/2)*atan(c*x**2)/(-12*I*c*
*6*x**10*(c**(-2))**(7/2) - 12*I*c**4*x**6*(c**(-2))**(7/2)) + I*b*c**5*x**
2*(c**(-2))**(7/2)/(-12*I*c**6*x**10*(c**(-2))**(7/2) - 12*I*c**4*x**6*(c**
(-2))**(7/2)) + 2*I*b*c**4*(c**(-2))**(7/2)*atan(c*x**2)/(-12*I*c**6*x**10*
(c**(-2))**(7/2) - 12*I*c**4*x**6*(c**(-2))**(7/2)) + 2*b*c**2*x**10*atan(c
*x**2)/(-12*I*c**6*x**10*(c**(-2))**(7/2) - 12*I*c**4*x**6*(c**(-2))**(7/2)
) + 2*b*x**6*atan(c*x**2)/(-12*I*c**6*x**10*(c**(-2))**(7/2) - 12*I*c**4*x*
*6*(c**(-2))**(7/2)), True))
```

Giac [A] time = 1.14157, size = 93, normalized size = 1.69

$$\frac{bc^7x^6\log\left(c^2x^4+1\right)-2\,bc^7x^6\log\left(cx^2\right)-bc^5x^2-2\,bc^4\arctan\left(cx^2\right)-2\,ac^4}{12\,c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(b*c^7*x^6*log(c^2*x^4 + 1) - 2*b*c^7*x^6*log(c*x^2) - b*c^5*x^2 - 2*b
*c^4*arctan(c*x^2) - 2*a*c^4)/(c^4*x^6)
```

3.68 $\int x^4 \left(a + b \tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=161

$$\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}c^{5/2}}-\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}c^{5/2}}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}c^{5/2}}+\frac{b\tan^{-1}\left(\sqrt{2}\sqrt{c}\right.}{5\sqrt{2}c^{5/2}}$$

```
[Out] (-2*b*x^3)/(15*c) + (x^5*(a + b*ArcTan[c*x^2]))/5 - (b*ArcTan[1 - Sqrt[2]*S
qrt[c]*x])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(5*Sqrt[
2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2)) -
 (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2))
```

Rubi [A] time = 0.112558, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 321, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}c^{5/2}}-\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}c^{5/2}}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}c^{5/2}}+\frac{b\tan^{-1}\left(\sqrt{2}\sqrt{c}\right.}{5\sqrt{2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x^3)/(15*c) + (x^5*(a + b*ArcTan[c*x^2]))/5 - (b*ArcTan[1 - Sqrt[2]*S
qrt[c]*x])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(5*Sqrt[
2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2)) -
 (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2))
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)dx &= \frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{5}(2bc)\int\frac{x^6}{1+c^2x^4}\,dx\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{(2b)\int\frac{x^2}{1+c^2x^4}\,dx}{5c}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\int\frac{1-cx^2}{1+c^2x^4}\,dx}{5c^2}+\frac{b\int\frac{1+cx^2}{1+c^2x^4}\,dx}{5c^2}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{10c^3}+\frac{b\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{10c^3}+\frac{b\int\frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x\ldots}}{10\sqrt{2}c^{5/2}\ldots}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}c^{5/2}}-\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^{2}\right)}{10\sqrt{2}c^{5/2}}\ldots\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}c^{5/2}}+\frac{b\tan^{-1}\left(1+\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}c^{5/2}}+\frac{b\,\mathrm{l}\ldots}{\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.0482874, size = 179, normalized size = 1.11

$$
\frac{ax^5}{5}+\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}c^{5/2}}-\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}c^{5/2}}+\frac{b\tan^{-1}\left(\frac{2\sqrt{c}x-\sqrt{2}}{\sqrt{2}}\right)}{5\sqrt{2}c^{5/2}}+\frac{b\tan^{-1}\left(\frac{2\sqrt{c}x+\sqrt{2}}{\sqrt{2}}\right)}{5\sqrt{2}c^{5/2}}-\frac{2bx^3}{15c}+\frac{1}{5}bx^5\ldots
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x^3)/(15*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^2])/5 + (b*ArcTan[(-Sqrt[
2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[(Sqrt[2] + 2*Sq
rt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x
^2])/(10*Sqrt[2]*c^(5/2)) - (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt
[2]*c^(5/2))
```

Maple [A] time = 0.037, size = 140, normalized size = 0.9

$$\frac{ax^5}{5}+\frac{bx^5\arctan\left(cx^2\right)}{5}-\frac{2\,bx^3}{15\,c}+\frac{b\sqrt{2}}{20\,c^3}\ln\left(\left(x^2-\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)\left(x^2+\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)^{-1}\right)\frac{1}{\sqrt[4]{c^{-2}}}+\frac{b\sqrt{2}}{10\,c^3}\arctan\Bigg(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctan(c*x^2)-2/15*b*x^3/c+1/20*b/c^3/(1/c^2)^(1/4)*2^(
1/2)*ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^
(1/2)+(1/c^2)^(1/2)))+1/10*b/c^3/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^
2)^(1/4)*x+1)+1/10*b/c^3/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)
*x-1)
```

Maxima [B] time = 1.52734, size = 375, normalized size = 2.33

$$\frac{1}{5}\,ax^5+\frac{1}{60}\left(12\,x^5\arctan\left(cx^2\right)-c\left(\frac{8\,x^3}{c^2}+\frac{3\left(\frac{\sqrt{2}\log\left(\sqrt{c^2}x^2+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c^2}x^2-\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\frac{2\sqrt{c^2}x-\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)\cdots}{2\sqrt{c^2}x+\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)\cdots}\right)}{\sqrt{c^2}\sqrt{-\sqrt{c^2}}}\cdots\right)}{c^2}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/60*(12*x^5*arctan(c*x^2) - c*(8*x^3/c^2 + 3*(sqrt(2)*log(sqrt
(c^2)*x^2 + sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(3/4) - sqrt(2)*log(sqrt(c^2)*
x^2 - sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(3/4) - sqrt(2)*log((2*sqrt(c^2)*x -
 sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*s
qrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4)))/(sqrt(c^2)*sqrt(-sqrt(c^2))) - sqrt
(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4))/(2
*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4)))/(sqrt(c^2)*
sqrt(-sqrt(c^2))))/c^2))*b
```

Fricas [B] time = 2.85234, size = 894, normalized size = 5.55

$$\frac{12\,bcx^5\arctan\left(cx^2\right)+12\,acx^5-8\,bx^3-12\,\sqrt{2}c\left(\frac{b^4}{c^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}b^3c^3x\left(\frac{b^4}{c^{10}}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{\sqrt{2}b^3c^7x\left(\frac{b^4}{c^{10}}\right)^{\frac{3}{4}}+b^4c^4\sqrt{\frac{b^4}{c^{10}}}+b^6x^2c^3\left(\frac{b^4}{c^{10}}\right)^{\frac{1}{4}}+b^4}}{b^4}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b*c*x^5*arctan(c*x^2) + 12*a*c*x^5 - 8*b*x^3 - 12*sqrt(2)*c*(b^4/c
^10)^(1/4)*arctan(-(sqrt(2)*b^3*c^3*x*(b^4/c^10)^(1/4) - sqrt(2)*sqrt(sqrt(
2)*b^3*c^7*x*(b^4/c^10)^(3/4) + b^4*c^4*sqrt(b^4/c^10) + b^6*x^2)*c^3*(b^4/
c^10)^(1/4) + b^4)/b^4) - 12*sqrt(2)*c*(b^4/c^10)^(1/4)*arctan(-(sqrt(2)*b^
3*c^3*x*(b^4/c^10)^(1/4) - sqrt(2)*sqrt(-sqrt(2)*b^3*c^7*x*(b^4/c^10)^(3/4)
 + b^4*c^4*sqrt(b^4/c^10) + b^6*x^2)*c^3*(b^4/c^10)^(1/4) - b^4)/b^4) - 3*s
qrt(2)*c*(b^4/c^10)^(1/4)*log(sqrt(2)*b^3*c^7*x*(b^4/c^10)^(3/4) + b^4*c^4*
sqrt(b^4/c^10) + b^6*x^2) + 3*sqrt(2)*c*(b^4/c^10)^(1/4)*log(-sqrt(2)*b^3*c
^7*x*(b^4/c^10)^(3/4) + b^4*c^4*sqrt(b^4/c^10) + b^6*x^2))/c
```

Sympy [A] time = 55.2367, size = 184, normalized size = 1.14

$$\begin{cases} \frac{ax^5}{5} + \frac{bx^5 \operatorname{atan}\left(cx^2\right)}{5} - \frac{2bx^3}{15c} - \frac{\sqrt[4]{-1}b \operatorname{atan}\left(cx^2\right)}{5c^8\left(\frac{1}{c^2}\right)^{\frac{11}{4}}} - \frac{(-1)^{\frac{3}{4}} b \log\left(x - \sqrt[4]{-1}\sqrt[4]{\frac{1}{c^2}}\right)}{5c^{13}\left(\frac{1}{c^2}\right)^{\frac{21}{4}}} + \frac{(-1)^{\frac{3}{4}} b \log\left(x^2 + i\sqrt{\frac{1}{c^2}}\right)}{10c^{13}\left(\frac{1}{c^2}\right)^{\frac{21}{4}}} + \frac{(-1)^{\frac{3}{4}} b \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}x}{\sqrt[4]{\frac{1}{c^2}}}\right)}{5c^{13}\left(\frac{1}{c^2}\right)^{\frac{21}{4}}} & \text{for } c \neq 0 \\ \frac{ax^5}{5} & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atan(c*x**2)/5 - 2*b*x**3/(15*c) - (-1)**(1/4)
*b*atan(c*x**2)/(5*c**8*(c**(-2))**(11/4)) - (-1)**(3/4)*b*log(x - (-1)**(1
/4)*(c**(-2))**(1/4))/(5*c**13*(c**(-2))**(21/4)) + (-1)**(3/4)*b*log(x**2
+ I*sqrt(c**(-2)))/(10*c**13*(c**(-2))**(21/4)) + (-1)**(3/4)*b*atan((-1)**
(3/4)*x/(c**(-2))**(1/4))/(5*c**13*(c**(-2))**(21/4)), Ne(c, 0)), (a*x**5/5
, True))
```

Giac [A] time = 1.30486, size = 228, normalized size = 1.42

$$\frac{1}{20}bc^9\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}} + \frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}} - \frac{\sqrt{2}\sqrt{|c|}\log\left(x^2 + \frac{\sqrt{2}x}{\sqrt{|c|}} + \frac{1}{|c}\right.}{c^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/20*b*c^9*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/c^12 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^12 - sqrt(2)*sqrt(abs(c))*log(x^2
+ sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^12 + sqrt(2)*sqrt(abs(c))*log(x^2 -
sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^12) + 1/15*(3*b*c*x^5*arctan(c*x^2) +
3*a*c*x^5 - 2*b*x^3)/c
```

3.69 $\int x^2 \left(a+b\tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=159

$$\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}+\frac{b\tan^{-1}\left(\sqrt{2}\sqrt{c}\right.}{3\sqrt{2}c^{3/2}}$$

```
[Out] (-2*b*x)/(3*c) + (x^3*(a + b*ArcTan[c*x^2]))/3 - (b*ArcTan[1 - Sqrt[2]*Sqrt
[c]*x])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*Sqrt[2]*
c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2)) + (b*
Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2))
```

Rubi [A] time = 0.10147, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}+\frac{b\tan^{-1}\left(\sqrt{2}\sqrt{c}\right.}{3\sqrt{2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x)/(3*c) + (x^3*(a + b*ArcTan[c*x^2]))/3 - (b*ArcTan[1 - Sqrt[2]*Sqrt
[c]*x])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*Sqrt[2]*
c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2)) + (b*
Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2))
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{3}(2bc)\int\frac{x^4}{1+c^2x^4}\,dx\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{(2b)\int\frac{1}{1+c^2x^4}\,dx}{3c}\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\int\frac{1-cx^2}{1+c^2x^4}\,dx}{3c}+\frac{b\int\frac{1+cx^2}{1+c^2x^4}\,dx}{3c}\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{6c^2}+\frac{b\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{6c^2}-\frac{b\int\frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}}{6\sqrt{2}c^{3/2}}\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right.}{6\sqrt{2}c^{3/2}}\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}+\frac{b\tan^{-1}\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}-\frac{b\,\mathrm{lo}}{}
\end{aligned}
$$

Mathematica [A] time = 0.0334661, size = 177, normalized size = 1.11

$$
\frac{ax^3}{3}-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}}+\frac{b\tan^{-1}\left(\frac{2\sqrt{c}x-\sqrt{2}}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}}+\frac{b\tan^{-1}\left(\frac{2\sqrt{c}x+\sqrt{2}}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}}+\frac{1}{3}bx^3\tan^{-1}\left(\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x)/(3*c) + (a*x^3)/3 + (b*x^3*ArcTan[c*x^2])/3 + (b*ArcTan[(-Sqrt[2]
+ 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[(Sqrt[2] + 2*Sqrt[
c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(6*Sqrt[2]*c^(3/2)) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c
^(3/2))
```

Maple [A] time = 0.026, size = 138, normalized size = 0.9

$$\frac{x^3a}{3}+\frac{bx^3\arctan\left(cx^2\right)}{3}-\frac{2\,bx}{3\,c}+\frac{b\sqrt{2}}{12\,c}\sqrt[4]{c^{-2}}\ln\left(\left(x^2+\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)\left(x^2-\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)^{-1}\right)+\frac{b\sqrt{2}}{6\,c}\sqrt[4]{c^{-2}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctan(c*x^2)-2/3*b*x/c+1/12*b/c*(1/c^2)^(1/4)*2^(1/2)*
ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)
+(1/c^2)^(1/2)))+1/6*b/c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)
*x+1)+1/6*b/c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)
```

Maxima [B] time = 1.53196, size = 354, normalized size = 2.23

$$\frac{1}{3}\,ax^3+\frac{1}{12}\left(4\,x^3\arctan\left(cx^2\right)+c\left(\frac{\frac{\sqrt{2}\log\left(\sqrt{c^2}x^2+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c^2}x^2-\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(\frac{2\sqrt{c^2}x-\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}}{2\sqrt{c^2}x+\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}}\right)}{\sqrt{-\sqrt{c^2}}}+\frac{\sqrt{2}\log}{}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arctan(c*x^2) + c*((sqrt(2)*log(sqrt(c^2)*x^2 + sqr
t(2)*(c^2)^(1/4)*x + 1)/(c^2)^(1/4) - sqrt(2)*log(sqrt(c^2)*x^2 - sqrt(2)*(
c^2)^(1/4)*x + 1)/(c^2)^(1/4) + sqrt(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-
sqrt(c^2)) + sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2))
 + sqrt(2)*(c^2)^(1/4)))/sqrt(-sqrt(c^2)) + sqrt(2)*log((2*sqrt(c^2)*x - sq
rt(2)*sqrt(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt
(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4)))/sqrt(-sqrt(c^2)))/c^2 - 8*x/c^2))*b
```

Fricas [B] time = 2.80276, size = 807, normalized size = 5.08

$$\frac{4\,bcx^3\arctan\left(cx^2\right)+4\,acx^3-4\,\sqrt{2}c\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}bc^5x\left(\frac{b^4}{c^6}\right)^{\frac{3}{4}}-\sqrt{2}\sqrt{b^2x^2+\sqrt{2}bcx\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}+c^2\sqrt{\frac{b^4}{c^6}}}\,c^5\left(\frac{b^4}{c^6}\right)^{\frac{3}{4}}+b^4}{b^4}\right)-4\,\sqrt{2}c\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*b*c*x^3*arctan(c*x^2) + 4*a*c*x^3 - 4*sqrt(2)*c*(b^4/c^6)^(1/4)*arc
tan(-(sqrt(2)*b*c^5*x*(b^4/c^6)^(3/4) - sqrt(2)*sqrt(b^2*x^2 + sqrt(2)*b*c*
x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6))*c^5*(b^4/c^6)^(3/4) + b^4)/b^4) - 4*
sqrt(2)*c*(b^4/c^6)^(1/4)*arctan(-(sqrt(2)*b*c^5*x*(b^4/c^6)^(3/4) - sqrt(2
)*sqrt(b^2*x^2 - sqrt(2)*b*c*x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6))*c^5*(b^
4/c^6)^(3/4) - b^4)/b^4) + sqrt(2)*c*(b^4/c^6)^(1/4)*log(b^2*x^2 + sqrt(2)*
b*c*x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6)) - sqrt(2)*c*(b^4/c^6)^(1/4)*log(
b^2*x^2 - sqrt(2)*b*c*x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6)) - 8*b*x)/c
```

Sympy [A] time = 28.3316, size = 1207, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**3/3, Eq(c, 0)), (x**3*(a - b*atan((-sqrt(2)/2 - sqrt(2)*I/2
)**(-2)))/3, Eq(c, -1/(x**2*(-sqrt(2)/2 - sqrt(2)*I/2)**2))), (x**3*(a - b*
atan((-sqrt(2)/2 + sqrt(2)*I/2)**(-2)))/3, Eq(c, -1/(x**2*(-sqrt(2)/2 + sqr
t(2)*I/2)**2))), (x**3*(a - b*atan((sqrt(2)/2 - sqrt(2)*I/2)**(-2)))/3, Eq(
c, -1/(x**2*(sqrt(2)/2 - sqrt(2)*I/2)**2))), (x**3*(a - b*atan((sqrt(2)/2 +
 sqrt(2)*I/2)**(-2)))/3, Eq(c, -1/(x**2*(sqrt(2)/2 + sqrt(2)*I/2)**2))), (-
2*(-1)**(3/4)*a*c**7*x**7*(c**(-2))**(7/4)/(-6*(-1)**(3/4)*c**7*x**4*(c**(-
2))**(7/4) - 6*(-1)**(3/4)*c**5*(c**(-2))**(7/4)) - 2*(-1)**(3/4)*a*c**5*x*
*3*(c**(-2))**(7/4)/(-6*(-1)**(3/4)*c**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3
/4)*c**5*(c**(-2))**(7/4)) + 2*I*b*c**15*x**4*(c**(-2))**(13/2)*atan(c*x**2
)/(-6*(-1)**(3/4)*c**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3/4)*c**5*(c**(-2))
**(7/4)) - 2*(-1)**(3/4)*b*c**7*x**7*(c**(-2))**(7/4)*atan(c*x**2)/(-6*(-1)
**(3/4)*c**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3/4)*c**5*(c**(-2))**(7/4)) +
 2*I*b*c**7*(c**(-2))**(7/2)*atan(c*x**2)/(-6*(-1)**(3/4)*c**7*x**4*(c**(-2
))**(7/4) - 6*(-1)**(3/4)*c**5*(c**(-2))**(7/4)) + 4*(-1)**(3/4)*b*c**6*x**
5*(c**(-2))**(7/4)/(-6*(-1)**(3/4)*c**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3/
4)*c**5*(c**(-2))**(7/4)) - 2*(-1)**(3/4)*b*c**5*x**3*(c**(-2))**(7/4)*atan
(c*x**2)/(-6*(-1)**(3/4)*c**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3/4)*c**5*(c
**(-2))**(7/4)) + 4*(-1)**(3/4)*b*c**4*x*(c**(-2))**(7/4)/(-6*(-1)**(3/4)*c
**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3/4)*c**5*(c**(-2))**(7/4)) - 2*b*c**2
*x**4*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(-6*(-1)**(3/4)*c**7*x**4*(c**(
-2))**(7/4) - 6*(-1)**(3/4)*c**5*(c**(-2))**(7/4)) + b*c**2*x**4*log(x**2 +
 I*sqrt(c**(-2)))/(-6*(-1)**(3/4)*c**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3/4
)*c**5*(c**(-2))**(7/4)) - 2*b*c**2*x**4*atan((-1)**(3/4)*x/(c**(-2))**(1/4
))/(-6*(-1)**(3/4)*c**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3/4)*c**5*(c**(-2)
)**(7/4)) - 2*b*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(-6*(-1)**(3/4)*c**7*
x**4*(c**(-2))**(7/4) - 6*(-1)**(3/4)*c**5*(c**(-2))**(7/4)) + b*log(x**2 +
 I*sqrt(c**(-2)))/(-6*(-1)**(3/4)*c**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3/4
)*c**5*(c**(-2))**(7/4)) - 2*b*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-6*(-1
)**(3/4)*c**7*x**4*(c**(-2))**(7/4) - 6*(-1)**(3/4)*c**5*(c**(-2))**(7/4)),
 True))
```

Giac [A] time = 1.23932, size = 223, normalized size = 1.4

$$\frac{1}{12}\,bc^5\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}}+\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{c^6\sqrt{|c|}}-\frac{\sqrt{2}\log\left(x^2\right.}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*c^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(
abs(c)))/(c^6*sqrt(abs(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/s
qrt(abs(c)))*sqrt(abs(c)))/(c^6*sqrt(abs(c))) + sqrt(2)*log(x^2 + sqrt(2)*x
/sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c))) - sqrt(2)*log(x^2 - sqrt(2)*x/
sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c)))) + 1/3*(b*c*x^3*arctan(c*x^2) +
 a*c*x^3 - 2*b*x)/c
```

3.70 $\int \left(a+b\tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=140

$$ax-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}+bx\tan^{-1}\left(cx^2\right)+\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\tan^{-1}\left(\sqrt{2}\sqrt{c}x\right.}{\sqrt{2}\sqrt{c}}$$

```
[Out] a*x + b*x*ArcTan[c*x^2] + (b*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c
]) - (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*Log[1 - Sqrt[
2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*Sqrt[c]) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x +
 c*x^2])/(2*Sqrt[2]*Sqrt[c])
```

Rubi [A] time = 0.104643, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5027, 297, 1162, 617, 204, 1165, 628}

$$ax-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}+bx\tan^{-1}\left(cx^2\right)+\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\tan^{-1}\left(\sqrt{2}\sqrt{c}x\right.}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^2] + (b*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c
]) - (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*Log[1 - Sqrt[
2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*Sqrt[c]) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x +
 c*x^2])/(2*Sqrt[2]*Sqrt[c])
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^{-1}\left(cx^2\right)\right)dx &= ax+b\int \tan^{-1}\left(cx^2\right)dx\\
&= ax+bx\tan^{-1}\left(cx^2\right)-(2bc)\int \frac{x^2}{1+c^2x^4}\,dx\\
&= ax+bx\tan^{-1}\left(cx^2\right)+b\int \frac{1-cx^2}{1+c^2x^4}\,dx-b\int \frac{1+cx^2}{1+c^2x^4}\,dx\\
&= ax+bx\tan^{-1}\left(cx^2\right)-\frac{b\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{2c}-\frac{b\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{2c}-\frac{b\int \frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}\sqrt{c}}-\frac{b\int \frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x}}{2\sqrt{2}\sqrt{c}}\\
&= ax+bx\tan^{-1}\left(cx^2\right)-\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}-\frac{b\,\text{Subst}\left(\int \frac{1}{-1-}\right.}{}\\
&= ax+bx\tan^{-1}\left(cx^2\right)+\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\tan^{-1}\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+c\right.}{2\sqrt{2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.042094, size = 107, normalized size = 0.76

$$ax+bx\tan^{-1}\left(cx^2\right)-\frac{b\left(\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)-\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)+2\tan^{-1}\left(\sqrt{2}\sqrt{c}x+1\right)\right.}{2\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^2] - (b*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c]*x] + 2*ArcTan[1
 + Sqrt[2]*Sqrt[c]*x] + Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2] - Log[1 + Sqrt[2
]*Sqrt[c]*x + c*x^2]))/(2*Sqrt[2]*Sqrt[c])
```

Maple [A] time = 0.023, size = 125, normalized size = 0.9

$$ax+bx\arctan\left(cx^2\right)-\frac{b\sqrt{2}}{4c}\ln\left(\left(x^2-\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)\left(x^2+\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)^{-1}\right)\frac{1}{\sqrt[4]{c^{-2}}}-\frac{b\sqrt{2}}{2c}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{c^{-2}}}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c*x^2),x)
```

```
[Out] a*x+b*x*arctan(c*x^2)-1/4*b/c/(1/c^2)^(1/4)*2^(1/2)*ln((x^2-(1/c^2)^(1/4)*x
*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))-1/2*b/
c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)-1/2*b/c/(1/c^2)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)
```

Maxima [B] time = 1.51946, size = 348, normalized size = 2.49

$$\frac{1}{4}\left(\left(c\left(\frac{\sqrt{2}\log\left(\sqrt{c^2}x^2+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c^2}x^2-\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\frac{2\sqrt{c^2}x-\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}}{2\sqrt{c^2}x+\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}}\right)}{\sqrt{c^2}\sqrt{-\sqrt{c^2}}}-\sqrt{2}\log\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(c*(sqrt(2)*log(sqrt(c^2)*x^2 + sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(3/4)
- sqrt(2)*log(sqrt(c^2)*x^2 - sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(3/4) - sqrt
(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4))/(2
*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4)))/(sqrt(c^2)*
sqrt(-sqrt(c^2))) - sqrt(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) -
 sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) - sqrt(2)*(
c^2)^(1/4)))/(sqrt(c^2)*sqrt(-sqrt(c^2)))) + 4*x*arctan(c*x^2))*b + a*x
```

Fricas [B] time = 2.82601, size = 768, normalized size = 5.49

$$bx\arctan\left(cx^2\right)+ax+\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}b^3cx+b^4-\sqrt{2}\sqrt{b^6x^2+\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{3}{4}}b^3cx+\sqrt{\frac{b^4}{c^2}}b^4}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}c}{b^4}\right)+\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="fricas")
```

```
[Out] b*x*arctan(c*x^2) + a*x + sqrt(2)*(b^4/c^2)^(1/4)*arctan(-(sqrt(2)*(b^4/c^2
)^(1/4)*b^3*c*x + b^4 - sqrt(2)*sqrt(b^6*x^2 + sqrt(2)*(b^4/c^2)^(3/4)*b^3*
c*x + sqrt(b^4/c^2)*b^4)*(b^4/c^2)^(1/4)*c)/b^4) + sqrt(2)*(b^4/c^2)^(1/4)*
arctan(-(sqrt(2)*(b^4/c^2)^(1/4)*b^3*c*x - b^4 - sqrt(2)*sqrt(b^6*x^2 - sqr
t(2)*(b^4/c^2)^(3/4)*b^3*c*x + sqrt(b^4/c^2)*b^4)*(b^4/c^2)^(1/4)*c)/b^4) +
 1/4*sqrt(2)*(b^4/c^2)^(1/4)*log(b^6*x^2 + sqrt(2)*(b^4/c^2)^(3/4)*b^3*c*x
+ sqrt(b^4/c^2)*b^4) - 1/4*sqrt(2)*(b^4/c^2)^(1/4)*log(b^6*x^2 - sqrt(2)*(b
^4/c^2)^(3/4)*b^3*c*x + sqrt(b^4/c^2)*b^4)
```

Sympy [A] time = 15.0671, size = 146, normalized size = 1.04

$$ax + b\begin{cases} -\frac{(-1)^{\frac{3}{4}}c^3\left(\frac{1}{c^2}\right)^{\frac{7}{4}}\log\left(x^2+i\sqrt{\frac{1}{c^2}}\right)}{2} + (-1)^{\frac{3}{4}}c\left(\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x-\sqrt[4]{-1}\sqrt[4]{\frac{1}{c^2}}\right) - (-1)^{\frac{3}{4}}c\left(\frac{1}{c^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}x}{\sqrt[4]{\frac{1}{c^2}}}\right) + x\operatorname{atan}\left(cx^2\right) + \frac{\sqrt[4]{-1}}{c} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c*x**2),x)
```

```
[Out] a*x + b*Piecewise((-(-1)**(3/4)*c**3*(c**(-2))**(7/4)*log(x**2 + I*sqrt(c**
(-2)))/2 + (-1)**(3/4)*c*(c**(-2))**(3/4)*log(x - (-1)**(1/4)*(c**(-2))**(1
/4)) - (-1)**(3/4)*c*(c**(-2))**(3/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))
+ x*atan(c*x**2) + (-1)**(1/4)*atan(c*x**2)/(c**4*(c**(-2))**(7/4)), Ne(c,
0)), (0, True))
```

Giac [A] time = 1.14414, size = 201, normalized size = 1.44

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2} + \frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2} - \frac{\sqrt{2}\sqrt{|c|}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{c^2} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="giac")
```

```
[Out] -1/4*(c*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(
c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 + sqr
t(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sqrt(2
)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 4*x*arctan(c*x^2))*b + a*x
```

3.71 $\int \frac{a+b\tan^{-1}(cx^2)}{x^2}\,dx$

Optimal. Leaf size=143

$$-\frac{a+b\tan^{-1}(cx^2)}{x}-\frac{b\sqrt{c}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}}-\frac{b\sqrt{c}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}+\frac{b\sqrt{c}\tan}{}$$

```
[Out] -((a + b*ArcTan[c*x^2])/x) - (b*Sqrt[c]*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/Sqrt
[2] + (b*Sqrt[c]*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/Sqrt[2] - (b*Sqrt[c]*Log[1
- Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sqrt[c]*Log[1 + Sqrt[2]*Sqrt
[c]*x + c*x^2])/(2*Sqrt[2])
```

Rubi [A] time = 0.0861341, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5033, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a+b\tan^{-1}(cx^2)}{x}-\frac{b\sqrt{c}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}}-\frac{b\sqrt{c}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}+\frac{b\sqrt{c}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x^2])/x) - (b*Sqrt[c]*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/Sqrt
[2] + (b*Sqrt[c]*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/Sqrt[2] - (b*Sqrt[c]*Log[1
- Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sqrt[c]*Log[1 + Sqrt[2]*Sqrt
[c]*x + c*x^2])/(2*Sqrt[2])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^2}\,dx &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}+(2bc)\int \frac{1}{1+c^2x^4}\,dx\\
&= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}+(bc)\int \frac{1-cx^2}{1+c^2x^4}\,dx+(bc)\int \frac{1+cx^2}{1+c^2x^4}\,dx\\
&= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}+\frac{1}{2}b\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx+\frac{1}{2}b\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx-\frac{\left(b\sqrt{c}\right)\int \frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}}\\
&= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}-\frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}+\frac{\left(b\sqrt{c}\right)\mathrm{Sub}}{}\\
&= -\frac{a+b\tan^{-1}\left(cx^2\right)}{x}-\frac{b\sqrt{c}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}+\frac{b\sqrt{c}\tan^{-1}\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}-\frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{\right.}}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.042499, size = 158, normalized size = 1.1

$$
-\frac{a}{x}-\frac{b\sqrt{c}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}}-\frac{b\tan^{-1}\left(cx^2\right)}{x}+\frac{b\sqrt{c}\tan^{-1}\left(\frac{2\sqrt{c}x-\sqrt{2}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{b\sqrt{c}\tan^{-1}\left(2\right.}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c*x^2])/x + (b*Sqrt[c]*ArcTan[(-Sqrt[2] + 2*Sqrt[c]*x)/S
qrt[2]])/Sqrt[2] + (b*Sqrt[c]*ArcTan[(Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/Sqrt
[2] - (b*Sqrt[c]*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sqrt[
c]*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2])
```

Maple [A] time = 0.025, size = 125, normalized size = 0.9

$$-\frac{a}{x}-\frac{b\arctan\left(cx^{2}\right)}{x}+\frac{bc\sqrt{2}}{2}\sqrt[4]{c^{-2}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{c^{-2}}}-1\right)+\frac{bc\sqrt{2}}{4}\sqrt[4]{c^{-2}}\ln\left(\left(x^{2}+\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)\left(x^{2}-\sqrt[4]{c^{-2}}x\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^2,x)
```

```
[Out] -a/x-b/x*arctan(c*x^2)+1/2*b*c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)
^(1/4)*x-1)+1/4*b*c*(1/c^2)^(1/4)*2^(1/2)*ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(
1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+1/2*b*c*(1/c^2)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)
```

Maxima [B] time = 1.52878, size = 339, normalized size = 2.37

$$\frac{1}{4}\left(\left(\left(\frac{\sqrt{2}\log\left(\sqrt{c^{2}}x^{2}+\sqrt{2}\left(c^{2}\right)^{\frac{1}{4}}x+1\right)}{\left(c^{2}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c^{2}}x^{2}-\sqrt{2}\left(c^{2}\right)^{\frac{1}{4}}x+1\right)}{\left(c^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(\frac{2\sqrt{c^{2}}x-\sqrt{2}\sqrt{-\sqrt{c^{2}}}+\sqrt{2}\left(c^{2}\right)^{\frac{1}{4}}}{2\sqrt{c^{2}}x+\sqrt{2}\sqrt{-\sqrt{c^{2}}}+\sqrt{2}\left(c^{2}\right)^{\frac{1}{4}}}\right)}{\sqrt{-\sqrt{c^{2}}}}+\frac{\sqrt{2}\log}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*((sqrt(2)*log(sqrt(c^2)*x^2 + sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(1/4) -
sqrt(2)*log(sqrt(c^2)*x^2 - sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(1/4) + sqrt(2
)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4))/(2*s
qrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4)))/sqrt(-sqrt(c^
2)) + sqrt(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) - sqrt(2)*(c^2)
^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4)))/s
qrt(-sqrt(c^2)))*c - 4*arctan(c*x^2)/x)*b - a/x
```

Fricas [B] time = 2.75745, size = 790, normalized size = 5.52

$$-\frac{4\sqrt{2}\left(b^{4}c^{2}\right)^{\frac{1}{4}}x\arctan\left(-\frac{b^{4}c^{2}+\sqrt{2}\left(b^{4}c^{2}\right)^{\frac{3}{4}}bcx-\sqrt{2}\left(b^{4}c^{2}\right)^{\frac{3}{4}}\sqrt{b^{2}c^{2}x^{2}+\sqrt{2}\left(b^{4}c^{2}\right)^{\frac{1}{4}}bcx+\sqrt{b^{4}c^{2}}}}{b^{4}c^{2}}\right)+4\sqrt{2}\left(b^{4}c^{2}\right)^{\frac{1}{4}}x\arctan\left(\frac{b^{4}c^{2}-\sqrt{2}\left(b^{4}c^{2}\right)^{\frac{3}{4}}b}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(b^4*c^2)^(1/4)*x*arctan(-(b^4*c^2 + sqrt(2)*(b^4*c^2)^(3/4
)*b*c*x - sqrt(2)*(b^4*c^2)^(3/4)*sqrt(b^2*c^2*x^2 + sqrt(2)*(b^4*c^2)^(1/4
)*b*c*x + sqrt(b^4*c^2)))/(b^4*c^2)) + 4*sqrt(2)*(b^4*c^2)^(1/4)*x*arctan((
b^4*c^2 - sqrt(2)*(b^4*c^2)^(3/4)*b*c*x + sqrt(2)*(b^4*c^2)^(3/4)*sqrt(b^2*
c^2*x^2 - sqrt(2)*(b^4*c^2)^(1/4)*b*c*x + sqrt(b^4*c^2)))/(b^4*c^2)) - sqrt
(2)*(b^4*c^2)^(1/4)*x*log(b^2*c^2*x^2 + sqrt(2)*(b^4*c^2)^(1/4)*b*c*x + sqr
t(b^4*c^2)) + sqrt(2)*(b^4*c^2)^(1/4)*x*log(b^2*c^2*x^2 - sqrt(2)*(b^4*c^2)
```

```
^(1/4)*b*c*x + sqrt(b^4*c^2)) + 4*b*arctan(c*x^2) + 4*a)/x
```

Sympy [A] time = 32.019, size = 1105, normalized size = 7.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**2,x)
```

```
[Out] Piecewise((-a/x, Eq(c, 0)), (-(a - b*atan((-sqrt(2)/2 - sqrt(2)*I/2)**(-2))
)/x, Eq(c, -1/(x**2*(-sqrt(2)/2 - sqrt(2)*I/2)**2))), (-(a - b*atan((-sqrt(
2)/2 + sqrt(2)*I/2)**(-2)))/x, Eq(c, -1/(x**2*(-sqrt(2)/2 + sqrt(2)*I/2)**2
))), (-(a - b*atan((sqrt(2)/2 - sqrt(2)*I/2)**(-2)))/x, Eq(c, -1/(x**2*(sqr
t(2)/2 - sqrt(2)*I/2)**2))), (-(a - b*atan((sqrt(2)/2 + sqrt(2)*I/2)**(-2))
)/x, Eq(c, -1/(x**2*(sqrt(2)/2 + sqrt(2)*I/2)**2))), (-2*(-1)**(1/4)*a*c**6
*x**4*(c**(-2))**(9/4)/(2*(-1)**(1/4)*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**
(1/4)*c**4*x*(c**(-2))**(9/4)) - 2*(-1)**(1/4)*a*c**4*(c**(-2))**(9/4)/(2*(
-1)**(1/4)*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2))**(9/
4)) - 2*I*b*c**7*x**5*(c**(-2))**(5/2)*log(x - (-1)**(1/4)*(c**(-2))**(1/4)
)/(2*(-1)**(1/4)*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2)
)**(9/4)) + I*b*c**7*x**5*(c**(-2))**(5/2)*log(x**2 + I*sqrt(c**(-2)))/(2*(
-1)**(1/4)*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2))**(9/
4)) - 2*I*b*c**7*x**5*(c**(-2))**(5/2)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))
/(2*(-1)**(1/4)*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2))
**(9/4)) - 2*(-1)**(1/4)*b*c**6*x**4*(c**(-2))**(9/4)*atan(c*x**2)/(2*(-1)*
*(1/4)*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2))**(9/4))
- 2*I*b*c**5*x*(c**(-2))**(5/2)*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(2*(-
1)**(1/4)*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2))**(9/4
)) + I*b*c**5*x*(c**(-2))**(5/2)*log(x**2 + I*sqrt(c**(-2)))/(2*(-1)**(1/4)
*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2))**(9/4)) - 2*I*
b*c**5*x*(c**(-2))**(5/2)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(2*(-1)**(1/
4)*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2))**(9/4)) - 2*
(-1)**(1/4)*b*c**4*(c**(-2))**(9/4)*atan(c*x**2)/(2*(-1)**(1/4)*c**6*x**5*(
c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2))**(9/4)) - 2*b*c**2*x**5*at
an(c*x**2)/(2*(-1)**(1/4)*c**6*x**5*(c**(-2))**(9/4) + 2*(-1)**(1/4)*c**4*x
*(c**(-2))**(9/4)) - 2*b*x*atan(c*x**2)/(2*(-1)**(1/4)*c**6*x**5*(c**(-2))*
*(9/4) + 2*(-1)**(1/4)*c**4*x*(c**(-2))**(9/4)), True))
```

Giac [A] time = 1.22791, size = 186, normalized size = 1.3

$$\frac{1}{4}\,bc\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{\sqrt{|c|}}-\frac{\sqrt{2}\log\left(x^2-\right.}{\sqrt{|}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(abs
(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(c
)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c)) +
 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/abs(
c))/sqrt(abs(c))) - (b*arctan(c*x^2) + a)/x
```

3.72 $\int \frac{a+b\tan^{-1}(cx^2)}{x^4}\,dx$

Optimal. Leaf size=159

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{bc^{3/2}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\text{t}}{}$$

```
[Out] (-2*b*c)/(3*x) - (a + b*ArcTan[c*x^2])/(3*x^3) + (b*c^(3/2)*ArcTan[1 - Sqrt
[2]*Sqrt[c]*x])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*
Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]) + (b*
c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2])
```

Rubi [A] time = 0.104142, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 325, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{bc^{3/2}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^4,x]
```


```
[Out] (-2*b*c)/(3*x) - (a + b*ArcTan[c*x^2])/(3*x^3) + (b*c^(3/2)*ArcTan[1 - Sqrt
[2]*Sqrt[c]*x])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*
Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]) + (b*
c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2])
```


Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^4}\,dx &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}(2bc)\int \frac{1}{x^2\left(1+c^2x^4\right)}\,dx\\ &= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{3}\left(2bc^3\right)\int \frac{x^2}{1+c^2x^4}\,dx\\ &= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}\left(bc^2\right)\int \frac{1-cx^2}{1+c^2x^4}\,dx-\frac{1}{3}\left(bc^2\right)\int \frac{1+cx^2}{1+c^2x^4}\,dx\\ &= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{6}(bc)\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx-\frac{1}{6}(bc)\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx-\frac{\left(bc^{3/2}\right)\int \cdots}{6\cdots}\\ &= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}-\frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}-\frac{\left(bc^{\cdots}\right)\cdots}{\cdots}\\ &= -\frac{2bc}{3x}-\frac{a+b\tan^{-1}\left(cx^2\right)}{3x^3}+\frac{bc^{3/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\tan^{-1}\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\log\left(1\cdots\right.}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.0517281, size = 177, normalized size = 1.11

$$-\frac{a}{3x^3}-\frac{bc^{3/2}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}}-\frac{bc^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}x-\sqrt{2}}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{bc^{3/2}\tan^{-1}\left(\frac{2\sqrt{c}x+\sqrt{2}}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{b\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^4,x]
```

```
[Out] -a/(3*x^3) - (2*b*c)/(3*x) - (b*ArcTan[c*x^2])/(3*x^3) - (b*c^(3/2)*ArcTan[
(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[(Sqrt[2]
 + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[c]*
x + c*x^2])/(6*Sqrt[2]) + (b*c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6
*Sqrt[2])
```

Maple [A] time = 0.029, size = 132, normalized size = 0.8

$$-\frac{a}{3x^3}-\frac{b\arctan\left(cx^2\right)}{3x^3}-\frac{bc\sqrt{2}}{12}\ln\left(\left(x^2-\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)\left(x^2+\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)^{-1}\right)\frac{1}{\sqrt[4]{c^{-2}}}-\frac{bc\sqrt{2}}{6}\arctan\left(x\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c*x^2)-1/12*b*c/(1/c^2)^(1/4)*2^(1/2)*ln((x^2-(
1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^
(1/2)))-1/6*b*c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)-1/6
*b*c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)-2/3*b*c/x
```

Maxima [B] time = 1.52514, size = 369, normalized size = 2.32

$$\frac{1}{12}\left(\left(\left(c^2\left(\frac{\sqrt{2}\log\left(\sqrt{c^2}x^2+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c^2}x^2-\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\frac{2\sqrt{c^2}x-\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}}{2\sqrt{c^2}x+\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}}\right)}{\sqrt{c^2}\sqrt{-\sqrt{c^2}}}-\frac{\sqrt{2}}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*((c^2*(sqrt(2)*log(sqrt(c^2)*x^2 + sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(3
/4) - sqrt(2)*log(sqrt(c^2)*x^2 - sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(3/4) -
sqrt(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4)
)/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4)))/(sqrt(c
^2)*sqrt(-sqrt(c^2))) - sqrt(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2
)) - sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) - sqrt(
2)*(c^2)^(1/4)))/(sqrt(c^2)*sqrt(-sqrt(c^2)))) - 8/x)*c - 4*arctan(c*x^2)/x
^3)*b - 1/3*a/x^3
```

Fricas [B] time = 2.8576, size = 900, normalized size = 5.66

$$4\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{b^4c^6+\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}b^3c^5x-\sqrt{2}\sqrt{b^6c^{10}x^2+\sqrt{b^4c^6}b^4c^6+\sqrt{2}\left(b^4c^6\right)^{\frac{3}{4}}b^3c^5x}\left(b^4c^6\right)^{\frac{1}{4}}}{b^4c^6}\right)+4\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}x^3\arctan\left(\frac{b^4c^6-\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(2)*(b^4*c^6)^(1/4)*x^3*arctan(-(b^4*c^6 + sqrt(2)*(b^4*c^6)^(1
/4)*b^3*c^5*x - sqrt(2)*sqrt(b^6*c^10*x^2 + sqrt(b^4*c^6)*b^4*c^6 + sqrt(2)
*(b^4*c^6)^(3/4)*b^3*c^5*x)*(b^4*c^6)^(1/4))/(b^4*c^6)) + 4*sqrt(2)*(b^4*c^
6)^(1/4)*x^3*arctan((b^4*c^6 - sqrt(2)*(b^4*c^6)^(1/4)*b^3*c^5*x + sqrt(2)*
sqrt(b^6*c^10*x^2 + sqrt(b^4*c^6)*b^4*c^6 - sqrt(2)*(b^4*c^6)^(3/4)*b^3*c^5
*x)*(b^4*c^6)^(1/4))/(b^4*c^6)) + sqrt(2)*(b^4*c^6)^(1/4)*x^3*log(b^6*c^10*
x^2 + sqrt(b^4*c^6)*b^4*c^6 + sqrt(2)*(b^4*c^6)^(3/4)*b^3*c^5*x) - sqrt(2)*
(b^4*c^6)^(1/4)*x^3*log(b^6*c^10*x^2 + sqrt(b^4*c^6)*b^4*c^6 - sqrt(2)*(b^4
*c^6)^(3/4)*b^3*c^5*x) - 8*b*c*x^2 - 4*b*arctan(c*x^2) - 4*a)/x^3
```

Sympy [A] time = 55.5763, size = 704, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*I*b)/(3*x**3), Eq(c, -I/x**2))
, (-(a + oo*I*b)/(3*x**3), Eq(c, I/x**2)), (-6*a*x**4/(18*x**7 + 18*x**3/c*
*2) - 6*a/(18*c**2*x**7 + 18*x**3) + 4*I*b*c**24*x**4*(c**(-2))**(27/2)/(18
*x**7/c**3 + 18*x**3/c**5) - 6*(-1)**(3/4)*b*c**18*x**7*(c**(-2))**(39/4)*a
tan((-1)**(3/4)*x/(c**(-2))**(1/4))/(18*x**7/c**3 + 18*x**3/c**5) - 4*I*b*c
**16*x**4*(c**(-2))**(19/2)/(18*x**7/c**3 + 18*x**3/c**5) - 6*(-1)**(3/4)*b
*c**16*x**3*(c**(-2))**(39/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(18*x**7
/c**3 + 18*x**3/c**5) + 6*(-1)**(3/4)*b*c**10*x**7*(c**(-2))**(23/4)*log(x
- (-1)**(1/4)*(c**(-2))**(1/4))/(18*x**7/c**3 + 18*x**3/c**5) - 3*(-1)**(3/
4)*b*c**10*x**7*(c**(-2))**(23/4)*log(x**2 + I*sqrt(c**(-2)))/(18*x**7/c**3
 + 18*x**3/c**5) + 6*(-1)**(3/4)*b*c**8*x**3*(c**(-2))**(23/4)*log(x - (-1)
**(1/4)*(c**(-2))**(1/4))/(18*x**7/c**3 + 18*x**3/c**5) - 3*(-1)**(3/4)*b*c
**8*x**3*(c**(-2))**(23/4)*log(x**2 + I*sqrt(c**(-2)))/(18*x**7/c**3 + 18*x
**3/c**5) + 6*(-1)**(1/4)*b*c**5*x**7*(c**(-2))**(13/4)*atan(c*x**2)/(18*x*
*7/c**3 + 18*x**3/c**5) + 6*(-1)**(1/4)*b*c**3*x**3*(c**(-2))**(13/4)*atan(
c*x**2)/(18*x**7/c**3 + 18*x**3/c**5) - 12*b*x**6/(18*x**7/c + 18*x**3/c**3
) - 6*b*x**4*atan(c*x**2)/(18*x**7 + 18*x**3/c**2) - 12*b*x**2/(18*c*x**7 +
 18*x**3/c) - 6*b*atan(c*x**2)/(18*c**2*x**7 + 18*x**3), True))
```

Giac [A] time = 1.26766, size = 215, normalized size = 1.35

$$-\frac{1}{12}bc^3\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}+\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{2}\sqrt{|c|}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/12*b*c^3*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(
abs(c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 +
 sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sq
rt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 1/3*(2*b*c*x^2 + b*arctan(c*x^2) +
```

```
a)/x^3
```

3.73 $\int \frac{a+b\tan^{-1}(cx^2)}{x^6}\,dx$

Optimal. Leaf size=159

$$-\frac{a+b\tan^{-1}(cx^2)}{5x^5}+\frac{bc^{5/2}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}}+\frac{bc^{5/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}-\frac{bc^{5/2}\tan}{}$$

```
[Out] (-2*b*c)/(15*x^3) - (a + b*ArcTan[c*x^2])/(5*x^5) + (b*c^(5/2)*ArcTan[1 - S
qrt[2]*Sqrt[c]*x])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/
(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]) -
 (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2])
```

Rubi [A] time = 0.102719, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 325, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a+b\tan^{-1}(cx^2)}{5x^5}+\frac{bc^{5/2}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}}+\frac{bc^{5/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}-\frac{bc^{5/2}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^6,x]
```

```
[Out] (-2*b*c)/(15*x^3) - (a + b*ArcTan[c*x^2])/(5*x^5) + (b*c^(5/2)*ArcTan[1 - S
qrt[2]*Sqrt[c]*x])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/
(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]) -
 (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2])
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^6}\,dx = -\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}(2bc)\int \frac{1}{x^4\left(1+c^2x^4\right)}\,dx$$
$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}-\frac{1}{5}\left(2bc^3\right)\int \frac{1}{1+c^2x^4}\,dx$$
$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}-\frac{1}{5}\left(bc^3\right)\int \frac{1-cx^2}{1+c^2x^4}\,dx-\frac{1}{5}\left(bc^3\right)\int \frac{1+cx^2}{1+c^2x^4}\,dx$$
$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}-\frac{1}{10}\left(bc^2\right)\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx-\frac{1}{10}\left(bc^2\right)\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx+$$
$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}-$$
$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}\left(cx^2\right)}{5x^5}+\frac{bc^{5/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}-\frac{bc^{5/2}\tan^{-1}\left(1+\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}+\frac{bc^{5/2}\,\mathrm{l}}{}$$

Mathematica [A] time = 0.0501376, size = 177, normalized size = 1.11

$$-\frac{a}{5x^5}+\frac{bc^{5/2}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}}-\frac{bc^{5/2}\tan^{-1}\left(\frac{2\sqrt{c}x-\sqrt{2}}{\sqrt{2}}\right)}{5\sqrt{2}}-\frac{bc^{5/2}\tan^{-1}\left(\frac{2\sqrt{c}x+\sqrt{2}}{\sqrt{2}}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^6,x]
```

```
[Out] -a/(5*x^5) - (2*b*c)/(15*x^3) - (b*ArcTan[c*x^2])/(5*x^5) - (b*c^(5/2)*ArcT
an[(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[(Sqrt
[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sqrt[
c]*x + c*x^2])/(10*Sqrt[2]) - (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(10*Sqrt[2])
```

Maple [A] time = 0.03, size = 138, normalized size = 0.9

$$-\frac{a}{5\,x^5}-\frac{b\arctan\left(cx^2\right)}{5\,x^5}-\frac{bc^3\sqrt{2}}{10}\sqrt[4]{c^{-2}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{c^{-2}}}-1\right)-\frac{bc^3\sqrt{2}}{20}\sqrt[4]{c^{-2}}\ln\left(\left(x^2+\sqrt[4]{c^{-2}}x\sqrt{2}+\sqrt{c^{-2}}\right)\left(x^2-\sqrt[4]{c^{-2}}x\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctan(c*x^2)-1/10*b*c^3*(1/c^2)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/c^2)^(1/4)*x-1)-1/20*b*c^3*(1/c^2)^(1/4)*2^(1/2)*ln((x^2+(1/c^2)
^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))
)-1/10*b*c^3*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)-2/15*b
*c/x^3
```

Maxima [B] time = 1.51966, size = 366, normalized size = 2.3

$$-\frac{1}{60}\left(\left(\frac{3\sqrt{2}c^2\log\left(\sqrt{c^2}x^2+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{1}{4}}}-\frac{3\sqrt{2}c^2\log\left(\sqrt{c^2}x^2-\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{1}{4}}}+\frac{3\sqrt{2}c^2\log\left(\frac{2\sqrt{c^2}x-\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}}{2\sqrt{c^2}x+\sqrt{2}\sqrt{-\sqrt{c^2}}+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}}\right)}{\sqrt{-\sqrt{c^2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*((3*sqrt(2)*c^2*log(sqrt(c^2)*x^2 + sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^
(1/4) - 3*sqrt(2)*c^2*log(sqrt(c^2)*x^2 - sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^
(1/4) + 3*sqrt(2)*c^2*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(
2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(
1/4)))/sqrt(-sqrt(c^2)) + 3*sqrt(2)*c^2*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-
sqrt(c^2)) - sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2))
 - sqrt(2)*(c^2)^(1/4)))/sqrt(-sqrt(c^2)) + 8/x^3)*c + 12*arctan(c*x^2)/x^5
)*b - 1/5*a/x^5
```

Fricas [B] time = 2.83075, size = 873, normalized size = 5.49

$$12\sqrt{2}\left(b^4c^{10}\right)^{\frac{1}{4}}x^5\arctan\left(-\frac{b^4c^{10}+\sqrt{2}\left(b^4c^{10}\right)^{\frac{3}{4}}bc^3x-\sqrt{2}\left(b^4c^{10}\right)^{\frac{3}{4}}\sqrt{b^2c^6x^2+\sqrt{2}\left(b^4c^{10}\right)^{\frac{1}{4}}bc^3x+\sqrt{b^4c^{10}}}}{b^4c^{10}}\right)+12\sqrt{2}\left(b^4c^{10}\right)^{\frac{1}{4}}x^5\arctan\left(\frac{b^4c^{10}-\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(12*sqrt(2)*(b^4*c^10)^(1/4)*x^5*arctan(-(b^4*c^10 + sqrt(2)*(b^4*c^10
)^(3/4)*b*c^3*x - sqrt(2)*(b^4*c^10)^(3/4)*sqrt(b^2*c^6*x^2 + sqrt(2)*(b^4*
c^10)^(1/4)*b*c^3*x + sqrt(b^4*c^10)))/(b^4*c^10)) + 12*sqrt(2)*(b^4*c^10)^
(1/4)*x^5*arctan((b^4*c^10 - sqrt(2)*(b^4*c^10)^(3/4)*b*c^3*x + sqrt(2)*(b^
4*c^10)^(3/4)*sqrt(b^2*c^6*x^2 - sqrt(2)*(b^4*c^10)^(1/4)*b*c^3*x + sqrt(b^
4*c^10)))/(b^4*c^10)) - 3*sqrt(2)*(b^4*c^10)^(1/4)*x^5*log(b^2*c^6*x^2 + sq
rt(2)*(b^4*c^10)^(1/4)*b*c^3*x + sqrt(b^4*c^10)) + 3*sqrt(2)*(b^4*c^10)^(1/
4)*x^5*log(b^2*c^6*x^2 - sqrt(2)*(b^4*c^10)^(1/4)*b*c^3*x + sqrt(b^4*c^10))
 - 8*b*c*x^2 - 12*b*arctan(c*x^2) - 12*a)/x^5
```

Sympy [A] time = 107.518, size = 1265, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**6,x)
```

```
[Out] Piecewise((-(a - b*atan((-sqrt(2)/2 - sqrt(2)*I/2)**(-2)))/(5*x**5), Eq(c,
-1/(x**2*(-sqrt(2)/2 - sqrt(2)*I/2)**2))), (-(a - b*atan((-sqrt(2)/2 + sqrt
(2)*I/2)**(-2)))/(5*x**5), Eq(c, -1/(x**2*(-sqrt(2)/2 + sqrt(2)*I/2)**2))),
 (-(a - b*atan((sqrt(2)/2 - sqrt(2)*I/2)**(-2)))/(5*x**5), Eq(c, -1/(x**2*(
sqrt(2)/2 - sqrt(2)*I/2)**2))), (-(a - b*atan((sqrt(2)/2 + sqrt(2)*I/2)**(-
2)))/(5*x**5), Eq(c, -1/(x**2*(sqrt(2)/2 + sqrt(2)*I/2)**2))), (-a/(5*x**5)
, Eq(c, 0)), (6*(-1)**(3/4)*a*c**15*x**4*(c**(-2))**(39/4)/(-30*(-1)**(3/4)
*c**15*x**9*(c**(-2))**(39/4) - 30*(-1)**(3/4)*c**13*x**5*(c**(-2))**(39/4)
) + 6*(-1)**(3/4)*a*c**13*(c**(-2))**(39/4)/(-30*(-1)**(3/4)*c**15*x**9*(c*
*(-2))**(39/4) - 30*(-1)**(3/4)*c**13*x**5*(c**(-2))**(39/4)) + 4*(-1)**(3/
4)*b*c**16*x**6*(c**(-2))**(39/4)/(-30*(-1)**(3/4)*c**15*x**9*(c**(-2))**(3
9/4) - 30*(-1)**(3/4)*c**13*x**5*(c**(-2))**(39/4)) + 6*(-1)**(3/4)*b*c**15
*x**4*(c**(-2))**(39/4)*atan(c*x**2)/(-30*(-1)**(3/4)*c**15*x**9*(c**(-2))*
*(39/4) - 30*(-1)**(3/4)*c**13*x**5*(c**(-2))**(39/4)) + 4*(-1)**(3/4)*b*c*
*14*x**2*(c**(-2))**(39/4)/(-30*(-1)**(3/4)*c**15*x**9*(c**(-2))**(39/4) -
30*(-1)**(3/4)*c**13*x**5*(c**(-2))**(39/4)) + 6*(-1)**(3/4)*b*c**13*(c**(-
2))**(39/4)*atan(c*x**2)/(-30*(-1)**(3/4)*c**15*x**9*(c**(-2))**(39/4) - 30
*(-1)**(3/4)*c**13*x**5*(c**(-2))**(39/4)) - 6*I*b*c**11*x**9*(c**(-2))**(1
3/2)*atan(c*x**2)/(-30*(-1)**(3/4)*c**15*x**9*(c**(-2))**(39/4) - 30*(-1)**
(3/4)*c**13*x**5*(c**(-2))**(39/4)) - 6*I*b*c**9*x**5*(c**(-2))**(13/2)*ata
n(c*x**2)/(-30*(-1)**(3/4)*c**15*x**9*(c**(-2))**(39/4) - 30*(-1)**(3/4)*c*
*13*x**5*(c**(-2))**(39/4)) + 6*b*x**9*log(x - (-1)**(1/4)*(c**(-2))**(1/4)
)/(-30*(-1)**(3/4)*c**17*x**9*(c**(-2))**(39/4) - 30*(-1)**(3/4)*c**15*x**5
*(c**(-2))**(39/4)) - 3*b*x**9*log(x**2 + I*sqrt(c**(-2)))/(-30*(-1)**(3/4)
*c**17*x**9*(c**(-2))**(39/4) - 30*(-1)**(3/4)*c**15*x**5*(c**(-2))**(39/4)
) + 6*b*x**9*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-30*(-1)**(3/4)*c**17*x*
*9*(c**(-2))**(39/4) - 30*(-1)**(3/4)*c**15*x**5*(c**(-2))**(39/4)) + 6*b*x
**5*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(-30*(-1)**(3/4)*c**19*x**9*(c**(
-2))**(39/4) - 30*(-1)**(3/4)*c**17*x**5*(c**(-2))**(39/4)) - 3*b*x**5*log(
x**2 + I*sqrt(c**(-2)))/(-30*(-1)**(3/4)*c**19*x**9*(c**(-2))**(39/4) - 30*
(-1)**(3/4)*c**17*x**5*(c**(-2))**(39/4)) + 6*b*x**5*atan((-1)**(3/4)*x/(c*
*(-2))**(1/4))/(-30*(-1)**(3/4)*c**19*x**9*(c**(-2))**(39/4) - 30*(-1)**(3/
4)*c**17*x**5*(c**(-2))**(39/4)), True))
```

Giac [A] time = 1.36396, size = 203, normalized size = 1.28

$$-\frac{1}{20}\,bc^3\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{\sqrt{|c|}}-\frac{\sqrt{2}\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt
(abs(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c
)) + 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/
abs(c))/sqrt(abs(c))) - 1/15*(2*b*c*x^2 + 3*b*arctan(c*x^2) + 3*a)/x^5
```

3.74 $\int x^7 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=124

$$\frac{abx^2}{4c^3}-\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{8c^4}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-\frac{bx^6\left(a+b\tan^{-1}\left(cx^2\right)\right)}{12c}+\frac{b^2x^4}{24c^2}-\frac{b^2\log\left(c^2x^4+1\right)}{6c^4}+\frac{b^2x^2 t}{}$$

```
[Out] (a*b*x^2)/(4*c^3) + (b^2*x^4)/(24*c^2) + (b^2*x^2*ArcTan[c*x^2])/(4*c^3) -
(b*x^6*(a + b*ArcTan[c*x^2]))/(12*c) - (a + b*ArcTan[c*x^2])^2/(8*c^4) + (x
^8*(a + b*ArcTan[c*x^2])^2)/8 - (b^2*Log[1 + c^2*x^4])/(6*c^4)
```

Rubi [C] time = 1.62698, antiderivative size = 731, normalized size of antiderivative = 5.9, number of steps used = 62, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {5035, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)}{16c^4}-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)}{16c^4}+\frac{abx^2}{8c^3}-\frac{bx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{32c^2}+\frac{1}{192}ib\left(-\frac{3\left(1-i\right.}{c^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (a*b*x^2)/(8*c^3) - (((23*I)/192)*b^2*x^2)/c^3 + (b^2*x^4)/(128*c^2) - (((7
*I)/576)*b^2*x^6)/c + (b^2*x^8)/256 - (3*b^2*(1 - I*c*x^2)^2)/(32*c^4) + (b
^2*(1 - I*c*x^2)^3)/(36*c^4) - (b^2*(1 - I*c*x^2)^4)/(256*c^4) - (b^2*Log[I
 - c*x^2])/(24*c^4) - (b^2*(1 - I*c*x^2)*Log[1 - I*c*x^2])/(16*c^4) - (b^2*
Log[1 - I*c*x^2]^2)/(32*c^4) - (b*x^4*((2*I)*a - b*Log[1 - I*c*x^2]))/(32*c
^2) + ((I/48)*b*x^6*((2*I)*a - b*Log[1 - I*c*x^2]))/c + (b*x^8*((2*I)*a - b
*Log[1 - I*c*x^2]))/64 + (x^8*(2*a + I*b*Log[1 - I*c*x^2])^2)/32 + (I/192)*
b*(2*a + I*b*Log[1 - I*c*x^2])*((48*(1 - I*c*x^2))/c^4 - (36*(1 - I*c*x^2)^
2)/c^4 + (16*(1 - I*c*x^2)^3)/c^4 - (3*(1 - I*c*x^2)^4)/c^4 - (12*Log[1 - I
*c*x^2])/c^4) + (b*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/(16
*c^4) + ((I/24)*b^2*x^6*Log[1 + I*c*x^2])/c - (b^2*(1 + I*c*x^2)*Log[1 + I*
c*x^2])/(8*c^4) - (b^2*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2])/(16*c^4) - (b
*x^8*((2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/16 + (b^2*Log[1 + I*c
*x^2]^2)/(32*c^4) - (b^2*x^8*Log[1 + I*c*x^2]^2)/32 + (5*b^2*Log[I + c*x^2]
)/(192*c^4) - (b^2*PolyLog[2, (1 - I*c*x^2)/2])/(16*c^4) - (b^2*PolyLog[2,
(1 + I*c*x^2)/2])/(16*c^4)
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
```

```
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^7 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^7\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^7\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{4}b^2x^7\right.$$

$$= \frac{1}{4}\int x^7\left(2a+ib\log\left(1-icx^2\right)\right)^2 dx+\frac{1}{2}b\int x^7\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right) dx$$

$$= \frac{1}{8}\operatorname{Subst}\left(\int x^3(2a+ib\log(1-icx))^2 dx, x, x^2\right)+\frac{1}{4}b\operatorname{Subst}\left(\int x^3(-2ia+b\log(1-icx))\right.$$

$$= \frac{1}{32}x^8\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{16}bx^8\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{32}b^2x^8\log$$

$$= \frac{1}{32}x^8\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{16}bx^8\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{32}b^2x^8\log$$

$$= \frac{1}{32}x^8\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{192}ib\left(2a+ib\log\left(1-icx^2\right)\right)\left(\frac{48\left(1-icx^2\right)}{c^4}-\frac{36\left(1-ic\right.}{c^4}\right.$$

$$= \frac{abx^2}{8c^3}-\frac{bx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{32c^2}+\frac{ibx^6\left(2ia-b\log\left(1-icx^2\right)\right)}{48c}+\frac{1}{64}bx^8\left(2ia-b\log\left(1\right.\right.$$

$$= \frac{abx^2}{8c^3}-\frac{bx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{32c^2}+\frac{ibx^6\left(2ia-b\log\left(1-icx^2\right)\right)}{48c}+\frac{1}{64}bx^8\left(2ia-b\log\left(1\right.\right.$$

$$= \frac{abx^2}{8c^3}-\frac{55ib^2x^2}{192c^3}-\frac{5b^2x^4}{384c^2}+\frac{ib^2x^6}{576c}+\frac{b^2x^8}{256}-\frac{3b^2\left(1-icx^2\right)^2}{32c^4}+\frac{b^2\left(1-icx^2\right)^3}{36c^4}-\frac{b^2\left(1-icx\right.}{256c^4}$$

$$= \frac{abx^2}{8c^3}-\frac{55ib^2x^2}{192c^3}-\frac{5b^2x^4}{384c^2}+\frac{ib^2x^6}{576c}+\frac{b^2x^8}{256}-\frac{3b^2\left(1-icx^2\right)^2}{32c^4}+\frac{b^2\left(1-icx^2\right)^3}{36c^4}-\frac{b^2\left(1-icx\right.}{256c^4}$$

Mathematica [A] time = 0.0909221, size = 121, normalized size = 0.98

$$\frac{cx^2\left(3a^2c^3x^6-2abc^2x^4+6ab+b^2cx^2\right)-2b\tan^{-1}\left(cx^2\right)\left(a\left(3-3c^4x^8\right)+bcx^2\left(c^2x^4-3\right)\right)-4b^2\log\left(c^2x^4+1\right)+3b^2\left(c^4x^8\right.}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (c*x^2*(6*a*b + b^2*c*x^2 - 2*a*b*c^2*x^4 + 3*a^2*c^3*x^6) - 2*b*(b*c*x^2*(
-3 + c^2*x^4) + a*(3 - 3*c^4*x^8))*ArcTan[c*x^2] + 3*b^2*(-1 + c^4*x^8)*Arc
Tan[c*x^2]^2 - 4*b^2*Log[1 + c^2*x^4])/(24*c^4)
```

Maple [A] time = 0.036, size = 151, normalized size = 1.2

$$\frac{x^8a^2}{8}+\frac{b^2x^8\left(\arctan\left(cx^2\right)\right)^2}{8}-\frac{b^2\arctan\left(cx^2\right)x^6}{12\,c}+\frac{b^2x^2\arctan\left(cx^2\right)}{4\,c^3}-\frac{b^2\left(\arctan\left(cx^2\right)\right)^2}{8\,c^4}+\frac{b^2x^4}{24\,c^2}-\frac{b^2\ln\left(c^2x^4+1\right)}{6\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] 1/8*x^8*a^2+1/8*b^2*x^8*arctan(c*x^2)^2-1/12*b^2*arctan(c*x^2)/c*x^6+1/4*b^
2*x^2*arctan(c*x^2)/c^3-1/8*b^2/c^4*arctan(c*x^2)^2+1/24*b^2*x^4/c^2-1/6*b^
2*ln(c^2*x^4+1)/c^4+1/4*a*b*x^8*arctan(c*x^2)-1/12*a*b/c*x^6+1/4*a*b*x^2/c^
3-1/4*a*b/c^4*arctan(c*x^2)
```

Maxima [A] time = 1.77938, size = 228, normalized size = 1.84

$$\frac{1}{8}\,b^2x^8\arctan\left(cx^2\right)^2+\frac{1}{8}\,a^2x^8+\frac{1}{12}\left(3\,x^8\arctan\left(cx^2\right)-c\left(\frac{c^2x^6-3\,x^2}{c^4}+\frac{3\,\arctan\left(cx^2\right)}{c^5}\right)\right)ab-\frac{1}{24}\left(2\,c\left(\frac{c^2x^6-3\,x^2}{c^4}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8*arctan(c*x^2)^2 + 1/8*a^2*x^8 + 1/12*(3*x^8*arctan(c*x^2) - c*(
(c^2*x^6 - 3*x^2)/c^4 + 3*arctan(c*x^2)/c^5))*a*b - 1/24*(2*c*((c^2*x^6 - 3
*x^2)/c^4 + 3*arctan(c*x^2)/c^5)*arctan(c*x^2) - (c^2*x^4 + 3*arctan(c*x^2)
^2 - 3*log(12*c^7*x^4 + 12*c^5) - log(c^2*x^4 + 1))/c^4)*b^2
```

Fricas [A] time = 2.99301, size = 301, normalized size = 2.43

$$\frac{3\,a^2c^4x^8-2\,abc^3x^6+b^2c^2x^4+6\,abcx^2+3\left(b^2c^4x^8-b^2\right)\arctan\left(cx^2\right)^2+6\,ab\arctan\left(\frac{1}{cx^2}\right)-4\,b^2\log\left(c^2x^4+1\right)+2\left(3\right.}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^2*c^4*x^8 - 2*a*b*c^3*x^6 + b^2*c^2*x^4 + 6*a*b*c*x^2 + 3*(b^2*c^
4*x^8 - b^2)*arctan(c*x^2)^2 + 6*a*b*arctan(1/(c*x^2)) - 4*b^2*log(c^2*x^4
+ 1) + 2*(3*a*b*c^4*x^8 - b^2*c^3*x^6 + 3*b^2*c*x^2)*arctan(c*x^2))/c^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19591, size = 196, normalized size = 1.58

$$\frac{3\,a^2cx^8+2\left(3\,cx^8\arctan\left(cx^2\right)-\frac{3\,\arctan\left(cx^2\right)}{c^3}-\frac{c^9x^6-3\,c^7x^2}{c^9}\right)ab+\left(3\,cx^8\arctan\left(cx^2\right)^2-\frac{2\,c^3x^6\arctan\left(cx^2\right)-c^2x^4-6\,cx^2\arctan\left(c\right.}{c^3}\right.}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*c*x^8 + 2*(3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x
^6 - 3*c^7*x^2)/c^9)*a*b + (3*c*x^8*arctan(c*x^2)^2 - (2*c^3*x^6*arctan(c*x
^2) - c^2*x^4 - 6*c*x^2*arctan(c*x^2) + 3*arctan(c*x^2)^2 + 4*log(c^2*x^4 +
 1))/c^3)*b^2)/c
```

3.75 $\int x^5 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=154

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{6c^3}-\frac{i\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{6c^3}-\frac{b\log\left(\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3c^3}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-$$

```
[Out] (b^2*x^2)/(6*c^2) - (b^2*ArcTan[c*x^2])/(6*c^3) - (b*x^4*(a + b*ArcTan[c*x^
2]))/(6*c) - ((I/6)*(a + b*ArcTan[c*x^2])^2)/c^3 + (x^6*(a + b*ArcTan[c*x^2
])^2)/6 - (b*(a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/(3*c^3) - ((I/6)*b
^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c^3
```

Rubi [B] time = 1.36568, antiderivative size = 647, normalized size of antiderivative = 4.2, number of steps used = 53, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {5035, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)}{12c^3}-\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)}{12c^3}-\frac{iabx^2}{6c^2}+\frac{1}{72}ib\left(\frac{2i\left(1-icx^2\right)^3}{c^3}-\frac{9i\left(1-icx^2\right)^2}{c^3}+\frac{18i\left(1\right.}{c}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] ((-I/6)*a*b*x^2)/c^2 + (19*b^2*x^2)/(72*c^2) - (((5*I)/144)*b^2*x^4)/c + (b
^2*x^6)/108 - ((I/16)*b^2*(1 - I*c*x^2)^2)/c^3 + ((I/108)*b^2*(1 - I*c*x^2)
^3)/c^3 + ((I/12)*b^2*Log[I - c*x^2])/c^3 + ((I/12)*b^2*(1 - I*c*x^2)*Log[1
 - I*c*x^2])/c^3 - ((I/24)*b^2*Log[1 - I*c*x^2]^2)/c^3 + ((I/24)*b*x^4*((2*
I)*a - b*Log[1 - I*c*x^2]))/c + (b*x^6*((2*I)*a - b*Log[1 - I*c*x^2]))/36 +
 (x^6*(2*a + I*b*Log[1 - I*c*x^2])^2)/24 + (I/72)*b*(2*a + I*b*Log[1 - I*c*
x^2])*(((18*I)*(1 - I*c*x^2))/c^3 - ((9*I)*(1 - I*c*x^2)^2)/c^3 + ((2*I)*(1
 - I*c*x^2)^3)/c^3 - ((6*I)*Log[1 - I*c*x^2])/c^3) - ((I/12)*b*((2*I)*a - b
*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/c^3 + ((I/12)*b^2*x^4*Log[1 + I*c*
x^2])/c - ((I/12)*b^2*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2])/c^3 - (b*x^6*(
(2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/12 - ((I/24)*b^2*Log[1 + I*
c*x^2]^2)/c^3 - (b^2*x^6*Log[1 + I*c*x^2]^2)/24 - ((I/72)*b^2*Log[I + c*x^2
])/c^3 + ((I/12)*b^2*PolyLog[2, (1 - I*c*x^2)/2])/c^3 - ((I/12)*b^2*PolyLog
[2, (1 + I*c*x^2)/2])/c^3
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
```

```
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^5 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^5\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^5\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{4}b^2x^5 \ldots\right.$$

$$=\frac{1}{4}\int x^5\left(2a+ib\log\left(1-icx^2\right)\right)^2dx+\frac{1}{2}b\int x^5\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)dx \ldots$$

$$=\frac{1}{8}\text{Subst}\left(\int x^2(2a+ib\log(1-icx))^2dx,x,x^2\right)+\frac{1}{4}b\,\text{Subst}\left(\int x^2(-2ia+b\log(1-icx)) \ldots\right.$$

$$=\frac{1}{24}x^6\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{24}b^2x^6\log \ldots$$

$$=\frac{1}{24}x^6\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{24}b^2x^6\log \ldots$$

$$=\frac{1}{24}x^6\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{72}ib\left(2a+ib\log\left(1-icx^2\right)\right)\left(\frac{18i\left(1-icx^2\right)}{c^3}-\frac{9i\left(1-icx \ldots\right.}{c^3} \ldots\right.$$

$$=-\frac{iabx^2}{6c^2}+\frac{ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{24c}+\frac{1}{36}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{1}{24}x^6\left(2a+ib\log \ldots\right.$$

$$=-\frac{iabx^2}{6c^2}+\frac{ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)}{24c}+\frac{1}{36}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{1}{24}x^6\left(2a+ib\log \ldots\right.$$

$$=-\frac{iabx^2}{6c^2}+\frac{13b^2x^2}{72c^2}+\frac{ib^2x^4}{144c}+\frac{b^2x^6}{108}-\frac{ib^2\left(1-icx^2\right)^2}{16c^3}+\frac{ib^2\left(1-icx^2\right)^3}{108c^3}+\frac{ib^2\left(1-icx^2\right)\log \ldots}{12c^3} \ldots$$

$$=-\frac{iabx^2}{6c^2}+\frac{13b^2x^2}{72c^2}+\frac{ib^2x^4}{144c}+\frac{b^2x^6}{108}-\frac{ib^2\left(1-icx^2\right)^2}{16c^3}+\frac{ib^2\left(1-icx^2\right)^3}{108c^3}+\frac{ib^2\left(1-icx^2\right)\log \ldots}{12c^3} \ldots$$

Mathematica [A] time = 0.288996, size = 141, normalized size = 0.92

$$\frac{ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}\left(cx^2\right)}\right)+a^2c^3x^6-abc^2x^4+ab\log\left(c^2x^4+1\right)-b\tan^{-1}\left(cx^2\right)\left(-2ac^3x^6+bc^2x^4+2b\log\left(1+e^{2i\tan \ldots}\right.\right.}{6c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*c*x^2 - a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(I + c^3*x^6)*ArcTan[c*x^2]^2
- b*ArcTan[c*x^2]*(b + b*c^2*x^4 - 2*a*c^3*x^6 + 2*b*Log[1 + E^((2*I)*ArcTa
n[c*x^2])]) + a*b*Log[1 + c^2*x^4] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^
2])])/(6*c^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] int(x^5*(a+b*arctan(c*x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2x^5\arctan\left(cx^2\right)^2+2\,abx^5\arctan\left(cx^2\right)+a^2x^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^5*arctan(c*x^2)^2 + 2*a*b*x^5*arctan(c*x^2) + a^2*x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral(x**5*(a + b*atan(c*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(cx^2\right)+a\right)^2x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*x^5, x)
```

3.76 $\int x^3 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=90

$$\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{4c^2}-\frac{abx^2}{2c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)^2+\frac{b^2\log\left(c^2x^4+1\right)}{4c^2}-\frac{b^2x^2\tan^{-1}\left(cx^2\right)}{2c}$$

```
[Out] -(a*b*x^2)/(2*c) - (b^2*x^2*ArcTan[c*x^2])/(2*c) + (a + b*ArcTan[c*x^2])^2/
(4*c^2) + (x^4*(a + b*ArcTan[c*x^2])^2)/4 + (b^2*Log[1 + c^2*x^4])/(4*c^2)
```

Rubi [C] time = 1.04173, antiderivative size = 612, normalized size of antiderivative = 6.8, number of steps used = 44, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)}{8c^2}-\frac{\left(1-icx^2\right)^2\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c^2}+\frac{\left(1-icx^2\right)\left(2a+ib\,\mathrm{lo}\right.}{8c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (-3*a*b*x^2)/(4*c) + (b^2*x^4)/16 + (b^2*(1 - I*c*x^2)^2)/(32*c^2) + (b^2*(
1 + I*c*x^2)^2)/(32*c^2) - (b^2*Log[I - c*x^2])/(16*c^2) + (3*b^2*(1 - I*c*
x^2)*Log[1 - I*c*x^2])/(8*c^2) + (b*x^4*((2*I)*a - b*Log[1 - I*c*x^2]))/16
+ ((I/16)*b*(1 - I*c*x^2)^2*(2*a + I*b*Log[1 - I*c*x^2]))/c^2 + ((1 - I*c*x
^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/(8*c^2) - ((1 - I*c*x^2)^2*(2*a + I*b*L
og[1 - I*c*x^2])^2)/(16*c^2) - (b*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I
*c*x^2)/2])/(8*c^2) - (b^2*x^4*Log[1 + I*c*x^2])/16 + (3*b^2*(1 + I*c*x^2)*
Log[1 + I*c*x^2])/(8*c^2) - (b^2*(1 + I*c*x^2)^2*Log[1 + I*c*x^2])/(16*c^2)
 + (b^2*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2])/(8*c^2) - (b*x^4*((2*I)*a -
b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/8 - (b^2*(1 + I*c*x^2)*Log[1 + I*c*x^
2]^2)/(8*c^2) + (b^2*(1 + I*c*x^2)^2*Log[1 + I*c*x^2]^2)/(16*c^2) - (b^2*Lo
g[I + c*x^2])/(16*c^2) + (b^2*PolyLog[2, (1 - I*c*x^2)/2])/(8*c^2) + (b^2*P
olyLog[2, (1 + I*c*x^2)/2])/(8*c^2)
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
```

```
d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
```

```
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)^2dx=\int\left(\frac{1}{4}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^3\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{4}b^2x^3\right.$$

$$=\frac{1}{4}\int x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2dx+\frac{1}{2}b\int x^3\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)dx$$

$$=\frac{1}{8}\operatorname{Subst}\left(\int x(2a+ib\log(1-icx))^2\,dx,x,x^2\right)+\frac{1}{4}b\operatorname{Subst}\left(\int x(-2ia+b\log(1-icx))\log\right.$$

$$=-\frac{1}{8}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)+\frac{1}{8}\operatorname{Subst}\left(\int\left(-\frac{i(2a+ib\log(1-icx))^2}{c}+\right.\right.$$

$$=-\frac{1}{8}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{i\operatorname{Subst}\left(\int(2a+ib\log(1-icx))^2\,dx,x,x^2\right)}{8c}$$

$$=-\frac{1}{8}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{8}b\operatorname{Subst}\left(\int x(-2ia+b\log(1-icx))\,dx,x\right.$$

$$=-\frac{abx^2}{4c}+\frac{1}{16}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c^2}-\frac{\left(1-icx^2\right)}{}$$

$$=-\frac{3abx^2}{4c}-\frac{3ib^2x^2}{8c}+\frac{b^2\left(1-icx^2\right)^2}{32c^2}+\frac{b^2\left(1+icx^2\right)^2}{32c^2}+\frac{1}{16}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{ib}{}$$

$$=-\frac{3abx^2}{4c}+\frac{b^2x^4}{16}+\frac{b^2\left(1-icx^2\right)^2}{32c^2}+\frac{b^2\left(1+icx^2\right)^2}{32c^2}-\frac{b^2\log\left(i-cx^2\right)}{16c^2}+\frac{3b^2\left(1-icx^2\right)\log\left(\right.}{8c^2}$$

Mathematica [A] time = 0.0640039, size = 85, normalized size = 0.94

$$\frac{2b\tan^{-1}\left(cx^2\right)\left(ac^2x^4+a-bcx^2\right)+acx^2\left(acx^2-2b\right)+b^2\log\left(c^2x^4+1\right)+b^2\left(c^2x^4+1\right)\tan^{-1}\left(cx^2\right)^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (a*c*x^2*(-2*b + a*c*x^2) + 2*b*(a - b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b
^2*(1 + c^2*x^4)*ArcTan[c*x^2]^2 + b^2*Log[1 + c^2*x^4])/(4*c^2)
```

Maple [A] time = 0.033, size = 113, normalized size = 1.3

$$\frac{a^2x^4}{4}+\frac{b^2x^4\left(\arctan\left(cx^2\right)\right)^2}{4}-\frac{b^2x^2\arctan\left(cx^2\right)}{2c}+\frac{b^2\left(\arctan\left(cx^2\right)\right)^2}{4c^2}+\frac{b^2\ln\left(c^2x^4+1\right)}{4c^2}+\frac{abx^4\arctan\left(cx^2\right)}{2}-\frac{ab}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctan(c*x^2)^2-1/2*b^2*x^2*arctan(c*x^2)/c+1/4*b^2
/c^2*arctan(c*x^2)^2+1/4*b^2*ln(c^2*x^4+1)/c^2+1/2*a*b*x^4*arctan(c*x^2)-1/
2*a*b*x^2/c+1/2*a*b/c^2*arctan(c*x^2)
```

Maxima [A] time = 1.75226, size = 170, normalized size = 1.89

$$\frac{1}{4}b^2x^4\arctan\left(cx^2\right)^2+\frac{1}{4}a^2x^4+\frac{1}{2}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)ab-\frac{1}{4}\left(2c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^2*x^4 + 1/2*(x^4*arctan(c*x^2) - c*(x^2
/c^2 - arctan(c*x^2)/c^3))*a*b - 1/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)*arc
tan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*b^2
```

Fricas [A] time = 2.77326, size = 227, normalized size = 2.52

$$\frac{a^2c^2x^4-2abcx^2+\left(b^2c^2x^4+b^2\right)\arctan\left(cx^2\right)^2-2ab\arctan\left(\frac{1}{cx^2}\right)+b^2\log\left(c^2x^4+1\right)+2\left(abc^2x^4-b^2cx^2\right)\arctan\left(c\right.}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*c^2*x^4 - 2*a*b*c*x^2 + (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - 2*a*
b*arctan(1/(c*x^2)) + b^2*log(c^2*x^4 + 1) + 2*(a*b*c^2*x^4 - b^2*c*x^2)*ar
```

```
ctan(c*x^2))/c^2
```

Sympy [A] time = 63.5517, size = 529, normalized size = 5.88

$$\begin{cases} \frac{a^2x^4}{4} \\ \frac{x^4(a-\infty ib)^2}{4} \\ \frac{x^4(a+\infty ib)^2}{4} \\ \frac{a^2c^4x^8}{4c^4x^4+4c^2} - \frac{a^2}{4c^4x^4+4c^2} + \frac{2abc^4x^8\operatorname{atan}(cx^2)}{4c^4x^4+4c^2} - \frac{2abc^3x^6}{4c^4x^4+4c^2} + \frac{4abc^2x^4\operatorname{atan}(cx^2)}{4c^4x^4+4c^2} - \frac{2abcx^2}{4c^4x^4+4c^2} + \frac{2ab\operatorname{atan}(cx^2)}{4c^4x^4+4c^2} + \frac{2ib^2c^5x^4\left(\frac{1}{c^2}\right)^{\frac{3}{2}}\operatorname{atan}(cx^2)}{4c^4x^4+4c^2} + \frac{b^2c}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4, Eq(c, 0)), (x**4*(a - oo*I*b)**2/4, Eq(c, -I/x**2))
, (x**4*(a + oo*I*b)**2/4, Eq(c, I/x**2)), (a**2*c**4*x**8/(4*c**4*x**4 + 4
*c**2) - a**2/(4*c**4*x**4 + 4*c**2) + 2*a*b*c**4*x**8*atan(c*x**2)/(4*c**4
*x**4 + 4*c**2) - 2*a*b*c**3*x**6/(4*c**4*x**4 + 4*c**2) + 4*a*b*c**2*x**4*
atan(c*x**2)/(4*c**4*x**4 + 4*c**2) - 2*a*b*c*x**2/(4*c**4*x**4 + 4*c**2) +
 2*a*b*atan(c*x**2)/(4*c**4*x**4 + 4*c**2) + 2*I*b**2*c**5*x**4*(c**(-2))**
(3/2)*atan(c*x**2)/(4*c**4*x**4 + 4*c**2) + b**2*c**4*x**8*atan(c*x**2)**2/
(4*c**4*x**4 + 4*c**2) - 2*b**2*c**3*x**6*atan(c*x**2)/(4*c**4*x**4 + 4*c**
2) + 2*b**2*c**2*x**4*log(x**2 + I*sqrt(c**(-2)))/(4*c**4*x**4 + 4*c**2) +
2*b**2*c**2*x**4*atan(c*x**2)**2/(4*c**4*x**4 + 4*c**2) - 2*b**2*c*x**2*ata
n(c*x**2)/(4*c**4*x**4 + 4*c**2) + 2*I*b**2*c*sqrt(c**(-2))*atan(c*x**2)/(4
*c**4*x**4 + 4*c**2) + 2*b**2*log(x**2 + I*sqrt(c**(-2)))/(4*c**4*x**4 + 4*
c**2) + b**2*atan(c*x**2)**2/(4*c**4*x**4 + 4*c**2), True))
```

Giac [A] time = 1.19708, size = 135, normalized size = 1.5

$$\frac{a^2cx^4 + \frac{2\left(c^2x^4\arctan\left(cx^2\right) - cx^2 + \arctan\left(cx^2\right)\right)ab}{c} + \frac{\left(c^2x^4\arctan\left(cx^2\right)^2 - 2\,cx^2\arctan\left(cx^2\right) + \arctan\left(cx^2\right)^2 + \log\left(c^2x^4+1\right)\right)b^2}{c}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*c*x^4 + 2*(c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*a*b/c +
(c^2*x^4*arctan(c*x^2)^2 - 2*c*x^2*arctan(c*x^2) + arctan(c*x^2)^2 + log(c^
2*x^4 + 1))*b^2/c)/c
```

3.77 $\int x\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=101

$$\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{2c}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^2+\frac{i\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{2c}+\frac{b\log\left(\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)}{c}$$

```
[Out] ((I/2)*(a + b*ArcTan[c*x^2])^2)/c + (x^2*(a + b*ArcTan[c*x^2])^2)/2 + (b*(a
 + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c + ((I/2)*b^2*PolyLog[2, 1 - 2/(
1 + I*c*x^2)])/c
```

Rubi [B] time = 0.553597, antiderivative size = 255, normalized size of antiderivative = 2.52, number of steps used = 28, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)}{4c}+\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)}{4c}+\frac{ib\log\left(\frac{1}{2}\left(1+icx^2\right)\right)\left(2ia-b\log\left(1-icx^2\right)\right)}{4c}-\frac{1}{4}bx^2$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] ((I/8)*(1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/c + ((I/4)*b*((2*I)*a
- b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/c + ((I/4)*b^2*Log[(1 - I*c*x^2
)/2]*Log[1 + I*c*x^2])/c - (b*x^2*((2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*
c*x^2])/4 + ((I/8)*b^2*(1 + I*c*x^2)*Log[1 + I*c*x^2]^2)/c - ((I/4)*b^2*Pol
yLog[2, (1 - I*c*x^2)/2])/c + ((I/4)*b^2*PolyLog[2, (1 + I*c*x^2)/2])/c
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{4}b^2x\,\mathrm{l}$$

$$=\frac{1}{4}\int x\left(2a+ib\log\left(1-icx^2\right)\right)^2 dx+\frac{1}{2}b\int x\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right) dx$$

$$=\frac{1}{8}\operatorname{Subst}\left(\int(2a+ib\log(1-icx))^2\,dx,x,x^2\right)+\frac{1}{4}b\operatorname{Subst}\left(\int(-2ia+b\log(1-icx))\log\right.$$

$$=-\frac{1}{4}bx^2\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)+\frac{i\operatorname{Subst}\left(\int(2a+ib\log(x))^2\,dx,x,1-icx\right.}{8c}$$

$$=\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}-\frac{1}{4}bx^2\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)+\frac{ib^2}{}$$

$$=-\frac{1}{2}iabx^2-\frac{b^2x^2}{4}+\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}-\frac{ib^2\left(1+icx^2\right)\log\left(1+icx^2\right)}{4c}-$$

$$=-\frac{1}{2}b^2x^2+\frac{ib^2\left(1-icx^2\right)\log\left(1-icx^2\right)}{4c}+\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}+\frac{ib\left(2ia\right.}{}$$

$$=-\frac{1}{4}b^2x^2+\frac{ib^2\left(1-icx^2\right)\log\left(1-icx^2\right)}{4c}+\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}+\frac{ib\left(2ia\right.}{}$$

$$=\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}+\frac{ib\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(\frac{1}{2}\left(1+icx^2\right)\right)}{4c}+\frac{ib}{}$$

Mathematica [A] time = 0.0869276, size = 107, normalized size = 1.06

$$\frac{-ib^2\operatorname{PolyLog}\left(2,-e^{2i\tan^{-1}\left(cx^2\right)}\right)+a\left(acx^2-b\log\left(c^2x^4+1\right)\right)+2b\tan^{-1}\left(cx^2\right)\left(acx^2+b\log\left(1+e^{2i\tan^{-1}\left(cx^2\right)}\right)\right)+b^2\left(c\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*(-I + c*x^2)*ArcTan[c*x^2]^2 + 2*b*ArcTan[c*x^2]*(a*c*x^2 + b*Log[1 +
E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2 - b*Log[1 + c^2*x^4]) - I*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x^2])])/(2*c)
```

Maple [A] time = 0.087, size = 146, normalized size = 1.5

$$\frac{x^2b^2\left(\arctan\left(cx^2\right)\right)^2}{2}+x^2ab\arctan\left(cx^2\right)-\frac{\frac{i}{2}\left(\arctan\left(cx^2\right)\right)^2b^2}{c}+\frac{a^2x^2}{2}+\frac{\arctan\left(cx^2\right)b^2}{c}\ln\left(\frac{\left(1+icx^2\right)^2}{c^2x^4+1}+1\right)-\frac{\frac{i}{2}b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] 1/2*x^2*b^2*arctan(c*x^2)^2+x^2*a*b*arctan(c*x^2)-1/2*I/c*arctan(c*x^2)^2*b
^2+1/2*a^2*x^2+1/c*arctan(c*x^2)*ln((1+I*c*x^2)^2/(c^2*x^4+1)+1)*b^2-1/2*I/
c*polylog(2,-(1+I*c*x^2)^2/(c^2*x^4+1))*b^2-1/2/c*a*b*ln(c^2*x^4+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2x^2 + \frac{1}{32}\left(4x^2\arctan\left(cx^2\right)^2 - x^2\log\left(c^2x^4+1\right)^2 + 384c^2\int\frac{x^5\arctan\left(cx^2\right)^2}{16\left(c^2x^4+1\right)}\,dx + 32c^2\int\frac{x^5\log\left(c^2x^4+1\right)^2}{16\left(c^2x^4+1\right)}\,dx + 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/32*(4*x^2*arctan(c*x^2)^2 - x^2*log(c^2*x^4 + 1)^2 + 384*c^
2*integrate(1/16*x^5*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 32*c^2*integrate(1
/16*x^5*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 128*c^2*integrate(1/16*x^5*l
og(c^2*x^4 + 1)/(c^2*x^4 + 1), x) + 4*arctan(c*x^2)^3/c - 256*c*integrate(1
/16*x^3*arctan(c*x^2)/(c^2*x^4 + 1), x) + 32*integrate(1/16*x*log(c^2*x^4 +
 1)^2/(c^2*x^4 + 1), x))*b^2 + 1/2*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1
))*a*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x\arctan\left(cx^2\right)^2 + 2\,abx\arctan\left(cx^2\right) + a^2x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*arctan(c*x^2)^2 + 2*a*b*x*arctan(c*x^2) + a^2*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + b\,\text{atan}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral(x*(a + b*atan(c*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(cx^2\right) + a\right)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*x, x)
```

3.78 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x}\,dx$

Optimal. Leaf size=151

$$-\frac{1}{2}ib\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{1}{2}ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{4}b^2\text{PolyL}$$

```
[Out] (a + b*ArcTan[c*x^2])^2*ArcTanh[1 - 2/(1 + I*c*x^2)] - (I/2)*b*(a + b*ArcTa
n[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + (I/2)*b*(a + b*ArcTan[c*x^2])*P
olyLog[2, -1 + 2/(1 + I*c*x^2)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 +
 (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^2)])/4
```

Rubi [A] time = 0.320927, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5031, 4850, 4988, 4884, 4994, 6610}

$$-\frac{1}{2}ib\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{1}{2}ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{4}b^2\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x,x]
```


```
[Out] (a + b*ArcTan[c*x^2])^2*ArcTanh[1 - 2/(1 + I*c*x^2)] - (I/2)*b*(a + b*ArcTa
n[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + (I/2)*b*(a + b*ArcTan[c*x^2])*P
olyLog[2, -1 + 2/(1 + I*c*x^2)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 +
 (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^2)])/4
```


Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx,x,x^2\right)$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-(2bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\tanh^{-1}\left(1-\right.}{1+c^2x^2}\right.$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)+(bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,d\right.$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)\operatorname{Li}_2\left(1-\frac{2}{1+icx^2}\right)+\frac{1}{2}$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)\operatorname{Li}_2\left(1-\frac{2}{1+icx^2}\right)+\frac{1}{2}$$

Mathematica [A] time = 0.0996191, size = 165, normalized size = 1.09

$$\frac{1}{4}b\left(2i\operatorname{PolyLog}\left(2,\frac{cx^2+i}{-cx^2+i}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-2i\operatorname{PolyLog}\left(2,\frac{cx^2+i}{cx^2-i}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+b\left(\operatorname{PolyLog}\left(3,\frac{cx^2+i}{-cx^2+}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^2*ArcTanh[1 + (2*I)/(-I + c*x^2)] + (b*((2*I)*(a + b*
ArcTan[c*x^2])*PolyLog[2, (I + c*x^2)/(I - c*x^2)] - (2*I)*(a + b*ArcTan[c*
x^2])*PolyLog[2, (I + c*x^2)/(-I + c*x^2)] + b*(PolyLog[3, (I + c*x^2)/(I -
 c*x^2)] - PolyLog[3, (I + c*x^2)/(-I + c*x^2)])))/4
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \frac{1}{16} \int \frac{12\, b^2 \arctan\left(cx^2\right)^2 + b^2 \log\left(c^2x^4 + 1\right)^2 + 32\, ab \arctan\left(cx^2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^2)^2 + b^2*log(c^2*x^4 + 1)^
2 + 32*a*b*arctan(c*x^2))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan\left(cx^2\right)^2 + 2\, ab \arctan\left(cx^2\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atan}\left(cx^2\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x, x)
```

3.79 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{2}ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)-\frac{1}{2}ic\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{2x^2}+bc\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\tan^{-1}\right.$$

```
[Out] (-I/2)*c*(a + b*ArcTan[c*x^2])^2 - (a + b*ArcTan[c*x^2])^2/(2*x^2) + b*c*(a
 + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)] - (I/2)*b^2*c*PolyLog[2, -1 +
2/(1 - I*c*x^2)]
```

Rubi [B] time = 0.65346, antiderivative size = 290, normalized size of antiderivative = 2.99, number of steps used = 24, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5035, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$\frac{1}{2}ib^2c\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{2}ib^2c\text{PolyLog}\left(2,icx^2\right)-\frac{1}{4}ib^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)+\frac{1}{4}ib^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

```
[Out] 2*a*b*c*Log[x] - ((1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/(8*x^2) + (
I/4)*b*c*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2] + (I/4)*b^2*c*
Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2] + (b*((2*I)*a - b*Log[1 - I*c*x^2])*L
og[1 + I*c*x^2])/(4*x^2) + (b^2*(1 + I*c*x^2)*Log[1 + I*c*x^2]^2)/(8*x^2) +
 (I/2)*b^2*c*PolyLog[2, (-I)*c*x^2] - (I/2)*b^2*c*PolyLog[2, I*c*x^2] - (I/
4)*b^2*c*PolyLog[2, (1 - I*c*x^2)/2] + (I/4)*b^2*c*PolyLog[2, (1 + I*c*x^2)
/2]
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^3}+\frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^3}-\frac{b^2\log^2\left(1+icx^2\right)}{4x^3}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^3}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^3}\,dx-\frac{1}{4}b^2$$

$$=\frac{1}{8}\,\mathrm{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^2}\,dx,x,x^2\right)+\frac{1}{4}b\,\mathrm{Subst}\left(\int \frac{(-2ia+b\log(1-icx))\log(1-}{x^2}\right.$$

$$=-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{4x^2}+\frac{b^2\left(1+icx\right.}{}$$

$$=abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{4x^2}+$$

$$=abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{4x^2}+$$

$$=2abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{1}{4}ibc\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(\frac{1}{2}\left(1+\right.\right.$$

$$=2abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{1}{4}ibc\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(\frac{1}{2}\left(1+\right.\right.$$

$$=2abc\log(x)-\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8x^2}+\frac{1}{4}ibc\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(\frac{1}{2}\left(1+\right.\right.$$

Mathematica [A] time = 0.148633, size = 127, normalized size = 1.31

$$\frac{1}{2}b^2c\left(-i\left(\tan^{-1}\left(cx^2\right)^2+\mathrm{PolyLog}\left(2,e^{2i\tan^{-1}\left(cx^2\right)}\right)\right)-\frac{\tan^{-1}\left(cx^2\right)^2}{cx^2}+2\tan^{-1}\left(cx^2\right)\log\left(1-e^{2i\tan^{-1}\left(cx^2\right)}\right)\right)-\frac{a^2}{2x^2}+abc\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + a*b*c*(-(ArcTan[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 + c^2*x
^4]/2) + (b^2*c*(-(ArcTan[c*x^2]^2/(c*x^2)) + 2*ArcTan[c*x^2]*Log[1 - E^((2
*I)*ArcTan[c*x^2])] - I*(ArcTan[c*x^2]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x^2
])])))/2
```

Maple [F] time = 0.337, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^3,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan\left(cx^2\right)^2 + 2\,ab\arctan\left(cx^2\right) + a^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\text{atan}\left(cx^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arctan\left(cx^2\right) + a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^3, x)
```

3.80 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{4}c^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-\frac{bc\left(a+b\tan^{-1}\left(cx^2\right)\right)}{2x^2}-\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{4x^4}-\frac{1}{4}b^2c^2\log\left(c^2x^4+1\right)+b^2c^2\log(x)$$

```
[Out] -(b*c*(a + b*ArcTan[c*x^2]))/(2*x^2) - (c^2*(a + b*ArcTan[c*x^2])^2)/4 - (a
 + b*ArcTan[c*x^2])^2/(4*x^4) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^4])
/4
```

Rubi [C] time = 1.13623, antiderivative size = 419, normalized size of antiderivative = 4.82, number of steps used = 46, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^2\right)\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^2\right)\right)+\frac{1}{8}bc^2\log\left(\frac{1}{2}\left(1+icx^2\right)\right)\left(2ia-b\log\left(1-icx^2\right)\right)-$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

```
[Out] b^2*c^2*Log[x] - (b^2*c^2*Log[I - c*x^2])/4 + ((I/8)*b*c*((2*I)*a - b*Log[1
 - I*c*x^2]))/x^2 - (b*c*(1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c*x^2]))/(8*x^2
) - (c^2*(2*a + I*b*Log[1 - I*c*x^2])^2)/16 - (2*a + I*b*Log[1 - I*c*x^2])^
2/(16*x^4) + (b*c^2*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/8
+ ((I/4)*b^2*c*Log[1 + I*c*x^2])/x^2 - (b^2*c^2*Log[(1 - I*c*x^2)/2]*Log[1
+ I*c*x^2])/8 + (b*((2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/(8*x^4)
 + (b^2*c^2*Log[1 + I*c*x^2]^2)/16 + (b^2*Log[1 + I*c*x^2]^2)/(16*x^4) - (b
^2*c^2*Log[I + c*x^2])/8 - (b^2*c^2*PolyLog[2, (1 - I*c*x^2)/2])/8 - (b^2*c
^2*PolyLog[2, (1 + I*c*x^2)/2])/8
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
```

```
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^5}+\frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^5}-\frac{b^2\log^2\left(1+ic\right.}{4x^5}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^5}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^5}\,dx-\frac{1}{4}$$

$$=\frac{1}{8}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^3}\,dx,x,x^2\right)+\frac{1}{4}b\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx))\log}{x^3}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16x^4}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{8x^4}+\frac{b^2\log^2\left(1+icx^2\right)}{16x^4}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16x^4}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{8x^4}+\frac{b^2\log^2\left(1+icx^2\right)}{16x^4}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16x^4}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{8x^4}+\frac{b^2\log^2\left(1+icx^2\right)}{16x^4}$$

$$=-\frac{1}{2}iabc^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^2\right)\right)}{8x^2}-\frac{bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)}{8x^2}-(2$$

$$=\frac{1}{4}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^2\right)\right)}{8x^2}-\frac{bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)}{8x^2}-\frac{1}{16}c^2$$

$$=\frac{1}{2}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^2\right)\right)}{8x^2}-\frac{bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)}{8x^2}-\frac{1}{16}c^2$$

Mathematica [A] time = 0.0769828, size = 98, normalized size = 1.13

$$-\frac{a^2+2b\tan^{-1}\left(cx^2\right)\left(ac^2x^4+a+bcx^2\right)+2abcx^2-4b^2c^2x^4\log(x)+b^2c^2x^4\log\left(c^2x^4+1\right)+b^2\left(c^2x^4+1\right)\tan^{-1}\left(cx^2\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

```
[Out] -(a^2 + 2*a*b*c*x^2 + 2*b*(a + b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b^2*(1
+ c^2*x^4)*ArcTan[c*x^2]^2 - 4*b^2*c^2*x^4*Log[x] + b^2*c^2*x^4*Log[1 + c^2
*x^4])/(4*x^4)
```

Maple [A] time = 0.037, size = 118, normalized size = 1.4

$$-\frac{a^2}{4x^4}-\frac{b^2\left(\arctan\left(cx^2\right)\right)^2}{4x^4}-\frac{b^2c\arctan\left(cx^2\right)}{2x^2}-\frac{b^2c^2\left(\arctan\left(cx^2\right)\right)^2}{4}-\frac{b^2c^2\ln\left(c^2x^4+1\right)}{4}+b^2c^2\ln\left(x\right)-\frac{ab\arctan\left(cx\right.}{2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-1/4*b^2/x^4*arctan(c*x^2)^2-1/2*b^2*c*arctan(c*x^2)/x^2-1/4*b^
2*c^2*arctan(c*x^2)^2-1/4*b^2*c^2*ln(c^2*x^4+1)+b^2*c^2*ln(x)-1/2*a*b/x^4*a
rctan(c*x^2)-1/2*c*a*b/x^2-1/2*a*b*c^2*arctan(c*x^2)
```

Maxima [A] time = 1.8367, size = 149, normalized size = 1.71

$$-\frac{1}{2}\left(\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c+\frac{\arctan\left(cx^2\right)}{x^4}\right)ab+\frac{1}{4}\left(\left(\arctan\left(cx^2\right)^2-\log\left(c^2x^4+1\right)+4\log\left(x\right)\right)c^2-2\left(c\arctan\left(cx^2\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a*b + 1/4*((arctan(c
*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2)*c*
arctan(c*x^2))*b^2 - 1/4*b^2*arctan(c*x^2)^2/x^4 - 1/4*a^2/x^4
```

Fricas [A] time = 2.73242, size = 258, normalized size = 2.97

$$\frac{2abc^2x^4\arctan\left(\frac{1}{cx^2}\right)-b^2c^2x^4\log\left(c^2x^4+1\right)+4b^2c^2x^4\log\left(x\right)-2abcx^2-\left(b^2c^2x^4+b^2\right)\arctan\left(cx^2\right)^2-a^2-2\left(b^2cx^2+\right.}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c^2*x^4*arctan(1/(c*x^2)) - b^2*c^2*x^4*log(c^2*x^4 + 1) + 4*b^2
*c^2*x^4*log(x) - 2*a*b*c*x^2 - (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - a^2 -
 2*(b^2*c*x^2 + a*b)*arctan(c*x^2))/x^4
```

Sympy [A] time = 86.8163, size = 1187, normalized size = 13.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4), Eq(c, 0)), (-(a - oo*I*b)**2/(4*x**4), Eq(c, -I/
x**2)), (-(a + oo*I*b)**2/(4*x**4), Eq(c, I/x**2)), (3*I*a**2*c**3*x**4*(c*
*(-2))**(3/2)/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c**(-2))**(3
/2)) + 3*I*a**2*c*(c**(-2))**(3/2)/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I
*c*x**4*(c**(-2))**(3/2)) + 6*I*a*b*c**5*x**8*(c**(-2))**(3/2)*atan(c*x**2)
/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c**(-2))**(3/2)) + 6*I*a*
b*c**4*x**6*(c**(-2))**(3/2)/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**
4*(c**(-2))**(3/2)) + 12*I*a*b*c**3*x**4*(c**(-2))**(3/2)*atan(c*x**2)/(-12
*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c**(-2))**(3/2)) + 6*I*a*b*c**
2*x**2*(c**(-2))**(3/2)/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c*
*(-2))**(3/2)) + 6*I*a*b*c*(c**(-2))**(3/2)*atan(c*x**2)/(-12*I*c**3*x**8*(
c**(-2))**(3/2) - 12*I*c*x**4*(c**(-2))**(3/2)) - 12*I*b**2*c**5*x**8*(c**(
-2))**(3/2)*log(x)/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c**(-2)
)**(3/2)) + 6*I*b**2*c**5*x**8*(c**(-2))**(3/2)*log(x**2 + I*sqrt(c**(-2)))
/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c**(-2))**(3/2)) + 3*I*b*
*2*c**5*x**8*(c**(-2))**(3/2)*atan(c*x**2)**2/(-12*I*c**3*x**8*(c**(-2))**(
3/2) - 12*I*c*x**4*(c**(-2))**(3/2)) + 6*I*b**2*c**4*x**6*(c**(-2))**(3/2)*
atan(c*x**2)/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c**(-2))**(3/
2)) - 12*I*b**2*c**3*x**4*(c**(-2))**(3/2)*log(x)/(-12*I*c**3*x**8*(c**(-2)
)**(3/2) - 12*I*c*x**4*(c**(-2))**(3/2)) + 6*I*b**2*c**3*x**4*(c**(-2))**(3
/2)*log(x**2 + I*sqrt(c**(-2)))/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*
x**4*(c**(-2))**(3/2)) + 6*I*b**2*c**3*x**4*(c**(-2))**(3/2)*atan(c*x**2)**
2/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c**(-2))**(3/2)) - 6*b**
2*c**2*x**8*atan(c*x**2)/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c
**(-2))**(3/2)) + 6*I*b**2*c**2*x**2*(c**(-2))**(3/2)*atan(c*x**2)/(-12*I*c
**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**4*(c**(-2))**(3/2)) + 3*I*b**2*c*(c**
(-2))**(3/2)*atan(c*x**2)**2/(-12*I*c**3*x**8*(c**(-2))**(3/2) - 12*I*c*x**
4*(c**(-2))**(3/2)) - 6*b**2*x**4*atan(c*x**2)/(-12*I*c**3*x**8*(c**(-2))**
(3/2) - 12*I*c*x**4*(c**(-2))**(3/2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(c x^2\right) + a\right)^2}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^5, x)
```

3.81 $\int x^2 \left(a + b \tan^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=1393

result too large to display

```
[Out] (-4*a*b*x)/(3*c) + ((2*I)/9)*a*b*x^3 + (4*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*
Sqrt[c]*x])/(3*c^(3/2)) + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/(
3*c^(3/2)) - (2*(-1)^(1/4)*a*b*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) -
 (4*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) - ((-1)^(3/4)
*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTa
n[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2
*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2
]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2*(
-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x
)])/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
+ (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]
*x))])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1
+ I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) -
 ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*
Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) - (((2*I)/3)*b^2*x*Log
[1 - I*c*x^2])/c - (b^2*x^3*Log[1 - I*c*x^2])/9 - ((-1)^(3/4)*b^2*ArcTanh[(
-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/(3*c^(3/2)) - (I/9)*b*x^3*(2*a + I*b
*Log[1 - I*c*x^2]) - ((-1)^(1/4)*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
Log[1 - I*c*x^2]))/(3*c^(3/2)) + (x^3*(2*a + I*b*Log[1 - I*c*x^2])^2)/12 +
(((2*I)/3)*b^2*x*Log[1 + I*c*x^2])/c - (I/3)*a*b*x^3*Log[1 + I*c*x^2] + ((-
1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2)) + (
(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2))
+ (b^2*x^3*Log[1 - I*c*x^2]*Log[1 + I*c*x^2])/6 - (b^2*x^3*Log[1 + I*c*x^2]
^2)/12 + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c
^(3/2)) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*
c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1 -
 2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1
- 2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1
 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/
2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - ((
1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2))
```

Rubi [A] time = 2.60206, antiderivative size = 1393, normalized size of antiderivative = 1., number of steps used = 86, number of rules used = 27, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.687, Rules used = {5035, 2457, 2476, 2448, 321, 203, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315, 6742, 206, 30, 2557, 205, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^2])^2,x]
```


```
[Out] (-4*a*b*x)/(3*c) + ((2*I)/9)*a*b*x^3 + (4*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*
Sqrt[c]*x])/(3*c^(3/2)) + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/(
3*c^(3/2)) - (2*(-1)^(1/4)*a*b*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) -
```

```
(4*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) - ((-1)^(3/4)
*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTa
n[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2
*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2
]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2*(
-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x
)])/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
+ (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]
*x))])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1
+ I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) -
 ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*
Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) - (((2*I)/3)*b^2*x*Log
[1 - I*c*x^2])/c - (b^2*x^3*Log[1 - I*c*x^2])/9 - ((-1)^(3/4)*b^2*ArcTanh[(
-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/(3*c^(3/2)) - (I/9)*b*x^3*(2*a + I*b
*Log[1 - I*c*x^2]) - ((-1)^(1/4)*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
Log[1 - I*c*x^2]))/(3*c^(3/2)) + (x^3*(2*a + I*b*Log[1 - I*c*x^2])^2)/12 +
(((2*I)/3)*b^2*x*Log[1 + I*c*x^2])/c - (I/3)*a*b*x^3*Log[1 + I*c*x^2] + ((-
1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2)) + (
(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2))
+ (b^2*x^3*Log[1 - I*c*x^2]*Log[1 + I*c*x^2])/6 - (b^2*x^3*Log[1 + I*c*x^2]
^2)/12 + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c
^(3/2)) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*
c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1 -
 2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1
- 2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1
 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/
2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - ((
1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2))
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
```

```
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx = \int\left(\frac{1}{4}x^2\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^2\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{4}b^2\right.$$

$$=\frac{1}{4}\int x^2\left(2a+ib\log\left(1-icx^2\right)\right)^2 dx+\frac{1}{2}b\int x^2\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}b^2x^3\log^2\left(1+icx^2\right)+\frac{1}{2}b\int\left(-2iax^2\log\left(1+icx^2\right)\right.$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}b^2x^3\log^2\left(1+icx^2\right)-(iab)\int x^2\log\left(1+icx^2\right)dx$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{3}iabx^3\log\left(1+icx^2\right)+\frac{1}{6}b^2x^3\log\left(1-icx^2\right)\log\left(1+i\right.$$

$$=-\frac{2abx}{3c}-\frac{1}{9}ibx^3\left(2a+ib\log\left(1-icx^2\right)\right)-\frac{\sqrt[4]{-1}b\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-ic\right.\right.}{3c^{3/2}}$$

$$=-\frac{4abx}{3c}-\frac{2ib^2x}{3c}+\frac{2}{9}iabx^3-\frac{ib^2x\log\left(1-icx^2\right)}{3c}-\frac{1}{9}ibx^3\left(2a+ib\log\left(1-icx^2\right)\right)-\frac{\sqrt[4]{-1}}{}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3-\frac{4b^2x^3}{27}+\frac{\sqrt[4]{-1}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{2\sqrt[4]{-1}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\right.}{3c^{3/2}}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3-\frac{4b^2x^3}{27}+\frac{8(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{9c^{3/2}}+\frac{\sqrt[4]{-1}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\right.}{3c^{3/2}}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3-\frac{4b^2x^3}{27}+\frac{8(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{9c^{3/2}}+\frac{\sqrt[4]{-1}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\right.}{3c^{3/2}}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{14(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{9c^{3/2}}+\frac{\sqrt[4]{-1}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{2}{}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{2\sqrt[4]{}}{}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{2\sqrt[4]{}}{}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{2\sqrt[4]{}}{}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{2\sqrt[4]{}}{}$$

Mathematica [F] time = 4.84914, size = 0, normalized size = 0.

$$\int x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] Integrate[x^2*(a + b*ArcTan[c*x^2])^2, x]
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int x^2 \left(a + b \arctan\left(c x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] int(x^2*(a+b*arctan(c*x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2 x^2 \arctan\left(c x^2\right)^2 + 2\, a b x^2 \arctan\left(c x^2\right) + a^2 x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c*x^2)^2 + 2*a*b*x^2*arctan(c*x^2) + a^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b \operatorname{atan}\left(c x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arctan\left(c x^2\right) + a\right)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*x^2, x)
```

3.82 $\int \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=1191

result too large to display

```
[Out] a^2*x - (2*(-1)^(3/4)*a*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] + ((-1)^(3/
4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/Sqrt[c] + (2*(-1)^(3/4)*a*b*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]
*x]^2)/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 -
(-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[
c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)
*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log
[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 +
 (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + I*a*b*x*Log[
1 - I*c*x^2] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2
])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2]
)/Sqrt[c] - (b^2*x*Log[1 - I*c*x^2]^2)/4 - I*a*b*x*Log[1 + I*c*x^2] - ((-1)
^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + ((-1)^(
1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + (b^2*x*L
og[1 - I*c*x^2]*Log[1 + I*c*x^2])/2 - (b^2*x*Log[1 + I*c*x^2]^2)/4 + ((-1)^
(3/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(3/
4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - ((-1)^(3/4)*
b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[
c]*x)])/(2*Sqrt[c]) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt
[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]
*x)])/Sqrt[c] - ((-1)^(1/4)*b^2*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[
c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqrt[c]) - ((-1)^(1/4)*b^2*PolyLog[2
, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*
Sqrt[c]) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*
x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sqrt[c])
```

Rubi [A] time = 1.77785, antiderivative size = 1191, normalized size of antiderivative = 1., number of steps used = 69, number of rules used = 23, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.917, Rules used = {5029, 2448, 321, 203, 2450, 2476, 2470, 12, 4920, 4854, 2402, 2315, 206, 2556, 205, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2,x]
```


```
[Out] a^2*x - (2*(-1)^(3/4)*a*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] + ((-1)^(3/
4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/Sqrt[c] + (2*(-1)^(3/4)*a*b*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]
*x]^2)/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 -
(-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[
c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)
```

```
*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log
[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 +
 (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + I*a*b*x*Log[
1 - I*c*x^2] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2
])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2]
)/Sqrt[c] - (b^2*x*Log[1 - I*c*x^2]^2)/4 - I*a*b*x*Log[1 + I*c*x^2] - ((-1)
^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + ((-1)^(
1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + (b^2*x*L
og[1 - I*c*x^2]*Log[1 + I*c*x^2])/2 - (b^2*x*Log[1 + I*c*x^2]^2)/4 + ((-1)^
(3/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(3/
4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - ((-1)^(3/4)*
b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[
c]*x)])/(2*Sqrt[c]) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt
[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]
*x)])/Sqrt[c] - ((-1)^(1/4)*b^2*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[
c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqrt[c]) - ((-1)^(1/4)*b^2*PolyLog[2
, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*
Sqrt[c]) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*
x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sqrt[c])
```

Rule 5029

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x],
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
```

]

Rubi steps

$$\int \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx = \int \left(a^2+iab\log\left(1-icx^2\right)-\frac{1}{4}b^2\log^2\left(1-icx^2\right)-iab\log\left(1+icx^2\right)+\frac{1}{2}b^2\log\left(1-icx^2\right)\log$$

$$= a^2x+(iab)\int \log\left(1-icx^2\right)\,dx-(iab)\int \log\left(1+icx^2\right)\,dx-\frac{1}{4}b^2\int \log^2\left(1-icx^2\right)\,dx-\frac{1}{4}b$$

$$= a^2x+iabx\log\left(1-icx^2\right)-\frac{1}{4}b^2x\log^2\left(1-icx^2\right)-iabx\log\left(1+icx^2\right)+\frac{1}{2}b^2x\log\left(1-icx^2\right)\text{lo}$$

$$= a^2x+iabx\log\left(1-icx^2\right)-\frac{1}{4}b^2x\log^2\left(1-icx^2\right)-iabx\log\left(1+icx^2\right)+\frac{1}{2}b^2x\log\left(1-icx^2\right)\text{lo}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+iabx\log\left(1-icx^2\right.$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+iabx\log\left(1-icx^2\right.$$

$$= a^2x-4b^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+iabx\log\left(1\right.$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}-\frac{2\sqrt[4]{-1}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)\right.}{\sqrt{c}}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left(\right.}{\sqrt{c}}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left(\right.}{\sqrt{c}}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left(\right.}{\sqrt{c}}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left(\right.}{\sqrt{c}}$$

$$= a^2x-\frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left(\right.}{\sqrt{c}}$$

Mathematica [C] time = 31.7773, size = 5466, normalized size = 4.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int \left(a + b \arctan \left(c x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2 \arctan \left(c x^2\right)^2 + 2\, a b \arctan \left(c x^2\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{atan}\left(c x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arctan \left(c x^2\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2, x)
```

3.83 $\int \frac{(a+b\tan^{-1}(cx^2))^2}{x^2}\,dx$

Optimal. Leaf size=1164

result too large to display

```
[Out] (-1)^(1/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2 - 2*(-1)^(1/4)*a*b*Sq
rt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x] - (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]^2 - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*L
og[2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/
4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)
^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]
*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)] - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^
(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^2*Sqrt[c]
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x))] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[
c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)]
- (-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1
)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2] - (-1)^(1/4)*b*Sqrt[c]*ArcTan[
(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*x^2]) - (2*a + I*b*Log[1 - I*c
*x^2])^2/(4*x) + (I*a*b*Log[1 + I*c*x^2])/x + (-1)^(3/4)*b^2*Sqrt[c]*ArcTan
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-
1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] - (b^2*Log[1 - I*c*x^2]*Log[1 + I*c*x^
2])/(2*x) + (b^2*Log[1 + I*c*x^2]^2)/(4*x) + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog
[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1
 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - ((-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1 - (
Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/2 + (-1)^(3/
4)*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^
2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]*x)] - ((-1)^(3/4)*b^2*Sq
rt[c]*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/2 - ((-1)^(3/4)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/
4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/2 - ((-1)^(1/4)*b^2*Sqrt[c]*Pol
yLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)
])/2
```

Rubi [A] time = 1.62168, antiderivative size = 1164, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2457, 203, 2470, 12, 4920, 4854, 2402, 2315, 2455, 6742, 206, 30, 2557, 205, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

$$\sqrt[4]{-1}\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 b^2 - (-1)^{3/4}\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 b^2 + \frac{\log^2\left(icx^2+1\right)b^2}{4x} - 2(-1)^{3/4}\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

```
[Out] (-1)^(1/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2 - 2*(-1)^(1/4)*a*b*Sq
rt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x] - (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]^2 - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*L
og[2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/
4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)
```

```
^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]
*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)] - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^
(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^2*Sqrt[c]
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x))] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[
c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)]
- (-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1
)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2] - (-1)^(1/4)*b*Sqrt[c]*ArcTan[
(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*x^2]) - (2*a + I*b*Log[1 - I*c
*x^2])^2/(4*x) + (I*a*b*Log[1 + I*c*x^2])/x + (-1)^(3/4)*b^2*Sqrt[c]*ArcTan
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-
1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] - (b^2*Log[1 - I*c*x^2]*Log[1 + I*c*x^
2])/(2*x) + (b^2*Log[1 + I*c*x^2]^2)/(4*x) + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog
[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1
 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - ((-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1 - (
Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/2 + (-1)^(3/
4)*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^
2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]*x)] - ((-1)^(3/4)*b^2*Sq
rt[c]*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/2 - ((-1)^(3/4)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/
4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/2 - ((-1)^(1/4)*b^2*Sqrt[c]*Pol
yLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)
])/2
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^2} + \frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^2} - \frac{b^2\log^2\left(1+icx^2\right)}{4x^2}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^2}\,dx + \frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^2}\,dx - \frac{1}{4}b^2$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x} + \frac{b^2\log^2\left(1+icx^2\right)}{4x} + \frac{1}{2}b\int \left(-\frac{2ia\log\left(1+icx^2\right)}{x^2} + \frac{b\log\left(1-icx\right.}{}\right.$$

$$= -\sqrt[4]{-1}b\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-icx^2\right)\right) - \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x} + (-1)^{3/4}b^2$$

$$= -\sqrt[4]{-1}b\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-icx^2\right)\right) - \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x} + \frac{iab\log\left(\right.}{}$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\tanh^{-1}\left((-\right.$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\tanh^{-1}\left((-\right.$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\tanh^{-1}\left((-\right.$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\tanh^{-1}\left((-\right.$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\tanh^{-1}\left((-\right.$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\tanh^{-1}\left((-\right.$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\tanh^{-1}\left((-\right.$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\tanh^{-1}\left((-\right.$$

Mathematica [B] time = 31.4029, size = 5434, normalized size = 4.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^2,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan\left(cx^2\right)^2 + 2\,ab \arctan\left(cx^2\right) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atan}\left(cx^2\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^2, x)
```

3.84 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=1360

result too large to display

```
[Out] (-2*a*b*c)/(3*x) - (4*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/
3 + ((-1)^(3/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(3/
4)*a*b*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - (4*(-1)^(1/4)*b^2*c^(3/2)
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x
]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2
*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^
(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/3 +
((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^
(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/3 + ((-1)^(1/4)*b^2*c^(3/
2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*S
qrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/3 - ((I/3)*b^2*c*Log[1 - I*c*x^2])/x - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/3 - (b*c*(2*a + I*b*Log[1 - I*c*x^
2]))/(3*x) - ((-1)^(3/4)*b*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
Log[1 - I*c*x^2]))/3 - (2*a + I*b*Log[1 - I*c*x^2])^2/(12*x^3) + ((I/3)*a*b
*Log[1 + I*c*x^2])/x^3 + (((2*I)/3)*b^2*c*Log[1 + I*c*x^2])/x - ((-1)^(1/4)
*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 + ((-1)^(1/4)
*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 - (b^2*Log[1
 - I*c*x^2]*Log[1 + I*c*x^2])/(6*x^3) + (b^2*Log[1 + I*c*x^2]^2)/(12*x^3) +
 ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 +
((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 - (
(-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1
 + (-1)^(1/4)*Sqrt[c]*x)])/6 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 +
 (-1)^(3/4)*Sqrt[c]*x)])/3 - ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 + (Sqrt[2
]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/6 - ((-1)^(1/4)*b^
2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/
4)*Sqrt[c]*x)])/6 - ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-
1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/6
```

Rubi [A] time = 2.22732, antiderivative size = 1360, normalized size of antiderivative = 1., number of steps used = 64, number of rules used = 25, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.562, Rules used = {5035, 2457, 2476, 2455, 203, 205, 2470, 12, 4920, 4854, 2402, 2315, 325, 6742, 206, 30, 2557, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^4,x]
```


```
[Out] (-2*a*b*c)/(3*x) - (4*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/
3 + ((-1)^(3/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(3/
4)*a*b*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - (4*(-1)^(1/4)*b^2*c^(3/2)
```

```
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x
]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2
*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^
(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/3 +
((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^
(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/3 + ((-1)^(1/4)*b^2*c^(3/
2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*S
qrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/3 - ((I/3)*b^2*c*Log[1 - I*c*x^2])/x - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/3 - (b*c*(2*a + I*b*Log[1 - I*c*x^
2]))/(3*x) - ((-1)^(3/4)*b*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
Log[1 - I*c*x^2]))/3 - (2*a + I*b*Log[1 - I*c*x^2])^2/(12*x^3) + ((I/3)*a*b
*Log[1 + I*c*x^2])/x^3 + (((2*I)/3)*b^2*c*Log[1 + I*c*x^2])/x - ((-1)^(1/4)
*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 + ((-1)^(1/4)
*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 - (b^2*Log[1
 - I*c*x^2]*Log[1 + I*c*x^2])/(6*x^3) + (b^2*Log[1 + I*c*x^2]^2)/(12*x^3) +
 ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 +
((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 - (
(-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1
 + (-1)^(1/4)*Sqrt[c]*x)])/6 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 +
 (-1)^(3/4)*Sqrt[c]*x)])/3 - ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 + (Sqrt[2
]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/6 - ((-1)^(1/4)*b^
2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/
4)*Sqrt[c]*x)])/6 - ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-
1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/6
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^4} + \frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^4} - \frac{b^2\log^2\left(1+ic\cdots\right.}{4x^4}\right.\cdots$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^4}\,dx + \frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^4}\,dx - \frac{1}{4}\cdots$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{12x^3} + \frac{b^2\log^2\left(1+icx^2\right)}{12x^3} + \frac{1}{2}b\int\left(-\frac{2ia\log\left(1+icx^2\right)}{x^4} + \frac{b\log\left(1-\cdots\right.}{\cdots}\right.\cdots$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{12x^3} + \frac{b^2\log^2\left(1+icx^2\right)}{12x^3} - (iab)\int \frac{\log\left(1+icx^2\right)}{x^4}\,dx + \frac{1}{2}b^2\int \frac{\log\cdots}{\cdots}$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{12x^3} + \frac{iab\log\left(1+icx^2\right)}{3x^3} - \frac{b^2\log\left(1-icx^2\right)\log\left(1+icx^2\right)}{6x^3} + \frac{b^2\log\cdots}{\cdots}$$

$$= -\frac{2abc}{3x} - \frac{bc\left(2a+ib\log\left(1-icx^2\right)\right)}{3x} - \frac{1}{3}(-1)^{3/4}bc^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-\cdots\right.\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}\sqrt[4]{-1}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{2}{3}(-1)^{3/4}abc^{3/2}\tanh^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{2}{3}\sqrt[4]{-1}\cdots$$

$$= -\frac{2abc}{3x} - \frac{2}{3}\sqrt[4]{-1}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3}\cdots$$

$$= -\frac{2abc}{3x} - \frac{2}{3}\sqrt[4]{-1}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3}\cdots$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3}\cdots$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3}\cdots$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3}\cdots$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3}\cdots$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3}\cdots$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3}\cdots$$

Mathematica [F] time = 2.67412, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^4,x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/x^4, x]
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(c x^2\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^4,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\arctan\left(c x^2\right)^2+2\,ab\arctan\left(c x^2\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{atan}\left(c x^2\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^4, x)
```

3.85 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=1444

result too large to display

```
[Out] (-2*a*b*c)/(15*x^3) + (((2*I)/5)*a*b*c^2)/x - (8*b^2*c^2)/(15*x) - (4*(-1)^
(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/15 - ((-1)^(1/4)*b^2*c^(5/2
)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*(-1)^(1/4)*a*b*c^(5/2)*ArcTanh[(-1
)^(3/4)*Sqrt[c]*x])/5 + (4*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c
]*x])/15 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*
(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*S
qrt[c]*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[
2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4
)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]
*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/5 + (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*A
rcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 +
(-1)^(3/4)*Sqrt[c]*x))])/5 - ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1
 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 - ((I/15)*b^2*c*Lo
g[1 - I*c*x^2])/x^3 - (b^2*c^2*Log[1 - I*c*x^2])/(5*x) + ((-1)^(3/4)*b^2*c^
(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/5 - (b*c*(2*a + I*b*L
og[1 - I*c*x^2]))/(15*x^3) - ((I/5)*b*c^2*(2*a + I*b*Log[1 - I*c*x^2]))/x +
 ((-1)^(1/4)*b*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*
x^2]))/5 - (2*a + I*b*Log[1 - I*c*x^2])^2/(20*x^5) + ((I/5)*a*b*Log[1 + I*c
*x^2])/x^5 + (((2*I)/15)*b^2*c*Log[1 + I*c*x^2])/x^3 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - (b^2*Log[1 - I*c*x^
2]*Log[1 + I*c*x^2])/(10*x^5) + (b^2*Log[1 + I*c*x^2]^2)/(20*x^5) - ((-1)^(
1/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/5 - ((-1)^(1
/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(1/
4)*b^2*c^(5/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^
(1/4)*Sqrt[c]*x)])/10 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^
(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(
3/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 + (Sqrt[2]*((-1)
^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/10 + ((-1)^(3/4)*b^2*c^(5
/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/10 + ((-1)^(1/4)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3
/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/10
```

Rubi [A] time = 2.42657, antiderivative size = 1444, normalized size of antiderivative = 1., number of steps used = 77, number of rules used = 25, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.562, Rules used = {5035, 2457, 2476, 2455, 325, 203, 205, 2470, 12, 4920, 4854, 2402, 2315, 6742, 206, 30, 2557, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

```
[Out] (-2*a*b*c)/(15*x^3) + (((2*I)/5)*a*b*c^2)/x - (8*b^2*c^2)/(15*x) - (4*(-1)^
(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/15 - ((-1)^(1/4)*b^2*c^(5/2
```

```
)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*(-1)^(1/4)*a*b*c^(5/2)*ArcTanh[(-1
)^(3/4)*Sqrt[c]*x])/5 + (4*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c
]*x])/15 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*
(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*S
qrt[c]*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[
2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4
)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]
*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/5 + (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*A
rcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 +
(-1)^(3/4)*Sqrt[c]*x))])/5 - ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1
 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 - ((I/15)*b^2*c*Lo
g[1 - I*c*x^2])/x^3 - (b^2*c^2*Log[1 - I*c*x^2])/(5*x) + ((-1)^(3/4)*b^2*c^
(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/5 - (b*c*(2*a + I*b*L
og[1 - I*c*x^2]))/(15*x^3) - ((I/5)*b*c^2*(2*a + I*b*Log[1 - I*c*x^2]))/x +
 ((-1)^(1/4)*b*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*
x^2]))/5 - (2*a + I*b*Log[1 - I*c*x^2])^2/(20*x^5) + ((I/5)*a*b*Log[1 + I*c
*x^2])/x^5 + (((2*I)/15)*b^2*c*Log[1 + I*c*x^2])/x^3 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - (b^2*Log[1 - I*c*x^
2]*Log[1 + I*c*x^2])/(10*x^5) + (b^2*Log[1 + I*c*x^2]^2)/(20*x^5) - ((-1)^(
1/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/5 - ((-1)^(1
/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(1/
4)*b^2*c^(5/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^
(1/4)*Sqrt[c]*x)])/10 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^
(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(
3/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 + (Sqrt[2]*((-1)
^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/10 + ((-1)^(3/4)*b^2*c^(5
/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/10 + ((-1)^(1/4)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3
/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/10
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
```

```
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^6} + \frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^6} - \frac{b^2\log^2\left(1+ic\right.}{4x^6}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^6}\,dx + \frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{x^6}\,dx - \frac{1}{4}$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5} + \frac{b^2\log^2\left(1+icx^2\right)}{20x^5} + \frac{1}{2}b\int\left(-\frac{2ia\log\left(1+icx^2\right)}{x^6} + \frac{b\log\left(1-\right.}{}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5} + \frac{b^2\log^2\left(1+icx^2\right)}{20x^5} - (iab)\int \frac{\log\left(1+icx^2\right)}{x^6}\,dx + \frac{1}{2}b^2\int \frac{\log}{}$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5} + \frac{iab\log\left(1+icx^2\right)}{5x^5} - \frac{b^2\log\left(1-icx^2\right)\log\left(1+icx^2\right)}{10x^5} + \frac{b^2\log}{}$$

$$= -\frac{2abc}{15x^3} - \frac{bc\left(2a+ib\log\left(1-icx^2\right)\right)}{15x^3} - \frac{ibc^2\left(2a+ib\log\left(1-icx^2\right)\right)}{5x} + \frac{1}{5}\sqrt[4]{-1}bc^{5/2}\tan^{-1}\left(\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{2}{5}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) + \frac{2}{5}(-1)^{3/4}b^2c^{5/2}\tanh^{-1}\left(\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{8}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\tan^{-1}\left((-1\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{8}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\tan^{-1}\left((-1\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{2}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\tan^{-1}\left((-1\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\tan^{-1}\left((-1\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\tan^{-1}\left((-1\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\tan^{-1}\left((-1\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\tan^{-1}\left((-1\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\tan^{-1}\left((-1\right.$$

Mathematica [F] time = 2.69961, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/x^6, x]
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(c x^{2}\right)\right)^{2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^6,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{2}\arctan\left(c x^{2}\right)^{2}+2\,ab\arctan\left(c x^{2}\right)+a^{2}}{x^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c x^{2}\right)\right)^{2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**6,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x**6, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arctan\left(cx^2\right)+a\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^6, x)
```

3.86 $\int x^3 \left(a + b\tan^{-1}\left(cx^2\right)\right)^3 dx$

Optimal. Leaf size=149

$$-\frac{3ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{4c^2}-\frac{3b^2\log\left(\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)}{2c^2}+\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{4c^2}-\frac{3ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{4c^2}+$$

```
[Out] (((-3*I)/4)*b*(a + b*ArcTan[c*x^2])^2)/c^2 - (3*b*x^2*(a + b*ArcTan[c*x^2])
^2)/(4*c) + (a + b*ArcTan[c*x^2])^3/(4*c^2) + (x^4*(a + b*ArcTan[c*x^2])^3)
/4 - (3*b^2*(a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/(2*c^2) - (((3*I)/4
)*b^3*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c^2
```

Rubi [B] time = 4.73595, antiderivative size = 951, normalized size of antiderivative = 6.38, number of steps used = 155, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2395, 2394, 2393, 2391, 2375, 2317, 2430, 2425}

$$\frac{3}{32}ib^2\left(2ia-b\log\left(1-icx^2\right)\right)\log^2\left(icx^2+1\right)x^4+\frac{3}{32}ib\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(icx^2+1\right)x^4+\frac{3b^2\left(2ia-b\log\left(1-ic\right.\right.}{8c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (((3*I)/64)*b^2*(1 - I*c*x^2)^2*((2*I)*a - b*Log[1 - I*c*x^2]))/c^2 + (((3*
I)/64)*b*(1 - I*c*x^2)^2*((2*I)*a - b*Log[1 - I*c*x^2])^2)/c^2 + (3*b^2*(1
- I*c*x^2)^2*(2*a + I*b*Log[1 - I*c*x^2]))/(64*c^2) - (((3*I)/16)*b*(1 - I*
c*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/c^2 + (((3*I)/64)*b*(1 - I*c*x^2)^2*
(2*a + I*b*Log[1 - I*c*x^2])^2)/c^2 + ((1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c
*x^2])^3)/(16*c^2) - ((1 - I*c*x^2)^2*(2*a + I*b*Log[1 - I*c*x^2])^3)/(32*c
^2) - (((3*I)/8)*b^2*((2*I)*a - b*Log[1 - I*c*x^2])*Log[(1 + I*c*x^2)/2])/c
^2 + (((3*I)/32)*b*((2*I)*a - b*Log[1 - I*c*x^2])^2*Log[(1 + I*c*x^2)/2])/c
^2 + (((3*I)/32)*b*(2*a + I*b*Log[1 - I*c*x^2])^2*Log[(1 + I*c*x^2)/2])/c^2
 - (((3*I)/8)*b^3*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2])/c^2 + (3*b^2*x^2*(
(2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2])/(8*c) + ((3*I)/32)*b*x^4*((
2*I)*a - b*Log[1 - I*c*x^2])^2*Log[1 + I*c*x^2] - (((3*I)/32)*b*(2*a + I*b*
Log[1 - I*c*x^2])^2*Log[1 + I*c*x^2])/c^2 - (((3*I)/16)*b^3*(1 + I*c*x^2)*L
og[1 + I*c*x^2]^2)/c^2 + ((3*I)/32)*b^2*x^4*((2*I)*a - b*Log[1 - I*c*x^2])*
Log[1 + I*c*x^2]^2 - (3*b^2*(2*a + I*b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2]^2
)/(32*c^2) + ((I/16)*b^3*(1 + I*c*x^2)*Log[1 + I*c*x^2]^3)/c^2 - ((I/32)*b^
3*(1 + I*c*x^2)^2*Log[1 + I*c*x^2]^3)/c^2 + (((3*I)/8)*b^3*PolyLog[2, (1 -
I*c*x^2)/2])/c^2 - (((3*I)/16)*b^2*((2*I)*a - b*Log[1 - I*c*x^2])*PolyLog[2
, (1 - I*c*x^2)/2])/c^2 - (3*b^2*(2*a + I*b*Log[1 - I*c*x^2])*PolyLog[2, (1
 - I*c*x^2)/2])/(16*c^2) - (((3*I)/8)*b^3*PolyLog[2, (1 + I*c*x^2)/2])/c^2
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
```

```
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
```

```
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^3 \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 dx = \int \left(\frac{1}{8}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^3+\frac{3}{8}ibx^3\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)-\frac{3}{8}$$

$$= \frac{1}{8}\int x^3\left(2a+ib\log\left(1-icx^2\right)\right)^3 dx+\frac{1}{8}(3ib)\int x^3\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+i\right.$$

$$= \frac{1}{16}\,\text{Subst}\left(\int x(2a+ib\log(1-icx))^3\,dx,x,x^2\right)+\frac{1}{16}(3ib)\,\text{Subst}\left(\int x(-2ia+b\log(1\right.$$

$$= \frac{3}{32}ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{32}ib^2x^4\left(2ia-b\log\left(1-icx^2\right)\right)\log^2\left(\right.$$

$$= \frac{3}{32}ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{32}ib^2x^4\left(2ia-b\log\left(1-icx^2\right)\right)\log^2\left(\right.$$

$$= \frac{3}{32}ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{32}ib^2x^4\left(2ia-b\log\left(1-icx^2\right)\right)\log^2\left(\right.$$

$$= \frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16c^2}-\frac{\left(1-icx^2\right)^2\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32c^2}+\frac{3ib\left(2ia-\right.}{}$$

$$= \frac{3ib\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{32c^2}-\frac{3ib\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c^2}+\frac{3ib\left(\right.}{}$$

$$= \frac{9iab^2x^2}{8c}+\frac{9b^3x^2}{16c}-\frac{3ib^3\left(1-icx^2\right)^2}{128c^2}+\frac{3ib^3\left(1+icx^2\right)^2}{128c^2}+\frac{3ib\left(1-icx^2\right)\left(2ia-b\log\left(1\right.\right.}{32c^2}$$

$$= \frac{9iab^2x^2}{8c}+\frac{9b^3x^2}{8c}-\frac{3ib^3\left(1-icx^2\right)^2}{128c^2}+\frac{3ib^3\left(1+icx^2\right)^2}{128c^2}-\frac{9ib^3\left(1-icx^2\right)\log\left(1-icx^2\right)}{16c^2}$$

$$= \frac{3iab^2x^2}{4c}+\frac{15b^3x^2}{16c}-\frac{9ib^3\left(1-icx^2\right)\log\left(1-icx^2\right)}{16c^2}+\frac{3ib^2\left(1-icx^2\right)^2\left(2ia-b\log\left(1-i\right.\right.}{64c^2}$$

$$= \frac{3iab^2x^2}{4c}+\frac{3b^3x^2}{4c}-\frac{3ib^3\left(1-icx^2\right)\log\left(1-icx^2\right)}{8c^2}+\frac{3ib^2\left(1-icx^2\right)^2\left(2ia-b\log\left(1-ic\right.\right.}{64c^2}$$

<u>Mathematica</u> [A] time = 0.311331, size = 170, normalized size = 1.14

$$\frac{3ib^3\text{PolyLog}\left(2,-e^{2i\tan^{-1}\left(cx^2\right)}\right)+a\left(acx^2\left(acx^2-3b\right)+3b^2\log\left(c^2x^4+1\right)\right)+3b^2\tan^{-1}\left(cx^2\right)^2\left(ac^2x^4+a+b\left(-cx^2+\right.\right.}{4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^4 + b*(I - c*x^2))*ArcTan[c*x^2]^2 + b^3*(1 + c^2*x^4)*
ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a - 2*b*c*x^2 + a*c^2*x^4) - 2*b^2*
Log[1 + E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2*(-3*b + a*c*x^2) + 3*b^2*Log
[1 + c^2*x^4]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])])/(4*c^2)
```

Maple [C] time = 0.733, size = 690, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^2))^3,x)
```

```
[Out] 3/4*a^2*b/c^2*arctan(c*x^2)-3/4/c*b*a^2*x^2+3/4/c^2*a*b^2*ln(c^2*x^4+1)-3/1
6/c^2*a*b^2*ln(1-I*c*x^2)^2+3/16*b^3/c*x^2*ln(1-I*c*x^2)^2+1/32*I*b^3*(c^2*
x^4+1)/c^2*ln(1+I*c*x^2)^3+3/4*I/c*b^2*Sum((ln(x-_alpha)*ln(1-I*c*x^2)+2*c*
(-1/2*ln(x-_alpha)*(ln((-1/2-1/2*I)*(I*(I/c)^(1/2)-(I/c)^(1/2)-x+_alpha)/(I
/c)^(1/2))+ln((1/2-1/2*I)*(I*(I/c)^(1/2)+(I/c)^(1/2)+x-_alpha)/(I/c)^(1/2))
)/c-1/2*(dilog((-1/2-1/2*I)*(I*(I/c)^(1/2)-(I/c)^(1/2)-x+_alpha)/(I/c)^(1/2
))+dilog((1/2-1/2*I)*(I*(I/c)^(1/2)+(I/c)^(1/2)+x-_alpha)/(I/c)^(1/2)))/c))
*b/c,_alpha=RootOf(c*_Z^2-RootOf(_Z^2+1,index=1)))-3/16*a*b^2*x^4*ln(1-I*c*
x^2)^2+3/16*I/c^2*b^3*ln(1-I*c*x^2)^2-3/4*I/c*a*b^2*x^2*ln(1-I*c*x^2)-1/32*
I*b^3*x^4*ln(1-I*c*x^2)^3-3/32*b^2*(I*x^4*b*ln(1-I*c*x^2)*c^2+2*a*c^2*x^4-2
*b*c*x^2+I*b*ln(1-I*c*x^2)+2*I*b+2*a)/c^2*ln(1+I*c*x^2)^2-1/32*I*b^3/c^2*ln
(1-I*c*x^2)^3+3/8*I*a^2*b*x^4*ln(1-I*c*x^2)+1/4*x^4*a^3+(3/32*I*b^3*(c^2*x^
4+1)/c^2*ln(1-I*c*x^2)^2+3/8*b^2*x^2*(a*c*x^2-b)/c*ln(1-I*c*x^2)-3/8*I*b*(a
^2*c^2*x^4-2*a*b*c*x^2+b^2*ln(1-I*c*x^2)+I*ln(1-I*c*x^2)*a*b)/c^2)*ln(1+I*c
*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{4}\,ab^2x^4\arctan\left(cx^2\right)^2+\frac{1}{4}\,a^3x^4+\frac{3}{4}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)a^2b-\frac{3}{4}\left(2\,c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^3*x^4 + 3/4*(x^4*arctan(c*x^2) - c*(x
^2/c^2 - arctan(c*x^2)/c^3))*a^2*b - 3/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)
*arctan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*a*b^2 + 1/
128*(4*x^4*arctan(c*x^2)^3 - 3*x^4*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 128*i
ntegrate(1/64*(12*c^2*x^7*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^5*arctan(
c*x^2)^2 + 56*(c^2*x^7 + x^3)*arctan(c*x^2)^3 + 3*(c*x^5 + 2*(c^2*x^7 + x^3
)*arctan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^3\arctan\left(cx^2\right)^3+3\,ab^2x^3\arctan\left(cx^2\right)^2+3\,a^2bx^3\arctan\left(cx^2\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c*x^2)^3 + 3*a*b^2*x^3*arctan(c*x^2)^2 + 3*a^2*b*x^
3*arctan(c*x^2) + a^3*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(a + b \operatorname{atan}\left(cx^2\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**2))**3,x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x**2))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arctan\left(cx^2\right) + a\right)^3 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*x^3, x)
```

3.87 $\int x \left(a+b \tan^{-1}\left(c x^2\right)\right)^3 \, dx$

Optimal. Leaf size=144

$$\frac{3 i b^2 \text{PolyLog}\left(2,1-\frac{2}{1+i c x^2}\right) \left(a+b \tan^{-1}\left(c x^2\right)\right)}{2 c}+\frac{3 b^3 \text{PolyLog}\left(3,1-\frac{2}{1+i c x^2}\right)}{4 c}+\frac{1}{2} x^2 \left(a+b \tan^{-1}\left(c x^2\right)\right)^3+\frac{i \left(a+b \tan^{-1}\right.}{2 c}$$

```
[Out] ((I/2)*(a + b*ArcTan[c*x^2])^3)/c + (x^2*(a + b*ArcTan[c*x^2])^3)/2 + (3*b*
(a + b*ArcTan[c*x^2])^2*Log[2/(1 + I*c*x^2)])/(2*c) + (((3*I)/2)*b^2*(a + b
*ArcTan[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c + (3*b^3*PolyLog[3, 1 -
2/(1 + I*c*x^2)])/(4*c)
```

Rubi [B] time = 2.545, antiderivative size = 545, normalized size of antiderivative = 3.78, number of steps used = 82, number of rules used = 23, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.643, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$-\frac{3 b^2 \text{PolyLog}\left(2,\frac{1}{2} \left(1-i c x^2\right)\right) \left(2 i a-b \log \left(1-i c x^2\right)\right)}{4 c}-\frac{3 b^3 \text{PolyLog}\left(3,\frac{1}{2} \left(1-i c x^2\right)\right)}{4 c}-\frac{3 b^3 \text{PolyLog}\left(3,\frac{1}{2} \left(1+i c x^2\right)\right)}{4 c}+$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (3*b*(1 - I*c*x^2)*((2*I)*a - b*Log[1 - I*c*x^2])^2)/(16*c) + (3*b*(1 - I*c
*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^2)/(16*c) + ((I/16)*(1 - I*c*x^2)*(2*a +
 I*b*Log[1 - I*c*x^2])^3)/c + (3*b*((2*I)*a - b*Log[1 - I*c*x^2])^2*Log[(1
+ I*c*x^2)/2])/(8*c) - (3*b*((2*I)*a - b*Log[1 - I*c*x^2])^2*Log[1 + I*c*x^
2])/(16*c) + ((3*I)/16)*b*x^2*((2*I)*a - b*Log[1 - I*c*x^2])^2*Log[1 + I*c*
x^2] + (3*b^3*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2]^2)/(8*c) + (3*b^2*((2*I
)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2]^2)/(16*c) + ((3*I)/16)*b^2*x^2*(
(2*I)*a - b*Log[1 - I*c*x^2])*Log[1 + I*c*x^2]^2 + (b^3*(1 + I*c*x^2)*Log[1
 + I*c*x^2]^3)/(16*c) - (3*b^2*((2*I)*a - b*Log[1 - I*c*x^2])*PolyLog[2, (1
 - I*c*x^2)/2])/(4*c) + (3*b^3*Log[1 + I*c*x^2]*PolyLog[2, (1 + I*c*x^2)/2]
)/(4*c) - (3*b^3*PolyLog[3, (1 - I*c*x^2)/2])/(4*c) - (3*b^3*PolyLog[3, (1
+ I*c*x^2)/2])/(4*c)
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(cx^2\right)\right)^3\,dx=\int\left(\frac{1}{8}x\left(2a+ib\log\left(1-icx^2\right)\right)^3+\frac{3}{8}ibx\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)-\frac{3}{8}ib^2\right.$$
$$=\frac{1}{8}\int x\left(2a+ib\log\left(1-icx^2\right)\right)^3\,dx+\frac{1}{8}(3ib)\int x\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right.$$
$$=\frac{1}{16}\mathrm{Subst}\left(\int(2a+ib\log(1-icx))^3\,dx,x,x^2\right)+\frac{1}{16}(3ib)\,\mathrm{Subst}\left(\int(-2ia+b\log(1-icx\right.$$
$$=\frac{3}{16}ibx^2\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{16}ib^2x^2\left(2ia-b\log\left(1-icx^2\right)\right)\log^2\left(1\right.$$
$$=\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16c}+\frac{3}{16}ibx^2\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+$$
$$=\frac{3b\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16c}+\frac{3}{16}ibx^2\left(\right.$$
$$=-\frac{3}{4}ab^2x^2-\frac{3}{8}ib^3x^2+\frac{3b\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i\left(1-icx^2\right)\left(2a+ib\log\left(1\right.\right.}{16c}$$
$$=-\frac{3}{4}ab^2x^2+\frac{3b^3\left(1-icx^2\right)\log\left(1-icx^2\right)}{8c}+\frac{3b\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+\frac{3b\left(\right.}{}$$
$$=\frac{3}{8}ib^3x^2+\frac{3b^3\left(1-icx^2\right)\log\left(1-icx^2\right)}{8c}+\frac{3b\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+\frac{3b\left(1-\right.}{}$$
$$=\frac{3b\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+\frac{3b\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i\left(1-\right.}{}$$
$$=\frac{3b\left(1-icx^2\right)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+\frac{3b\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i\left(1-\right.}{}$$

Mathematica [A] time = 0.102436, size = 224, normalized size = 1.56

$$-6ib^2\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}\left(cx^2\right)}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+3b^3\mathrm{PolyLog}\left(3,-e^{2i\tan^{-1}\left(cx^2\right)}\right)-3a^2b\log\left(c^2x^4+1\right)+6a^2bcx^2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (2*a^3*c*x^2 + 6*a^2*b*c*x^2*ArcTan[c*x^2] - (6*I)*a*b^2*ArcTan[c*x^2]^2 +
6*a*b^2*c*x^2*ArcTan[c*x^2]^2 - (2*I)*b^3*ArcTan[c*x^2]^3 + 2*b^3*c*x^2*Arc
Tan[c*x^2]^3 + 12*a*b^2*ArcTan[c*x^2]*Log[1 + E^((2*I)*ArcTan[c*x^2])] + 6*
b^3*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^2])] - 3*a^2*b*Log[1 + c^2*
x^4] - (6*I)*b^2*(a + b*ArcTan[c*x^2])*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])]
 + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])])/(4*c)
```

Maple [B] time = 0.125, size = 306, normalized size = 2.1

$$\frac{x^2a^3}{2}-\frac{\frac{i}{2}b^3\left(\arctan\left(cx^2\right)\right)^3}{c}+\frac{b^3\left(\arctan\left(cx^2\right)\right)^3x^2}{2}+\frac{3\,b^3\left(\arctan\left(cx^2\right)\right)^2}{2\,c}\ln\left(\frac{\left(1+icx^2\right)^2}{c^2x^4+1}+1\right)-\frac{\frac{3\,i}{2}b^3\arctan\left(cx^2\right)}{c}\mathrm{pol}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^2))^3,x)
```

```
[Out] 1/2*x^2*a^3-1/2*I/c*b^3*arctan(c*x^2)^3+1/2*b^3*arctan(c*x^2)^3*x^2+3/2/c*b
^3*arctan(c*x^2)^2*ln((1+I*c*x^2)^2/(c^2*x^4+1)+1)-3/2*I/c*b^3*arctan(c*x^2
)*polylog(2,-(1+I*c*x^2)^2/(c^2*x^4+1))+3/4/c*b^3*polylog(3,-(1+I*c*x^2)^2/
(c^2*x^4+1))-3/2*I/c*arctan(c*x^2)^2*a*b^2+3/2*arctan(c*x^2)^2*x^2*a*b^2+3/
c*arctan(c*x^2)*ln((1+I*c*x^2)^2/(c^2*x^4+1)+1)*a*b^2-3/2*I/c*polylog(2,-(1
+I*c*x^2)^2/(c^2*x^4+1))*a*b^2+3/2*a^2*b*x^2*arctan(c*x^2)-3/4/c*a^2*b*ln(c
^2*x^4+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\,b^3x^2\arctan\left(cx^2\right)^3-\frac{3}{64}\,b^3x^2\arctan\left(cx^2\right)\log\left(c^2x^4+1\right)^2+\frac{7\,b^3\arctan\left(cx^2\right)^4}{64\,c}+28\,b^3c^2\int\frac{x^5\arctan\left(cx^2\right)^3}{32\left(c^2x^4+1\right)}\,dx+3\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^2*arctan(c*x^2)^3 - 3/64*b^3*x^2*arctan(c*x^2)*log(c^2*x^4 + 1)^
2 + 7/64*b^3*arctan(c*x^2)^4/c + 28*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2
)^3/(c^2*x^4 + 1), x) + 3*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c^2*
x^4 + 1)^2/(c^2*x^4 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^5*arctan(c*x^2
)^2/(c^2*x^4 + 1), x) + 12*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c^2
*x^4 + 1)/(c^2*x^4 + 1), x) + 1/2*a^3*x^2 + 1/2*a*b^2*arctan(c*x^2)^3/c - 1
2*b^3*c*integrate(1/32*x^3*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 3*b^3*c*inte
grate(1/32*x^3*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3*b^3*integrate(1/32*
x*arctan(c*x^2)*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3/4*(2*c*x^2*arctan(
c*x^2) - log(c^2*x^4 + 1))*a^2*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x\arctan\left(cx^2\right)^3+3\,ab^2x\arctan\left(cx^2\right)^2+3\,a^2bx\arctan\left(cx^2\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c*x^2)^3 + 3*a*b^2*x*arctan(c*x^2)^2 + 3*a^2*b*x*arct
an(c*x^2) + a^3*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{atan}\left(cx^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**2))**3,x)
```

```
[Out] Integral(x*(a + b*atan(c*x**2))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(cx^2\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*x, x)
```

3.88 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x}\,dx$

Optimal. Leaf size=229

$$-\frac{3}{4}b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{3}{4}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{3}{4}ib\text{PolyLog}$$

```
[Out] (a + b*ArcTan[c*x^2])^3*ArcTanh[1 - 2/(1 + I*c*x^2)] - ((3*I)/4)*b*(a + b*A
rcTan[c*x^2])^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + ((3*I)/4)*b*(a + b*ArcTan
[c*x^2])^2*PolyLog[2, -1 + 2/(1 + I*c*x^2)] - (3*b^2*(a + b*ArcTan[c*x^2])*
PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 + (3*b^2*(a + b*ArcTan[c*x^2])*PolyLog[3
, -1 + 2/(1 + I*c*x^2)])/4 + ((3*I)/8)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^2)]
- ((3*I)/8)*b^3*PolyLog[4, -1 + 2/(1 + I*c*x^2)]
```

Rubi [A] time = 0.528502, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5031, 4850, 4988, 4884, 4994, 4998, 6610}

$$-\frac{3}{4}b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{3}{4}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{3}{4}ib\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^3*ArcTanh[1 - 2/(1 + I*c*x^2)] - ((3*I)/4)*b*(a + b*A
rcTan[c*x^2])^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + ((3*I)/4)*b*(a + b*ArcTan
[c*x^2])^2*PolyLog[2, -1 + 2/(1 + I*c*x^2)] - (3*b^2*(a + b*ArcTan[c*x^2])*
PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 + (3*b^2*(a + b*ArcTan[c*x^2])*PolyLog[3
, -1 + 2/(1 + I*c*x^2)])/4 + ((3*I)/8)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^2)]
- ((3*I)/8)*b^3*PolyLog[4, -1 + 2/(1 + I*c*x^2)]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx, x, x^2\right)$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-(3bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}}{1+c^2x^2}\right.$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)+\frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\right.}{1+c^2x^2}\right.$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{3}{4}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx^2}\right)$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{3}{4}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx^2}\right)$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{3}{4}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx^2}\right)$$

Mathematica [A] time = 0.195245, size = 245, normalized size = 1.07

$$\frac{3}{8}ib\left(2\text{PolyLog}\left(2,\frac{cx^2+i}{-cx^2+i}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-2\text{PolyLog}\left(2,\frac{cx^2+i}{cx^2-i}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)^2+b\left(-2i\text{PolyLog}\left(3,\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^3*ArcTanh[1 + (2*I)/(-I + c*x^2)] + ((3*I)/8)*b*(2*(a
 + b*ArcTan[c*x^2])^2*PolyLog[2, (I + c*x^2)/(I - c*x^2)] - 2*(a + b*ArcTan
[c*x^2])^2*PolyLog[2, (I + c*x^2)/(-I + c*x^2)] + b*((-2*I)*(a + b*ArcTan[c
*x^2])*PolyLog[3, (I + c*x^2)/(I - c*x^2)] + (2*I)*(a + b*ArcTan[c*x^2])*Po
lyLog[3, (I + c*x^2)/(-I + c*x^2)] + b*(-PolyLog[4, (I + c*x^2)/(I - c*x^2)
] + PolyLog[4, (I + c*x^2)/(-I + c*x^2)])))
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^3/x,x)
```

```
[Out] int((a+b*arctan(c*x^2))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3\log(x)+\frac{1}{32}\int \frac{28\,b^3\arctan\left(cx^2\right)^3+3\,b^3\arctan\left(cx^2\right)\log\left(c^2x^4+1\right)^2+96\,ab^2\arctan\left(cx^2\right)^2+96\,a^2b\arctan\left(cx^2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^2)^3 + 3*b^3*arctan(c*x^2)*l
og(c^2*x^4 + 1)^2 + 96*a*b^2*arctan(c*x^2)^2 + 96*a^2*b*arctan(c*x^2))/x, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\arctan\left(cx^2\right)^3+3\,ab^2\arctan\left(cx^2\right)^2+3\,a^2b\arctan\left(cx^2\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**3/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arctan\left(cx^2\right) + a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3/x, x)
```

3.89 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=138

$$-\frac{3}{2}ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{3}{4}b^3c\text{PolyLog}\left(3,-1+\frac{2}{1-icx^2}\right)-\frac{1}{2}ic\left(a+b\tan^{-1}\left(cx^2\right)\right)^3-$$

```
[Out] (-I/2)*c*(a + b*ArcTan[c*x^2])^3 - (a + b*ArcTan[c*x^2])^3/(2*x^2) + (3*b*c
*(a + b*ArcTan[c*x^2])^2*Log[2 - 2/(1 - I*c*x^2)])/2 - ((3*I)/2)*b^2*c*(a +
 b*ArcTan[c*x^2])*PolyLog[2, -1 + 2/(1 - I*c*x^2)] + (3*b^3*c*PolyLog[3, -1
 + 2/(1 - I*c*x^2)])/4
```

Rubi [F] time = 0.83101, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

```
[Out] (3*b*c*Log[I*c*x^2]*(2*a + I*b*Log[1 - I*c*x^2])^2)/16 - ((1 - I*c*x^2)*(2*
a + I*b*Log[1 - I*c*x^2])^3)/(16*x^2) - (3*b^3*c*Log[(-I)*c*x^2]*Log[1 + I*
c*x^2]^2)/16 - ((I/16)*b^3*(1 + I*c*x^2)*Log[1 + I*c*x^2]^3)/x^2 + ((3*I)/8
)*b^2*c*(2*a + I*b*Log[1 - I*c*x^2])*PolyLog[2, 1 - I*c*x^2] - (3*b^3*c*Log
[1 + I*c*x^2]*PolyLog[2, 1 + I*c*x^2])/8 + (3*b^3*c*PolyLog[3, 1 - I*c*x^2]
)/8 + (3*b^3*c*PolyLog[3, 1 + I*c*x^2])/8 + ((3*I)/16)*b*Defer[Subst][Defer
[Int][(((-2*I)*a + b*Log[1 - I*c*x])^2*Log[1 + I*c*x])/x^2, x], x, x^2] - (
(3*I)/16)*b^2*Defer[Subst][Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x])*Log[1
+ I*c*x]^2)/x^2, x], x, x^2]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{8x^3} + \frac{3ib\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)}{8x^3} - \frac{3ib^2\left(-2ia\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{x^3}\,dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)}{x^3}\,dx$$

$$= \frac{1}{16}\,\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^3}{x^2}\,dx, x, x^2\right) + \frac{1}{16}(3ib)\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx}{x^2}\right.$$

$$= -\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2} - \frac{ib^3\left(1+icx^2\right)\log^3\left(1+icx^2\right)}{16x^2} + \frac{1}{16}(3ib)\,\text{Subst}\left(\int\right.$$

$$= \frac{3}{16}bc\log\left(icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2 - \frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2} - \frac{3}{16}b^3c\log$$

$$= \frac{3}{16}bc\log\left(icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2 - \frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2} - \frac{3}{16}b^3c\log$$

$$= \frac{3}{16}bc\log\left(icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2 - \frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2} - \frac{3}{16}b^3c\log$$

$$= \frac{3}{16}bc\log\left(icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2 - \frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16x^2} - \frac{3}{16}b^3c\log$$

Mathematica [A] time = 0.429957, size = 239, normalized size = 1.73

$$\frac{1}{4}\left(6ab^2c\left(\tan^{-1}\left(cx^2\right)\left(\left(-\frac{1}{cx^2}-i\right)\tan^{-1}\left(cx^2\right)+2\log\left(1-e^{2i\tan^{-1}\left(cx^2\right)}\right)\right)-i\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(cx^2\right)}\right)\right)+2b^3c\left(3i\tan\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

```
[Out] ((-2*a^3)/x^2 - (6*a^2*b*ArcTan[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b*c
*Log[1 + c^2*x^4] + 6*a*b^2*c*(ArcTan[c*x^2]*((-I - 1/(c*x^2))*ArcTan[c*x^2
] + 2*Log[1 - E^((2*I)*ArcTan[c*x^2])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^
2])]) + 2*b^3*c*((-I/8)*Pi^3 + I*ArcTan[c*x^2]^3 - ArcTan[c*x^2]^3/(c*x^2)
+ 3*ArcTan[c*x^2]^2*Log[1 - E^((-2*I)*ArcTan[c*x^2])] + (3*I)*ArcTan[c*x^2]
*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x^
2])])/2))/4
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^3/x^3,x)
```

```
[Out] int((a+b*arctan(c*x^2))^3/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3 \arctan\left(cx^2\right)^3 + 3\,ab^2 \arctan\left(cx^2\right)^2 + 3\,a^2 b \arctan\left(cx^2\right) + a^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atan}\left(cx^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**3/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3/x^3, x)
```

3.90 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx$

Optimal. Leaf size=149

$$-\frac{3}{4}ib^3c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)+\frac{3}{2}b^2c^2\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{3}{4}ibc^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^2-\frac{1}{4}c^2$$

```
[Out] ((-3*I)/4)*b*c^2*(a + b*ArcTan[c*x^2])^2 - (3*b*c*(a + b*ArcTan[c*x^2])^2)/
(4*x^2) - (c^2*(a + b*ArcTan[c*x^2])^3)/4 - (a + b*ArcTan[c*x^2])^3/(4*x^4)
 + (3*b^2*c^2*(a + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)])/2 - ((3*I)/4)
*b^3*c^2*PolyLog[2, -1 + 2/(1 - I*c*x^2)]
```

Rubi [F] time = 1.65887, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

```
[Out] (3*a*b^2*c^2*Log[x])/4 - (3*b*c*(1 - I*c*x^2)*(2*a + I*b*Log[1 - I*c*x^2])^
2)/(32*x^2) + ((3*I)/32)*b*c^2*Log[I*c*x^2]*(2*a + I*b*Log[1 - I*c*x^2])^2
- (c^2*(2*a + I*b*Log[1 - I*c*x^2])^3)/32 - (2*a + I*b*Log[1 - I*c*x^2])^3/
(32*x^4) + (3*b^3*c*(1 + I*c*x^2)*Log[1 + I*c*x^2]^2)/(32*x^2) + ((3*I)/32)
*b^3*c^2*Log[(-I)*c*x^2]*Log[1 + I*c*x^2]^2 - (I/32)*b^3*c^2*Log[1 + I*c*x^
2]^3 - ((I/32)*b^3*Log[1 + I*c*x^2]^3)/x^4 + ((3*I)/16)*b^3*c^2*PolyLog[2,
(-I)*c*x^2] - ((3*I)/16)*b^3*c^2*PolyLog[2, I*c*x^2] - (3*b^2*c^2*(2*a + I*
b*Log[1 - I*c*x^2])*PolyLog[2, 1 - I*c*x^2])/16 + ((3*I)/16)*b^3*c^2*Log[1
+ I*c*x^2]*PolyLog[2, 1 + I*c*x^2] + ((3*I)/16)*b^3*c^2*PolyLog[3, 1 - I*c*
x^2] - ((3*I)/16)*b^3*c^2*PolyLog[3, 1 + I*c*x^2] + ((3*I)/16)*b*Defer[Subs
t][Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x])^2*Log[1 + I*c*x])/x^3, x], x,
x^2] - ((3*I)/16)*b^2*Defer[Subst][Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x]
)*Log[1 + I*c*x]^2)/x^3, x], x, x^2]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^5}dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{8x^5}+\frac{3ib\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)}{8x^5}-\frac{3ib^2\left(-2ia+b\right.}{}\right.$$

$$=\frac{1}{8}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{x^5}dx+\frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)}{x^5}dx-\frac{1}{8}$$

$$=\frac{1}{16}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^3}{x^3}dx,x,x^2\right)+\frac{1}{16}(3ib)\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx))^2}{x^3}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4}-\frac{ib^3\log^3\left(1+icx^2\right)}{32x^4}+\frac{1}{16}(3ib)\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx))}{x^3}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4}-\frac{ib^3\log^3\left(1+icx^2\right)}{32x^4}+\frac{1}{32}(3ib)\,\text{Subst}\left(\int \frac{(2a+ib\log(x))^2}{x\left(-\frac{i}{c}+\frac{ix}{c}\right)^2}dx,x,\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4}-\frac{ib^3\log^3\left(1+icx^2\right)}{32x^4}+\frac{1}{32}(3ib)\,\text{Subst}\left(\int \frac{(2a+ib\log(x))^2}{\left(-\frac{i}{c}+\frac{ix}{c}\right)^2}dx,x,\right.$$

$$=-\frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2}-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4}+\frac{3b^3c\left(1+icx^2\right)\log^2\left(1\right.}{32x^2}$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2}+\frac{3}{32}ibc^2\log\left(icx^2\right)\left(2a+ib\log\left(1-\right.\right.$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2}+\frac{3}{32}ibc^2\log\left(icx^2\right)\left(2a+ib\log\left(1-\right.\right.$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2}+\frac{3}{32}ibc^2\log\left(icx^2\right)\left(2a+ib\log\left(1-\right.\right.$$

Mathematica [A] time = 0.334389, size = 196, normalized size = 1.32

$$-\frac{3ib^3c^2x^4\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(cx^2\right)}\right)+a\left(a\left(a+3bcx^2\right)-6b^2c^2x^4\log\left(\frac{cx^2}{\sqrt{c^2x^4+1}}\right)\right)+3b^2\tan^{-1}\left(cx^2\right)^2\left(ac^2x^4+a+bcx^2\left(1+\right.\right.}{4x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

```
[Out] -(3*b^2*(a + a*c^2*x^4 + b*c*x^2*(1 + I*c*x^2))*ArcTan[c*x^2]^2 + b^3*(1 +
c^2*x^4)*ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a + 2*b*c*x^2 + a*c^2*x^4)
 - 2*b^2*c^2*x^4*Log[1 - E^((2*I)*ArcTan[c*x^2])]) + a*(a*(a + 3*b*c*x^2) -
 6*b^2*c^2*x^4*Log[(c*x^2)/Sqrt[1 + c^2*x^4]]) + (3*I)*b^3*c^2*x^4*PolyLog[
2, E^((2*I)*ArcTan[c*x^2])])/(4*x^4)
```

Maple [F] time = 0.661, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^5}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^3/x^5,x)
```

```
[Out] int((a+b*arctan(c*x^2))^3/x^5,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\arctan\left(cx^2\right)^3+3\,ab^2\arctan\left(cx^2\right)^2+3\,a^2b\arctan\left(cx^2\right)+a^3}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**3/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(cx^2\right)+a\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3/x^5, x)
```

3.91 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3, x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

Rubi [A] time = 0.0233255, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx = \int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx$$

Mathematica [A] time = 1.78784, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

Maple [A] time = 0.237, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\arctan\left(cx^2\right)^3 + 3\,ab^2\arctan\left(cx^2\right)^2 + 3\,a^2b\arctan\left(cx^2\right) + a^3\right)(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(cx^2\right) + a\right)^3(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*(d*x)^m, x)
```

3.92 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2,x\right)$$

[Out] Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]

Rubi [A] time = 0.0235031, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx$$

Verification is Not applicable to the result.

[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]

[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx=\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx$$

Mathematica [A] time = 1.17481, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]

[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]

Maple [A] time = 0.213, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a+b*arctan(c*x^2))^2,x)

[Out] int((d*x)^m*(a+b*arctan(c*x^2))^2,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\arctan\left(cx^2\right)^2 + 2\,ab\arctan\left(cx^2\right) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(cx^2\right) + a\right)^2 (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*(d*x)^m, x)
```

3.93 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=75

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{4},\frac{m+7}{4},-c^2x^4\right)}{d^3(m+1)(m+3)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*Hy
pergeometric2F1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)])/(d^3*(1 + m)*(3 + m))
```

Rubi [A] time = 0.0406812, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5033, 16, 364}

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};-c^2x^4\right)}{d^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*Hy
pergeometric2F1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)])/(d^3*(1 + m)*(3 + m))
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right) dx &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int \frac{x(dx)^{1+m}}{1+c^2x^4}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int \frac{(dx)^{2+m}}{1+c^2x^4}\,dx}{d^2(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};-c^2x^4\right)}{d^3(1+m)(3+m)}\end{aligned}$$

Mathematica [A] time = 0.0587734, size = 65, normalized size = 0.87

$$-\frac{x(dx)^m\left(2bcx^2\text{Hypergeometric2F1}\left(1,\frac{m+3}{4},\frac{m+7}{4},-c^2x^4\right)-(m+3)\left(a+b\tan^{-1}\left(cx^2\right)\right)\right)}{(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -((x*(d*x)^m*(-((3 + m)*(a + b*ArcTan[c*x^2])) + 2*b*c*x^2*Hypergeometric2F
1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)]))/((1 + m)*(3 + m)))
```

Maple [F] time = 0.209, size = 0, normalized size = 0.

$$\int (dx)^m\left(a+b\arctan\left(cx^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\arctan\left(cx^2\right)+a\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(cx^2\right)+a\right)(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

3.94 $\int \frac{(dx)^m}{a+b\tan^{-1}(cx^2)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

Rubi [A] time = 0.0268812, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)}\,dx=\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)}\,dx$$

Mathematica [A] time = 0.312356, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

Maple [A] time = 0.145, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\arctan\left(cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

```
[Out] int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \arctan(cx^2) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b \arctan(cx^2) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \arctan(cx^2) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

3.95 $\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

Rubi [A] time = 0.0255398, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx$$

Mathematica [A] time = 0.32898, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

Maple [A] time = 0.171, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\arctan\left(cx^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2d^mx^4+d^m\right)x^m-\left(b^2cx\arctan\left(cx^2\right)+abcx\right)\int\frac{\left(\left(c^2d^mm+3\,c^2d^m\right)x^4+d^mm-d^m\right)x^m}{b^2cx^2\arctan\left(cx^2\right)+abcx^2}\,dx}{2\left(b^2cx\arctan\left(cx^2\right)+abcx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((c^2*d^m*x^4 + d^m)*x^m - 2*(b^2*c*x*arctan(c*x^2) + a*b*c*x)*integra
te(1/2*((c^2*d^m*m + 3*c^2*d^m)*x^4 + d^m*m - d^m)*x^m/(b^2*c*x^2*arctan(c*
x^2) + a*b*c*x^2), x))/(b^2*c*x*arctan(c*x^2) + a*b*c*x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^2\arctan\left(cx^2\right)^2+2\,ab\arctan\left(cx^2\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^m}{\left(b\arctan\left(cx^2\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a)^2, x)
```

3.96 $\int x^{11} \left(a + b \tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=54

$$\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{bx^3}{12c^3}-\frac{b\tan^{-1}\left(cx^3\right)}{12c^4}-\frac{bx^9}{36c}$$

```
[Out] (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*ArcTan[c*x^3])/(12*c^4) + (x^12*(a +
 b*ArcTan[c*x^3]))/12
```

Rubi [A] time = 0.0388067, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 302, 203}

$$\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{bx^3}{12c^3}-\frac{b\tan^{-1}\left(cx^3\right)}{12c^4}-\frac{bx^9}{36c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*ArcTan[c*x^3])/(12*c^4) + (x^12*(a +
 b*ArcTan[c*x^3]))/12
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^{11}\left(a+b\tan^{-1}\left(cx^3\right)\right)dx=\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{4}(bc)\int\frac{x^{14}}{1+c^2x^6}dx$$
$$=\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\frac{x^4}{1+c^2x^2}dx,x,x^3\right)$$
$$=\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)dx,x,x^3\right)$$
$$=\frac{bx^3}{12c^3}-\frac{bx^9}{36c}+\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1+c^2x^2}dx,x,x^3\right)}{12c^3}$$
$$=\frac{bx^3}{12c^3}-\frac{bx^9}{36c}-\frac{b\tan^{-1}\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)$$

Mathematica [A] time = 0.0080439, size = 59, normalized size = 1.09

$$\frac{ax^{12}}{12}+\frac{bx^3}{12c^3}-\frac{b\tan^{-1}\left(cx^3\right)}{12c^4}-\frac{bx^9}{36c}+\frac{1}{12}bx^{12}\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) - (b*x^9)/(36*c) + (a*x^12)/12 - (b*ArcTan[c*x^3])/(12*c^4
) + (b*x^12*ArcTan[c*x^3])/12
```

Maple [A] time = 0.026, size = 50, normalized size = 0.9

$$\frac{x^{12}a}{12}+\frac{bx^{12}\arctan\left(cx^3\right)}{12}-\frac{bx^9}{36\,c}+\frac{bx^3}{12\,c^3}-\frac{b\arctan\left(cx^3\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/12*x^12*a+1/12*b*x^12*arctan(c*x^3)-1/36*b*x^9/c+1/12*b*x^3/c^3-1/12*b*ar
ctan(c*x^3)/c^4
```

Maxima [A] time = 1.50242, size = 73, normalized size = 1.35

$$\frac{1}{12}\,ax^{12}+\frac{1}{36}\left(3\,x^{12}\arctan\left(cx^3\right)-c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/12*a*x^12 + 1/36*(3*x^12*arctan(c*x^3) - c*((c^2*x^9 - 3*x^3)/c^4 + 3*arc
tan(c*x^3)/c^5))*b
```

Fricas [A] time = 2.61917, size = 113, normalized size = 2.09

$$\frac{3\,ac^4x^{12} - bc^3x^9 + 3\,bcx^3 + 3\left(bc^4x^{12} - b\right)\arctan\left(cx^3\right)}{36\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/36*(3*a*c^4*x^12 - b*c^3*x^9 + 3*b*c*x^3 + 3*(b*c^4*x^12 - b)*arctan(c*x^
3))/c^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atan(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11677, size = 81, normalized size = 1.5

$$\frac{3\,acx^{12} + \left(3\,cx^{12}\arctan\left(cx^3\right) - \frac{3\,\arctan\left(cx^3\right)}{c^3} - \frac{c^9x^9 - 3\,c^7x^3}{c^9}\right)b}{36\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/36*(3*a*c*x^12 + (3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9*x^9
 - 3*c^7*x^3)/c^9)*b)/c
```

3.97 $\int x^8 \left(a+b\tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=47

$$\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^2x^6+1\right)}{18c^3}-\frac{bx^6}{18c}$$

```
[Out] -(b*x^6)/(18*c) + (x^9*(a + b*ArcTan[c*x^3]))/9 + (b*Log[1 + c^2*x^6])/(18*
c^3)
```

Rubi [A] time = 0.036474, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5033, 266, 43}

$$\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^2x^6+1\right)}{18c^3}-\frac{bx^6}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -(b*x^6)/(18*c) + (x^9*(a + b*ArcTan[c*x^3]))/9 + (b*Log[1 + c^2*x^6])/(18*
c^3)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^8\left(a+b\tan^{-1}\left(cx^3\right)\right) dx &= \frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{3}(bc)\int\frac{x^{11}}{1+c^2x^6}\,dx\\
&= \frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\,\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^6\right)\\
&= \frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\,\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)dx,x,x^6\right)\\
&= -\frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1+c^2x^6\right)}{18c^3}
\end{aligned}$$

Mathematica [A] time = 0.0134892, size = 52, normalized size = 1.11

$$\frac{ax^9}{9}+\frac{b\log\left(c^2x^6+1\right)}{18c^3}-\frac{bx^6}{18c}+\frac{1}{9}bx^9\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -(b*x^6)/(18*c) + (a*x^9)/9 + (b*x^9*ArcTan[c*x^3])/9 + (b*Log[1 + c^2*x^6]
)/(18*c^3)
```

Maple [A] time = 0.023, size = 45, normalized size = 1.

$$\frac{x^9a}{9}+\frac{bx^9\arctan\left(cx^3\right)}{9}-\frac{bx^6}{18\,c}+\frac{b\ln\left(c^2x^6+1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/9*x^9*a+1/9*b*x^9*arctan(c*x^3)-1/18*b*x^6/c+1/18*b*ln(c^2*x^6+1)/c^3
```

Maxima [A] time = 1.05226, size = 65, normalized size = 1.38

$$\frac{1}{9}\,ax^9+\frac{1}{18}\left(2\,x^9\arctan\left(cx^3\right)-\left(\frac{x^6}{c^2}-\frac{\log\left(c^2x^6+1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9 + 1/18*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c)
*b
```

Fricas [A] time = 2.65607, size = 115, normalized size = 2.45

$$\frac{2\,bc^3x^9\arctan\left(cx^3\right)+2\,ac^3x^9-bc^2x^6+b\log\left(c^2x^6+1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/18*(2*b*c^3*x^9*arctan(c*x^3) + 2*a*c^3*x^9 - b*c^2*x^6 + b*log(c^2*x^6 +
 1))/c^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atan(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13653, size = 63, normalized size = 1.34

$$\frac{2\,acx^9+\left(2\,cx^9\arctan\left(cx^3\right)-x^6+\frac{\log\left(c^2x^6+1\right)}{c^2}\right)b}{18\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/18*(2*a*c*x^9 + (2*c*x^9*arctan(c*x^3) - x^6 + log(c^2*x^6 + 1)/c^2)*b)/c
```

3.98 $\int x^5 \left(a+b\tan^{-1}\left(cx^3\right)\right) \, dx$

Optimal. Leaf size=43

$$\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\tan^{-1}\left(cx^3\right)}{6c^2}-\frac{bx^3}{6c}$$

```
[Out] -(b*x^3)/(6*c) + (b*ArcTan[c*x^3])/(6*c^2) + (x^6*(a + b*ArcTan[c*x^3]))/6
```

Rubi [A] time = 0.029365, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 321, 203}

$$\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\tan^{-1}\left(cx^3\right)}{6c^2}-\frac{bx^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -(b*x^3)/(6*c) + (b*ArcTan[c*x^3])/(6*c^2) + (x^6*(a + b*ArcTan[c*x^3]))/6
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= \frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}(bc)\int\frac{x^8}{1+c^2x^6}\,dx\\
&= \frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int\frac{x^2}{1+c^2x^2}\,dx,x,x^3\right)\\
&= -\frac{bx^3}{6c}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\,\mathrm{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{6c}\\
&= -\frac{bx^3}{6c}+\frac{b\tan^{-1}\left(cx^3\right)}{6c^2}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.0052992, size = 48, normalized size = 1.12

$$\frac{ax^6}{6}+\frac{b\tan^{-1}\left(cx^3\right)}{6c^2}-\frac{bx^3}{6c}+\frac{1}{6}bx^6\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -(b*x^3)/(6*c) + (a*x^6)/6 + (b*ArcTan[c*x^3])/(6*c^2) + (b*x^6*ArcTan[c*x^
3])/6
```

Maple [A] time = 0.023, size = 41, normalized size = 1.

$$\frac{x^6a}{6}+\frac{bx^6\arctan\left(cx^3\right)}{6}-\frac{bx^3}{6c}+\frac{b\arctan\left(cx^3\right)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctan(c*x^3)-1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2
```

Maxima [A] time = 1.48283, size = 58, normalized size = 1.35

$$\frac{1}{6}ax^6+\frac{1}{6}\left(x^6\arctan\left(cx^3\right)-c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/6*(x^6*arctan(c*x^3) - c*(x^3/c^2 - arctan(c*x^3)/c^3))*b
```

Fricas [A] time = 2.58688, size = 85, normalized size = 1.98

$$\frac{ac^2x^6-bcx^3+\left(bc^2x^6+b\right)\arctan\left(cx^3\right)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*c^2*x^6 - b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3))/c^2
```

Sympy [A] time = 166.108, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{atan}\left(cx^3\right)}{6} - \frac{bx^3}{6c} + \frac{b \operatorname{atan}\left(cx^3\right)}{6c^2} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x**3)/6 - b*x**3/(6*c) + b*atan(c*x**3)
/(6*c**2), Ne(c, 0)), (a*x**6/6, True))
```

Giac [A] time = 1.10493, size = 58, normalized size = 1.35

$$\frac{acx^6 + \frac{\left(c^2x^6 \arctan\left(cx^3\right) - cx^3 + \arctan\left(cx^3\right)\right)b}{c}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/6*(a*c*x^6 + (c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*b/c)/c
```

3.99 $\int x^2 \left(a+b\tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=36

$$\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\log\left(c^2x^6+1\right)}{6c}$$

```
[Out] (x^3*(a + b*ArcTan[c*x^3]))/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Rubi [A] time = 0.0206472, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5033, 260}

$$\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\log\left(c^2x^6+1\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (x^3*(a + b*ArcTan[c*x^3]))/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)-(bc)\int\frac{x^5}{1+c^2x^6}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\log\left(1+c^2x^6\right)}{6c}\end{aligned}$$

Mathematica [A] time = 0.007222, size = 41, normalized size = 1.14

$$\frac{ax^3}{3}-\frac{b\log\left(c^2x^6+1\right)}{6c}+\frac{1}{3}bx^3\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*ArcTan[c*x^3])/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Maple [A] time = 0.021, size = 36, normalized size = 1.

$$\frac{x^3 a}{3}+\frac{b x^3 \arctan\left(c x^3\right)}{3}-\frac{b \ln\left(c^2 x^6+1\right)}{6 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctan(c*x^3)-1/6*b*ln(c^2*x^6+1)/c
```

Maxima [A] time = 1.00259, size = 51, normalized size = 1.42

$$\frac{1}{3}\,a x^3+\frac{\left(2\,c x^3 \arctan\left(c x^3\right)-\log\left(c^2 x^6+1\right)\right) b}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b/c
```

Fricas [A] time = 2.66065, size = 89, normalized size = 2.47

$$\frac{2\,b c x^3 \arctan\left(c x^3\right)+2\,a c x^3-b \log\left(c^2 x^6+1\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c*x^3*arctan(c*x^3) + 2*a*c*x^3 - b*log(c^2*x^6 + 1))/c
```

Sympy [A] time = 83.1445, size = 808, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((x**3*(a - oo*I*b)/3, Eq(c, -I/x**3)), (x**3*(a + oo*I*b)/3, Eq(c
, I/x**3)), (a*x**3/3, Eq(c, 0)), (-I*a*c**17*x**9*(c**(-2))**(15/2)/(-3*I*
c**17*x**6*(c**(-2))**(15/2) - 3*I*c**15*(c**(-2))**(15/2)) - I*a*c**15*x**
3*(c**(-2))**(15/2)/(-3*I*c**17*x**6*(c**(-2))**(15/2) - 3*I*c**15*(c**(-2)
)**(15/2)) - I*b*c**17*x**9*(c**(-2))**(15/2)*atan(c*x**3)/(-3*I*c**17*x**6
*(c**(-2))**(15/2) - 3*I*c**15*(c**(-2))**(15/2)) + I*b*c**16*x**6*(c**(-2)
)**(15/2)*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(-3*I*c**17*x**6*(c**(-2))*
*(15/2) - 3*I*c**15*(c**(-2))**(15/2)) + I*b*c**16*x**6*(c**(-2))**(15/2)*l
og(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/
```

```
3))/(-3*I*c**17*x**6*(c**(-2))**(15/2) - 3*I*c**15*(c**(-2))**(15/2)) - 2*I
*b*c**16*x**6*(c**(-2))**(15/2)*log(2)/(-3*I*c**17*x**6*(c**(-2))**(15/2) -
 3*I*c**15*(c**(-2))**(15/2)) - I*b*c**15*x**3*(c**(-2))**(15/2)*atan(c*x**
3)/(-3*I*c**17*x**6*(c**(-2))**(15/2) - 3*I*c**15*(c**(-2))**(15/2)) + I*b*
c**14*(c**(-2))**(15/2)*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(-3*I*c**17*x
**6*(c**(-2))**(15/2) - 3*I*c**15*(c**(-2))**(15/2)) + I*b*c**14*(c**(-2))*
*(15/2)*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(
-2))**(1/3))/(-3*I*c**17*x**6*(c**(-2))**(15/2) - 3*I*c**15*(c**(-2))**(15/
2)) - 2*I*b*c**14*(c**(-2))**(15/2)*log(2)/(-3*I*c**17*x**6*(c**(-2))**(15/
2) - 3*I*c**15*(c**(-2))**(15/2)) + b*c*x**6*atan(c*x**3)/(-3*I*c**17*x**6*
(c**(-2))**(15/2) - 3*I*c**15*(c**(-2))**(15/2)) + b*atan(c*x**3)/(-3*I*c**
18*x**6*(c**(-2))**(15/2) - 3*I*c**16*(c**(-2))**(15/2)), True))
```

Giac [A] time = 1.161, size = 54, normalized size = 1.5

$$\frac{2\,acx^3 + \left(2\,cx^3\arctan\left(cx^3\right) - \log\left(c^2x^6 + 1\right)\right)b}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/6*(2*a*c*x^3 + (2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b)/c
```

3.100 $\int \frac{a+b\tan^{-1}(cx^3)}{x}\,dx$

Optimal. Leaf size=39

$$\frac{1}{6}ib\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{6}ib\text{PolyLog}\left(2,icx^3\right)+a\log(x)$$

```
[Out] a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

Rubi [A] time = 0.0503424, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5031, 4848, 2391}

$$\frac{1}{6}ib\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{6}ib\text{PolyLog}\left(2,icx^3\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x,x]
```

```
[Out] a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{a+b\tan^{-1}(cx)}{x}\,dx,x,x^3\right)\\ &= a\log(x)+\frac{1}{6}(ib)\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,x^3\right)-\frac{1}{6}(ib)\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx,x,x^3\right)\\ &= a\log(x)+\frac{1}{6}ib\text{Li}_2\left(-icx^3\right)-\frac{1}{6}ib\text{Li}_2\left(icx^3\right)\end{aligned}$$

Mathematica [A] time = 0.0047405, size = 39, normalized size = 1.

$$\frac{1}{6}ib\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{6}ib\text{PolyLog}\left(2,icx^3\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x,x]
```

```
[Out] a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

Maple [C] time = 0.086, size = 63, normalized size = 1.6

$$a\ln(x) + b\ln(x)\arctan\left(cx^3\right) - \frac{b}{2c}\sum_{\_R1=\mathrm{RootOf}\left(c^2\_Z^6+1\right)} \frac{1}{\_R1^3}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right) + \mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x,x)
```

```
[Out] a*ln(x)+b*ln(x)*arctan(c*x^3)-1/2*b/c*sum(1/_R1^3*(ln(x)*ln((_R1-x)/_R1)+di
log((_R1-x)/_R1)),_R1=RootOf(_Z^6*c^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int \frac{\arctan\left(cx^3\right)}{x}\,dx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x^3)/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\arctan\left(cx^3\right) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^3) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{atan}\left(cx^3\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x,x)
```

```
[Out] Integral((a + b*atan(c*x**3))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan\left(cx^3\right) + a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)/x, x)
```

3.101 $\int \frac{a+b\tan^{-1}(cx^3)}{x^4}\,dx$

Optimal. Leaf size=39

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}-\frac{1}{6}bc\log\left(c^2x^6+1\right)+bc\log(x)$$

```
[Out] -(a + b*ArcTan[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 + c^2*x^6])/6
```

Rubi [A] time = 0.0245742, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5033, 266, 36, 29, 31}

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}-\frac{1}{6}bc\log\left(c^2x^6+1\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^4,x]
```

```
[Out] -(a + b*ArcTan[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 + c^2*x^6])/6
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^4}\,dx &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}+(bc)\int \frac{1}{x\left(1+c^2x^6\right)}\,dx\\ &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}+\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx,x,x^6\right)\\ &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}+\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^6\right)-\frac{1}{6}\left(bc^3\right)\mathrm{Subst}\left(\int \frac{1}{1+c^2x}\,dx,x,x^6\right)\\ &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)\end{aligned}$$

Mathematica [A] time = 0.0067643, size = 44, normalized size = 1.13

$$-\frac{a}{3x^3}-\frac{1}{6}bc\log\left(c^2x^6+1\right)-\frac{b\tan^{-1}\left(cx^3\right)}{3x^3}+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^4,x]

[Out] -a/(3*x^3) - (b*ArcTan[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 + c^2*x^6]
)/6
```

Maple [A] time = 0.026, size = 39, normalized size = 1.

$$-\frac{a}{3\,x^3}-\frac{b\arctan\left(cx^3\right)}{3\,x^3}-\frac{bc\ln\left(c^2x^6+1\right)}{6}+bc\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^4,x)

[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c*x^3)-1/6*b*c*ln(c^2*x^6+1)+b*c*ln(x)
```

Maxima [A] time = 0.993088, size = 55, normalized size = 1.41

$$-\frac{1}{6}\left(c\left(\log\left(c^2x^6+1\right)-\log\left(x^6\right)\right)+\frac{2\arctan\left(cx^3\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="maxima")

[Out] -1/6*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*b - 1/3*a/x^3
```

Fricas [A] time = 3.27881, size = 111, normalized size = 2.85

$$-\frac{bcx^3\log\left(c^2x^6+1\right)-6\,bcx^3\log(x)+2\,b\arctan\left(cx^3\right)+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*x^3*log(c^2*x^6 + 1) - 6*b*c*x^3*log(x) + 2*b*arctan(c*x^3) + 2*a
)/x^3
```

Sympy [A] time = 155.503, size = 505, normalized size = 12.95

$$\begin{cases} -\frac{a-\infty ib}{3x^3} \\ -\frac{a+\infty ib}{3x^3} \\ -\frac{a}{3x^3} \\ -\frac{ax^6}{3x^9+\frac{3x^3}{c^2}} - \frac{a}{3c^2x^9+3x^3} + \frac{ibc^{28}x^9\left(\frac{1}{c^2}\right)^{\frac{29}{2}}\operatorname{atan}\left(cx^3\right)}{\frac{3x^9}{c^2}+\frac{3x^3}{c^4}} + \frac{ibc^{26}x^3\left(\frac{1}{c^2}\right)^{\frac{29}{2}}\operatorname{atan}\left(cx^3\right)}{\frac{3x^9}{c^2}+\frac{3x^3}{c^4}} + \frac{3bx^9\log\left(x\right)}{\frac{3x^9}{c}+\frac{3x^3}{c^3}} - \frac{bx^9\log\left(x-\sqrt[6]{-1}\sqrt[6]{\frac{1}{c^2}}\right)}{\frac{3x^9}{c}+\frac{3x^3}{c^3}} - \frac{bx^9\log\left(4x^2+4\sqrt[6]{-1}x\sqrt[6]{\frac{1}{c^2}} \cdots\right.}{\frac{3x^9}{c}+\frac{3x^3}{c^3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**4,x)
```

```
[Out] Piecewise((-(a - oo*I*b)/(3*x**3), Eq(c, -I/x**3)), (-(a + oo*I*b)/(3*x**3)
, Eq(c, I/x**3)), (-a/(3*x**3), Eq(c, 0)), (-a*x**6/(3*x**9 + 3*x**3/c**2)
- a/(3*c**2*x**9 + 3*x**3) + I*b*c**28*x**9*(c**(-2))**(29/2)*atan(c*x**3)/
(3*x**9/c**2 + 3*x**3/c**4) + I*b*c**26*x**3*(c**(-2))**(29/2)*atan(c*x**3)
/(3*x**9/c**2 + 3*x**3/c**4) + 3*b*x**9*log(x)/(3*x**9/c + 3*x**3/c**3) - b
*x**9*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(3*x**9/c + 3*x**3/c**3) - b*x*
*9*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))*
*(1/3))/(3*x**9/c + 3*x**3/c**3) + 2*b*x**9*log(2)/(3*x**9/c + 3*x**3/c**3)
 - b*x**6*atan(c*x**3)/(3*x**9 + 3*x**3/c**2) + 3*b*x**3*log(x)/(3*c*x**9 +
 3*x**3/c) - b*x**3*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(3*c*x**9 + 3*x**
3/c) - b*x**3*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)
*(c**(-2))**(1/3))/(3*c*x**9 + 3*x**3/c) + 2*b*x**3*log(2)/(3*c*x**9 + 3*x*
*3/c) - b*atan(c*x**3)/(3*c**2*x**9 + 3*x**3), True))
```

Giac [A] time = 1.11099, size = 81, normalized size = 2.08

$$-\frac{bc^3x^3\log\left(c^2x^6+1\right) - 2\,bc^3x^3\log\left(cx^3\right) + 2\,bc^2\arctan\left(cx^3\right) + 2\,ac^2}{6\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(b*c^3*x^3*log(c^2*x^6 + 1) - 2*b*c^3*x^3*log(c*x^3) + 2*b*c^2*arctan(
c*x^3) + 2*a*c^2)/(c^2*x^3)
```

3.102 $\int \frac{a+b\tan^{-1}(cx^3)}{x^7}\,dx$

Optimal. Leaf size=41

$$-\frac{a+b\tan^{-1}(cx^3)}{6x^6}-\frac{1}{6}bc^2\tan^{-1}(cx^3)-\frac{bc}{6x^3}$$

```
[Out] -(b*c)/(6*x^3) - (b*c^2*ArcTan[c*x^3])/6 - (a + b*ArcTan[c*x^3])/(6*x^6)
```

Rubi [A] time = 0.0266583, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 275, 325, 203}

$$-\frac{a+b\tan^{-1}(cx^3)}{6x^6}-\frac{1}{6}bc^2\tan^{-1}(cx^3)-\frac{bc}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^7,x]
```

```
[Out] -(b*c)/(6*x^3) - (b*c^2*ArcTan[c*x^3])/6 - (a + b*ArcTan[c*x^3])/(6*x^6)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^7}\,dx = -\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}+\frac{1}{2}(bc)\int \frac{1}{x^4\left(1+c^2x^6\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx,x,x^3\right)$$
$$= -\frac{bc}{6x^3}-\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}-\frac{1}{6}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx,x,x^3\right)$$
$$= -\frac{bc}{6x^3}-\frac{1}{6}bc^2\tan^{-1}\left(cx^3\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{6x^6}$$

Mathematica [C] time = 0.0063388, size = 48, normalized size = 1.17

$$-\frac{bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^6\right)}{6x^3}-\frac{a}{6x^6}-\frac{b\tan^{-1}\left(cx^3\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^7,x]
```

```
[Out] -a/(6*x^6) - (b*ArcTan[c*x^3])/(6*x^6) - (b*c*Hypergeometric2F1[-1/2, 1, 1/
2, -(c^2*x^6)])/(6*x^3)
```

Maple [A] time = 0.028, size = 39, normalized size = 1.

$$-\frac{a}{6x^6}-\frac{b\arctan\left(cx^3\right)}{6x^6}-\frac{bc^2\arctan\left(cx^3\right)}{6}-\frac{bc}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctan(c*x^3)-1/6*b*c^2*arctan(c*x^3)-1/6*b*c/x^3
```

Maxima [A] time = 1.50501, size = 47, normalized size = 1.15

$$-\frac{1}{6}\left(\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c+\frac{\arctan\left(cx^3\right)}{x^6}\right)b-\frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*b - 1/6*a/x^6
```

Fricas [A] time = 3.13239, size = 76, normalized size = 1.85

$$-\frac{bcx^3+\left(bc^2x^6+b\right)\arctan\left(cx^3\right)+a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3) + a)/x^6
```

Sympy [A] time = 153.635, size = 42, normalized size = 1.02

$$-\frac{a}{6x^6}-\frac{bc^2\operatorname{atan}\left(cx^3\right)}{6}-\frac{bc}{6x^3}-\frac{b\operatorname{atan}\left(cx^3\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**7,x)
```

```
[Out] -a/(6*x**6) - b*c**2*atan(c*x**3)/6 - b*c/(6*x**3) - b*atan(c*x**3)/(6*x**6
)
```

Giac [B] time = 1.16607, size = 100, normalized size = 2.44

$$\frac{bc^5ix^6\log\left(cix^3+1\right)-bc^5ix^6\log\left(-cix^3+1\right)-2\,bc^4x^3-2\,bc^3\arctan\left(cx^3\right)-2\,ac^3}{12\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(b*c^5*i*x^6*log(c*i*x^3 + 1) - b*c^5*i*x^6*log(-c*i*x^3 + 1) - 2*b*c^
4*x^3 - 2*b*c^3*arctan(c*x^3) - 2*a*c^3)/(c^3*x^6)
```

3.103 $\int \frac{a+b\tan^{-1}(cx^3)}{x^{10}}\,dx$

Optimal. Leaf size=55

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{18}bc^3\log\left(c^2x^6+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{18x^6}$$

```
[Out] -(b*c)/(18*x^6) - (a + b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^6])/18
```

Rubi [A] time = 0.0350834, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5033, 266, 44}

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{18}bc^3\log\left(c^2x^6+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{18x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^10,x]
```

```
[Out] -(b*c)/(18*x^6) - (a + b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^6])/18
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^{10}}\,dx = -\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{3}(bc)\int \frac{1}{x^7\left(1+c^2x^6\right)}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{18}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x\right)}\,dx,x,x^6\right)$$
$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{18}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx,x,x^6\right)$$
$$= -\frac{bc}{18x^6}-\frac{a+b\tan^{-1}\left(cx^3\right)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(1+c^2x^6\right)$$

Mathematica [A] time = 0.0135117, size = 60, normalized size = 1.09

$$-\frac{a}{9x^9}+\frac{1}{18}bc^3\log\left(c^2x^6+1\right)-\frac{1}{3}bc^3\log(x)-\frac{bc}{18x^6}-\frac{b\tan^{-1}\left(cx^3\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^10,x]
```

```
[Out] -a/(9*x^9) - (b*c)/(18*x^6) - (b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3
+ (b*c^3*Log[1 + c^2*x^6])/18
```

Maple [A] time = 0.027, size = 51, normalized size = 0.9

$$-\frac{a}{9\,x^9}-\frac{b\arctan\left(cx^3\right)}{9\,x^9}+\frac{bc^3\ln\left(c^2x^6+1\right)}{18}-\frac{bc}{18\,x^6}-\frac{bc^3\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^10,x)
```

```
[Out] -1/9*a/x^9-1/9*b/x^9*arctan(c*x^3)+1/18*b*c^3*ln(c^2*x^6+1)-1/18*b*c/x^6-1/
3*b*c^3*ln(x)
```

Maxima [A] time = 1.0209, size = 72, normalized size = 1.31

$$\frac{1}{18}\left(\left(c^2\log\left(c^2x^6+1\right)-c^2\log\left(x^6\right)-\frac{1}{x^6}\right)c-\frac{2\arctan\left(cx^3\right)}{x^9}\right)b-\frac{a}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="maxima")
```

```
[Out] 1/18*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9
)*b - 1/9*a/x^9
```

Fricas [A] time = 3.32517, size = 130, normalized size = 2.36

$$\frac{bc^3x^9\log\left(c^2x^6+1\right)-6\,bc^3x^9\log\left(x\right)-bcx^3-2\,b\arctan\left(cx^3\right)-2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(b*c^3*x^9*log(c^2*x^6 + 1) - 6*b*c^3*x^9*log(x) - b*c*x^3 - 2*b*arcta
n(c*x^3) - 2*a)/x^9
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11312, size = 93, normalized size = 1.69

$$\frac{bc^7x^9\log\left(c^2x^6+1\right)-2\,bc^7x^9\log\left(cx^3\right)-bc^5x^3-2\,bc^4\arctan\left(cx^3\right)-2\,ac^4}{18\,c^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="giac")
```

```
[Out] 1/18*(b*c^7*x^9*log(c^2*x^6 + 1) - 2*b*c^7*x^9*log(c*x^3) - b*c^5*x^3 - 2*b
*c^4*arctan(c*x^3) - 2*a*c^4)/(c^4*x^9)
```

3.104 $\int x^3 \left(a + b \tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=174

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}-\frac{b\tan^{-1}}{}$$

```
[Out] (-3*b*x)/(4*c) + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (x^4*(a + b*ArcTan[c*x
^3]))/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan[Sqrt[3]
 + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/
3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/(16*c^(4/3))
```

Rubi [A] time = 0.324721, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 321, 209, 634, 618, 204, 628, 203}

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}-\frac{b\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x)/(4*c) + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (x^4*(a + b*ArcTan[c*x
^3]))/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan[Sqrt[3]
 + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/
3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/(16*c^(4/3))
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{4}(3bc)\int\frac{x^6}{1+c^2x^6}\,dx\\
&= -\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{(3b)\int\frac{1}{1+c^2x^6}\,dx}{4c}\\
&= -\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{4c}+\frac{b\int\frac{1-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{4c}+\frac{b\int\frac{1+\frac{1}{2}\sqrt{3}\sqrt[3]{c}
\\
&= -\frac{3bx}{4c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\left(\sqrt{3}b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{16c^{4/3}}+\frac{\left(\sqrt{3}b\right)\int
\\
&= -\frac{3bx}{4c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}+\frac{\sqrt{3}b
\\
&= -\frac{3bx}{4c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{8c^{4/3}}+\frac{b\tan^{-1}\left(\sqrt{3}-
\end{aligned}$$

Mathematica [A] time = 0.0428507, size = 179, normalized size = 1.03

$$\frac{ax^4}{4}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{8c^{4/3}}+\frac{b\tan$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x)/(4*c) + (a*x^4)/4 + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (b*x^4*Arc
Tan[c*x^3])/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan[S
qrt[3] + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x +
 c^(2/3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3
)*x^2])/(16*c^(4/3))
```

Maple [A] time = 0.092, size = 165, normalized size = 1.

$$\frac{x^4 a}{4}+\frac{b x^4 \arctan\left(c x^3\right)}{4}-\frac{3 b x}{4 c}+\frac{b\sqrt{3}}{16 c}\sqrt[6]{c^{-2}}\ln\left(x^2+\sqrt{3}\sqrt[6]{c^{-2}}x+\sqrt[3]{c^{-2}}\right)+\frac{b}{8 c}\sqrt[6]{c^{-2}}\arctan\left(2\frac{x}{\sqrt[6]{c^{-2}}}+\sqrt{3}\right)-\frac{b\sqrt{3}}{16 c}\sqrt[6]{c^{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctan(c*x^3)-3/4*b*x/c+1/16*b/c*3^(1/2)*(1/c^2)^(1/6)*
ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/8*b/c*(1/c^2)^(1/6)*arctan(
2*x/(1/c^2)^(1/6)+3^(1/2))-1/16*b/c*3^(1/2)*(1/c^2)^(1/6)*ln(x^2-3^(1/2)*(1
/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/8*b/c*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)
-3^(1/2))+1/4*b/c*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))
```

Maxima [B] time = 1.53178, size = 396, normalized size = 2.28

$$\frac{1}{4}ax^4+\frac{1}{16}\left(4x^4\arctan\left(cx^3\right)+c\left(\frac{\frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2+\sqrt{3}\left(c^2\right)^{\frac{1}{6}}x+1\right)}{\left(c^2\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2-\sqrt{3}\left(c^2\right)^{\frac{1}{6}}x+1\right)}{\left(c^2\right)^{\frac{1}{6}}}+\frac{\log\left(\frac{2\left(c^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(c^2\right)^{\frac{1}{6}}-\sqrt{-\left(c^2\right)^{\frac{1}{3}}}}{2\left(c^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(c^2\right)^{\frac{1}{6}}+\sqrt{-\left(c^2\right)^{\frac{1}{3}}}}\right)}{\sqrt{-\left(c^2\right)^{\frac{1}{3}}}}+\cdots}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/16*(4*x^4*arctan(c*x^3) + c*((sqrt(3)*log((c^2)^(1/3)*x^2 + s
qrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(1/6) - sqrt(3)*log((c^2)^(1/3)*x^2 - sqrt(
3)*(c^2)^(1/6)*x + 1)/(c^2)^(1/6) + log((2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1
/6) - sqrt(-(c^2)^(1/3)))/(2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1/6) + sqrt(-(c
^2)^(1/3))))/sqrt(-(c^2)^(1/3)) + log((2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6
) - sqrt(-(c^2)^(1/3)))/(2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6) + sqrt(-(c^2
)^(1/3))))/sqrt(-(c^2)^(1/3)) + 2*log(((c^2)^(1/3)*x - sqrt(-(c^2)^(1/3)))/
((c^2)^(1/3)*x + sqrt(-(c^2)^(1/3))))/sqrt(-(c^2)^(1/3)))/c^2 - 12*x/c^2))*
b
```

Fricas [B] time = 3.14076, size = 948, normalized size = 5.45

$$4\,bcx^4\arctan\left(cx^3\right)+4\,acx^4+\sqrt{3}c\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}\log\left(b^2x^2+\sqrt{3}bcx\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}+c^2\left(\frac{b^6}{c^8}\right)^{\frac{1}{3}}\right)-\sqrt{3}c\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}\log\left(b^2x^2-\sqrt{3}bcx\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}+c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b*c*x^4*arctan(c*x^3) + 4*a*c*x^4 + sqrt(3)*c*(b^6/c^8)^(1/6)*log(b
^2*x^2 + sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^(1/3)) - sqrt(3)*c*(
b^6/c^8)^(1/6)*log(b^2*x^2 - sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^
(1/3)) - 4*c*(b^6/c^8)^(1/6)*arctan(-(2*b*c^7*x*(b^6/c^8)^(5/6) - 2*sqrt(b^
2*x^2 + sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^(1/3))*c^7*(b^6/c^8)^
(5/6) + sqrt(3)*b^6)/b^6) - 4*c*(b^6/c^8)^(1/6)*arctan(-(2*b*c^7*x*(b^6/c^8
)^(5/6) - 2*sqrt(b^2*x^2 - sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^(1
/3))*c^7*(b^6/c^8)^(5/6) - sqrt(3)*b^6)/b^6) - 8*c*(b^6/c^8)^(1/6)*arctan(-
(b*c^7*x*(b^6/c^8)^(5/6) - sqrt(b^2*x^2 + c^2*(b^6/c^8)^(1/3))*c^7*(b^6/c^8
)^(5/6))/b^6) - 12*b*x)/c
```

Sympy [A] time = 111.885, size = 1287, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((x**4*(a - oo*I*b)/4, Eq(c, -I/x**3)), (x**4*(a + oo*I*b)/4, Eq(c
, I/x**3)), (a*x**4/4, Eq(c, 0)), (4*I*a*c**42*x**10*(c**(-2))**(73/2)/(16*
I*c**42*x**6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(73/2)) + 4*I*a*c**4
0*x**4*(c**(-2))**(73/2)/(16*I*c**42*x**6*(c**(-2))**(73/2) + 16*I*c**40*(c
**(-2))**(73/2)) - 2*(-1)**(2/3)*sqrt(3)*b*c**71*x**6*(c**(-2))**(155/3)*at
an(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(16*I*c**42*x*
*6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(73/2)) - 2*(-1)**(2/3)*sqrt(3
)*b*c**69*(c**(-2))**(155/3)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/
6)) - sqrt(3)/3)/(16*I*c**42*x**6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-2))*
*(73/2)) + 4*I*b*c**42*x**10*(c**(-2))**(73/2)*atan(c*x**3)/(16*I*c**42*x**
6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(73/2)) - 12*I*b*c**41*x**7*(c*
*(-2))**(73/2)/(16*I*c**42*x**6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(
73/2)) + 4*I*b*c**40*x**4*(c**(-2))**(73/2)*atan(c*x**3)/(16*I*c**42*x**6*(
c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(73/2)) - 12*I*b*c**39*x*(c**(-2))
**(73/2)/(16*I*c**42*x**6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(73/2))
 - 3*(-1)**(2/3)*b*c**33*x**6*(c**(-2))**(98/3)*log(4*x**2 - 4*(-1)**(1/6)*
x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(16*I*c**42*x**6*(c**(
-2))**(73/2) + 16*I*c**40*(c**(-2))**(73/2)) + 3*(-1)**(2/3)*b*c**33*x**6*(
c**(-2))**(98/3)*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1
/3)*(c**(-2))**(1/3))/(16*I*c**42*x**6*(c**(-2))**(73/2) + 16*I*c**40*(c**(
-2))**(73/2)) - 2*(-1)**(2/3)*sqrt(3)*b*c**33*x**6*(c**(-2))**(98/3)*atan(2
*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/(16*I*c**42*x**6*(
c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(73/2)) - 3*(-1)**(2/3)*b*c**31*(c
**(-2))**(98/3)*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/
3)*(c**(-2))**(1/3))/(16*I*c**42*x**6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-
```

```
2))**(73/2)) + 3*(-1)**(2/3)*b*c**31*(c**(-2))**(98/3)*log(4*x**2 + 4*(-1)*
*(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(16*I*c**42*x**
6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(73/2)) - 2*(-1)**(2/3)*sqrt(3)
*b*c**31*(c**(-2))**(98/3)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)
) + sqrt(3)/3)/(16*I*c**42*x**6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(
73/2)) + 4*(-1)**(1/6)*b*c**14*x**6*(c**(-2))**(139/6)*atan(c*x**3)/(16*I*c
**42*x**6*(c**(-2))**(73/2) + 16*I*c**40*(c**(-2))**(73/2)) + 4*(-1)**(1/6)
*b*(c**(-2))**(103/6)*atan(c*x**3)/(16*I*c**42*x**6*(c**(-2))**(73/2) + 16*
I*c**40*(c**(-2))**(73/2)), True))
```

Giac [A] time = 1.23519, size = 225, normalized size = 1.29

$$\frac{1}{16}\,bc^7\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^8|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\left(2\,x-\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right.}{c^8|c|^{\frac{1}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/16*b*c^7*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^8
*abs(c)^(1/3)) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))
/(c^8*abs(c)^(1/3)) + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(
c^8*abs(c)^(1/3)) + 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^
8*abs(c)^(1/3)) + 4*arctan(x*abs(c)^(1/3))/(c^8*abs(c)^(1/3))) + 1/4*(b*c*x
^4*arctan(c*x^3) + a*c*x^4 - 3*b*x)/c
```

3.105 $\int \left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=101

$$ax+\frac{b\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+bx\tan^{-1}\left(cx^3\right)$$

```
[Out] a*x + b*x*ArcTan[c*x^3] + (Sqrt[3]*b*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(
2*c^(1/3)) + (b*Log[1 + c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 - c^(2/3)*x^2
+ c^(4/3)*x^4])/(4*c^(1/3))
```

Rubi [A] time = 0.0980866, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5027, 275, 292, 31, 634, 617, 204, 628}

$$ax+\frac{b\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+bx\tan^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^3] + (Sqrt[3]*b*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(
2*c^(1/3)) + (b*Log[1 + c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 - c^(2/3)*x^2
+ c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= ax+b\int \tan^{-1}\left(cx^3\right)dx\\ &= ax+bx\tan^{-1}\left(cx^3\right)-(3bc)\int \frac{x^3}{1+c^2x^6}dx\\ &= ax+bx\tan^{-1}\left(cx^3\right)-\frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{x}{1+c^2x^3}dx,x,x^2\right)\\ &= ax+bx\tan^{-1}\left(cx^3\right)+\frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}dx,x,x^2\right)-\frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/}}\right.\\ &= ax+bx\tan^{-1}\left(cx^3\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\,\text{Subst}\left(\int \frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}dx,x,x^2\right)}{4\sqrt[3]{c}}-\frac{1}{4}\left(3b\sqrt[3]{c}\right)\\ &= ax+bx\tan^{-1}\left(cx^3\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{-3-}\right.}{2}\\ &= ax+bx\tan^{-1}\left(cx^3\right)+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}\right.}{4\sqrt[3]{c}}\end{aligned}$$

Mathematica [A] time = 0.0404928, size = 131, normalized size = 1.3

$$ax-\frac{b\left(-2\log\left(c^{2/3}x^2+1\right)+\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)+\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)-2\sqrt{3}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)-2\sqrt{3}\right.}{4\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*c^(1/3)*x] - 2*
Sqrt[3]*ArcTan[Sqrt[3] + 2*c^(1/3)*x] - 2*Log[1 + c^(2/3)*x^2] + Log[1 - Sq
rt[3]*c^(1/3)*x + c^(2/3)*x^2] + Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]))
/(4*c^(1/3))
```

Maple [A] time = 0.023, size = 98, normalized size = 1.

$$ax+bx\arctan\left(cx^{3}\right)+\frac{b}{2c}\ln\left(x^{2}+\sqrt[3]{c^{-2}}\right)\frac{1}{\sqrt[3]{c^{-2}}}-\frac{b}{4c}\ln\left(x^{4}-\sqrt[3]{c^{-2}}x^{2}+\left(c^{-2}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^{-2}}}-\frac{b\sqrt{3}}{2c}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{x^{2}}{\sqrt[3]{c^{-2}}}-1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c*x^3),x)
```

```
[Out] a*x+b*x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/(
1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))-1/2*b*3^(1/2)/c/(1/c^2
)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2-1))
```

Maxima [A] time = 1.49389, size = 143, normalized size = 1.42

$$-\frac{1}{4}\left(\left(c\left(\frac{2\sqrt{3}\left(c^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(c^{2}\right)^{\frac{1}{3}}\left(2x^{2}-\frac{1}{c^{2}}^{\frac{1}{3}}\right)\right)}{c^{2}}+\frac{\left(c^{2}\right)^{\frac{1}{3}}\log\left(x^{4}-\frac{1}{c^{2}}^{\frac{1}{3}}x^{2}+\frac{1}{c^{2}}^{\frac{2}{3}}\right)}{c^{2}}-\frac{2\left(c^{2}\right)^{\frac{1}{3}}\log\left(x^{2}+\frac{1}{c^{2}}^{\frac{1}{3}}\right)}{c^{2}}\right)-4x\arctan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2*sqrt(3)*(c^2)^(1/3)*arctan(1/3*sqrt(3)*(c^2)^(1/3)*(2*x^2 - (c^(
-2))^(1/3)))/c^2 + (c^2)^(1/3)*log(x^4 - (c^(-2))^(1/3)*x^2 + (c^(-2))^(2/3
))/c^2 - 2*(c^2)^(1/3)*log(x^2 + (c^(-2))^(1/3))/c^2) - 4*x*arctan(c*x^3))*
b + a*x
```

Fricas [A] time = 3.30625, size = 632, normalized size = 6.26

$$\left[\frac{4bcx\arctan\left(cx^{3}\right)+\sqrt{3}bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\log\left(\frac{2c^{2}x^{6}-3c^{\frac{2}{3}}x^{2}-\sqrt{3}\left(2c^{\frac{5}{3}}x^{4}+cx^{2}-c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}}-1}{c^{2}x^{6}+1}\right)+4acx-bc^{\frac{2}{3}}\log\left(c^{2}x^{4}-c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right)+2bc^{\frac{2}{3}}}{4c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*c*x*arctan(c*x^3) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6 -
 3*c^(2/3)*x^2 - sqrt(3)*(2*c^(5/3)*x^4 + c*x^2 - c^(1/3))*sqrt(-1/c^(2/3))
 - 1)/(c^2*x^6 + 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2
/3)) + 2*b*c^(2/3)*log(c*x^2 + c^(1/3)))/c, 1/4*(4*b*c*x*arctan(c*x^3) + 2*
sqrt(3)*b*c^(2/3)*arctan(-1/3*sqrt(3)*(2*c*x^2 - c^(1/3))/c^(1/3)) + 4*a*c*
x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2
+ c^(1/3)))/c]
```

Sympy [A] time = 55.8924, size = 1703, normalized size = 16.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c*x**3),x)
```

```
[Out] a*x + b*Piecewise((-I*x*atanh(x**3*sqrt(x**(-6))), Eq(c, -sqrt(-1/x**6)) |
Eq(c, -sqrt(1/(x**6*(1/2 - sqrt(3)*I/2)**3))) | Eq(c, -sqrt(1/(x**6*(1/2 +
sqrt(3)*I/2)**3)))), (0, Eq(c, 0)), (2*(-1)**(1/3)*sqrt(3)*c**39*x**6*(c**(
-2))**(73/3)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)
/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2
))**(71/3)) - 2*(-1)**(1/3)*sqrt(3)*c**39*x**6*(c**(-2))**(73/3)*atan(2*(-1
)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/(-4*(-1)**(2/3)*c**38*
x**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2))**(71/3)) - 3*(-1)**(
1/3)*c**37*x**6*(c**(-2))**(70/3)*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(
1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))
**(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2))**(71/3)) + 6*(-1)**(1/3)*c**37*x**
6*(c**(-2))**(70/3)*log(2)/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4
*(-1)**(2/3)*c**36*(c**(-2))**(71/3)) + 2*(-1)**(1/3)*sqrt(3)*c**37*(c**(-2
))**(73/3)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(
-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2))
**(71/3)) - 2*(-1)**(1/3)*sqrt(3)*c**37*(c**(-2))**(73/3)*atan(2*(-1)**(5/6
)*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/(-4*(-1)**(2/3)*c**38*x**6*(c
**(-2))**(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2))**(71/3)) - 3*(-1)**(1/3)*c*
*35*(c**(-2))**(70/3)*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1
)**(1/3)*(c**(-2))**(1/3))/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4
*(-1)**(2/3)*c**36*(c**(-2))**(71/3)) + 6*(-1)**(1/3)*c**35*(c**(-2))**(70/
3)*log(2)/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)*c**3
6*(c**(-2))**(71/3)) + 4*(-1)**(1/3)*c**33*x**6*(c**(-2))**(64/3)*log(x - (
-1)**(1/6)*(c**(-2))**(1/6))/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) -
 4*(-1)**(2/3)*c**36*(c**(-2))**(71/3)) + (-1)**(1/3)*c**31*x**6*(c**(-2))*
*(61/3)*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(
-2))**(1/3))/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)*c
**36*(c**(-2))**(71/3)) - 2*(-1)**(1/3)*c**31*x**6*(c**(-2))**(61/3)*log(2)
/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2
))**(71/3)) + 4*(-1)**(1/3)*c**31*(c**(-2))**(64/3)*log(x - (-1)**(1/6)*(c*
*(-2))**(1/6))/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)
*c**36*(c**(-2))**(71/3)) + (-1)**(1/3)*c**29*(c**(-2))**(61/3)*log(4*x**2
+ 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(-4*(-
1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2))**(71
/3)) - 2*(-1)**(1/3)*c**29*(c**(-2))**(61/3)*log(2)/(-4*(-1)**(2/3)*c**38*x
**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2))**(71/3)) - 4*(-1)**(2
/3)*c**26*x**7*(c**(-2))**(53/3)*atan(c*x**3)/(-4*(-1)**(2/3)*c**38*x**6*(c
**(-2))**(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2))**(71/3)) - 4*(-1)**(2/3)*c*
*24*x*(c**(-2))**(53/3)*atan(c*x**3)/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**
(71/3) - 4*(-1)**(2/3)*c**36*(c**(-2))**(71/3)) - 4*(-1)**(5/6)*c**14*(c**(
-2))**(77/6)*atan(c*x**3)/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4*
(-1)**(2/3)*c**36*(c**(-2))**(71/3)) - 4*(-1)**(5/6)*x**6*(c**(-2))**(29/6)
*atan(c*x**3)/(-4*(-1)**(2/3)*c**38*x**6*(c**(-2))**(71/3) - 4*(-1)**(2/3)*
c**36*(c**(-2))**(71/3)), True))
```

Giac [A] time = 1.1522, size = 128, normalized size = 1.27

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{2}-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{2}}+\frac{|c|^{\frac{2}{3}}\log\left(x^{4}-\frac{x^{2}}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{2}}-\frac{2\log\left(x^{2}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right)-4x\arctan\left(cx^{3}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="giac")
```

```
[Out] -1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))
*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/
3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 4*x*arctan(c*x^3))*b
+ a*x
```

3.106 $\int \frac{a+b\tan^{-1}(cx^3)}{x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{a+b\tan^{-1}(cx^3)}{2x^2}-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}x\right)$$

```
[Out] (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (a + b*ArcTan[c*x^3])/(2*x^2) - (b*c^(2/3
)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*
x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8 + (S
qrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```

Rubi [A] time = 0.294441, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5033, 209, 634, 618, 204, 628, 203}

$$-\frac{a+b\tan^{-1}(cx^3)}{2x^2}-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^3,x]
```


```
[Out] (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (a + b*ArcTan[c*x^3])/(2*x^2) - (b*c^(2/3
)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*
x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8 + (S
qrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```


Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^3}\,dx = -\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(3bc)\int \frac{1}{1+c^2x^6}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{1+c^{2/3}x^2}\,dx+\frac{1}{2}(bc)\int \frac{1-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx+\frac{1}{2}(bc)\int \frac{}{1+}$$
$$= \frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\left(\sqrt{3}bc^{2/3}\right)\int \frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx+\frac{1}{8}\left(\sqrt{3}bc^{2/3}\right.$$
$$= \frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(\right.$$
$$= \frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{4}bc^{2/3}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)+\frac{1}{4}bc^{2/3}\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{}\right.$$

Mathematica [A] time = 0.0478968, size = 170, normalized size = 1.03

$$-\frac{a}{2x^2}-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}x\right)-\frac{1}{4}bc^{2/3}\tan^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^3,x]
```

```
[Out] -a/(2*x^2) + (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (b*ArcTan[c*x^3])/(2*x^2) -
(b*c^(2/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] + 2
*c^(1/3)*x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2
])/8 + (Sqrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```

Maple [A] time = 0.056, size = 148, normalized size = 0.9

$$-\frac{a}{2x^2}-\frac{b\arctan\left(cx^3\right)}{2x^2}+\frac{bc\sqrt{3}}{8}\sqrt[6]{c^{-2}}\ln\left(x^2+\sqrt{3}\sqrt[6]{c^{-2}}x+\sqrt[3]{c^{-2}}\right)+\frac{bc}{4}\sqrt[6]{c^{-2}}\arctan\left(2\frac{x}{\sqrt[6]{c^{-2}}}+\sqrt{3}\right)-\frac{bc\sqrt{3}}{8}\sqrt[6]{c^{-2}}\ln\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c*x^3)+1/8*b*c*3^(1/2)*(1/c^2)^(1/6)*ln(x^2+3^(
1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/4*b*c*(1/c^2)^(1/6)*arctan(2*x/(1/c^2
)^(1/6)+3^(1/2))-1/8*b*c*3^(1/2)*(1/c^2)^(1/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)
*x+(1/c^2)^(1/3))+1/4*b*c*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))+1
/2*b*c*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))
```

Maxima [B] time = 1.52117, size = 381, normalized size = 2.31

$$\frac{1}{8}\left(\left(\left(\frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2+\sqrt{3}\left(c^2\right)^{\frac{1}{6}}x+1\right)}{\left(c^2\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2-\sqrt{3}\left(c^2\right)^{\frac{1}{6}}x+1\right)}{\left(c^2\right)^{\frac{1}{6}}}+\frac{\log\left(\frac{2\left(c^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(c^2\right)^{\frac{1}{6}}-\sqrt{-\left(c^2\right)^{\frac{1}{3}}}}{2\left(c^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(c^2\right)^{\frac{1}{6}}+\sqrt{-\left(c^2\right)^{\frac{1}{3}}}}\right)}{\sqrt{-\left(c^2\right)^{\frac{1}{3}}}}+\frac{\log\left(\frac{2(\cdots}{2(\cdots}\right.}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*((sqrt(3)*log((c^2)^(1/3)*x^2 + sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(1/6)
- sqrt(3)*log((c^2)^(1/3)*x^2 - sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(1/6) + lo
g((2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1/6) - sqrt(-(c^2)^(1/3)))/(2*(c^2)^(1/
3)*x + sqrt(3)*(c^2)^(1/6) + sqrt(-(c^2)^(1/3))))/sqrt(-(c^2)^(1/3)) + log(
(2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6) - sqrt(-(c^2)^(1/3)))/(2*(c^2)^(1/3)
*x - sqrt(3)*(c^2)^(1/6) + sqrt(-(c^2)^(1/3))))/sqrt(-(c^2)^(1/3)) + 2*log(
((c^2)^(1/3)*x - sqrt(-(c^2)^(1/3)))/((c^2)^(1/3)*x + sqrt(-(c^2)^(1/3))))/
sqrt(-(c^2)^(1/3)))*c - 4*arctan(c*x^3)/x^2)*b - 1/2*a/x^2
```

Fricas [B] time = 3.58416, size = 1224, normalized size = 7.42

$$\frac{\sqrt{3}\left(b^6c^4\right)^{\frac{1}{6}}x^2\log\left(4b^2c^2x^2+4\sqrt{3}\left(b^6c^4\right)^{\frac{1}{6}}bcx+4\left(b^6c^4\right)^{\frac{1}{3}}\right)-\sqrt{3}\left(b^6c^4\right)^{\frac{1}{6}}x^2\log\left(4b^2c^2x^2-4\sqrt{3}\left(b^6c^4\right)^{\frac{1}{6}}bcx+4\left(b^6\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(3)*(b^6*c^4)^(1/6)*x^2*log(4*b^2*c^2*x^2 + 4*sqrt(3)*(b^6*c^4)^(
1/6)*b*c*x + 4*(b^6*c^4)^(1/3)) - sqrt(3)*(b^6*c^4)^(1/6)*x^2*log(4*b^2*c^2
*x^2 - 4*sqrt(3)*(b^6*c^4)^(1/6)*b*c*x + 4*(b^6*c^4)^(1/3)) + sqrt(3)*(b^6*
c^4)^(1/6)*x^2*log(b^2*c^2*x^2 + sqrt(3)*(b^6*c^4)^(1/6)*b*c*x + (b^6*c^4)^
(1/3)) - sqrt(3)*(b^6*c^4)^(1/6)*x^2*log(b^2*c^2*x^2 - sqrt(3)*(b^6*c^4)^(1
/6)*b*c*x + (b^6*c^4)^(1/3)) - 8*(b^6*c^4)^(1/6)*x^2*arctan(-(sqrt(3)*b^6*c
^4 + 2*(b^6*c^4)^(5/6)*b*c*x - 2*(b^6*c^4)^(5/6)*sqrt(b^2*c^2*x^2 + sqrt(3)
*(b^6*c^4)^(1/6)*b*c*x + (b^6*c^4)^(1/3)))/(b^6*c^4)) - 8*(b^6*c^4)^(1/6)*x
^2*arctan((sqrt(3)*b^6*c^4 - 2*(b^6*c^4)^(5/6)*b*c*x + 2*(b^6*c^4)^(5/6)*sq
rt(b^2*c^2*x^2 - sqrt(3)*(b^6*c^4)^(1/6)*b*c*x + (b^6*c^4)^(1/3)))/(b^6*c^4
)) - 16*(b^6*c^4)^(1/6)*x^2*arctan(-((b^6*c^4)^(5/6)*b*c*x - (b^6*c^4)^(5/6
```

```
)*sqrt(b^2*c^2*x^2 + (b^6*c^4)^(1/3)))/(b^6*c^4)) - 8*b*arctan(c*x^3) - 8*a
)/x^2
```

Sympy [A] time = 124.381, size = 311, normalized size = 1.88

$$\begin{cases} -\frac{a}{2x^2} - \frac{\sqrt[6]{-1}\sqrt{3}bc^7\left(\frac{1}{c^2}\right)^{\frac{19}{6}}\operatorname{atan}\left(\frac{2(-1)^{\frac{5}{6}}\sqrt{3}x}{3\sqrt[6]{\frac{1}{c^2}}} - \frac{\sqrt{3}}{3}\right)}{4} - \frac{\sqrt[6]{-1}\sqrt{3}bc^7\left(\frac{1}{c^2}\right)^{\frac{19}{6}}\operatorname{atan}\left(\frac{2(-1)^{\frac{5}{6}}\sqrt{3}x}{3\sqrt[6]{\frac{1}{c^2}}} + \frac{\sqrt{3}}{3}\right)}{4} - \frac{b\operatorname{atan}\left(cx^3\right)}{2x^2} + \frac{3\sqrt[6]{-1}b\log\left(4x^2 + 4\sqrt[6]{-1}x\sqrt[6]{\frac{1}{c^2}} + 4\sqrt[3]{-1}\sqrt[3]{\ldots}\right.}{8c^7\left(\frac{1}{c^2}\right)^{\frac{23}{6}}} \\ -\frac{a}{2x^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - (-1)**(1/6)*sqrt(3)*b*c**7*(c**(-2))**(19/6)*atan(
2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/4 - (-1)**(1/6)*s
qrt(3)*b*c**7*(c**(-2))**(19/6)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**
(1/6)) + sqrt(3)/3)/4 - b*atan(c*x**3)/(2*x**2) + 3*(-1)**(1/6)*b*log(4*x**
2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(8*c
**7*(c**(-2))**(23/6)) - 3*(-1)**(1/6)*b*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(
-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(8*c**11*(c**(-2))**(35/6)) -
 (-1)**(2/3)*b*atan(c*x**3)/(2*c**24*(c**(-2))**(37/3)), Ne(c, 0)), (-a/(2*
x**2), True))
```

Giac [A] time = 1.24397, size = 185, normalized size = 1.12

$$\frac{1}{8}\left(\frac{\sqrt{3}\log\left(x^2 + \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}} - \frac{\sqrt{3}\log\left(x^2 - \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}} + \frac{2\arctan\left(\left(2x + \frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}} + \frac{2\arctan\left(\left(2x - \frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}} + \frac{4\,\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3
) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3)
 + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 2*arc
tan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 4*arctan(x*ab
s(c)^(1/3))/abs(c)^(1/3))*b*c - 1/2*(b*arctan(c*x^3) + a)/x^2
```

3.107 $\int \frac{a+b\tan^{-1}(cx^3)}{x^6}\,dx$

Optimal. Leaf size=115

$$-\frac{a+b\tan^{-1}(cx^3)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(c^{2/3}x^2+1\right)-\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)+\frac{1}{10}\sqrt{3}bc^{5/3}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{3}{10}$$

```
[Out] (-3*b*c)/(10*x^2) - (a + b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(5/3)*ArcT
an[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 -
 (b*c^(5/3)*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/20
```

Rubi [A] time = 0.0913579, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5033, 275, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{a+b\tan^{-1}(cx^3)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(c^{2/3}x^2+1\right)-\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)+\frac{1}{10}\sqrt{3}bc^{5/3}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{3}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^6,x]
```

```
[Out] (-3*b*c)/(10*x^2) - (a + b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(5/3)*ArcT
an[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 -
 (b*c^(5/3)*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/20
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^6}\,dx &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{5}(3bc)\int \frac{1}{x^3\left(1+c^2x^6\right)}\,dx\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}(3bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x^3\right)}\,dx,x,x^2\right)\\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}\left(3bc^3\right)\text{Subst}\left(\int \frac{x}{1+c^2x^3}\,dx,x,x^2\right)\\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}\left(bc^{7/3}\right)\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)-\frac{1}{10}\left(bc^{7/3}\right)\text{Subst}\left(\int \frac{1}{}\right.\\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}\left(bc^{5/3}\right)\text{Subst}\left(\int \frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\right.\\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)-\frac{1}{10}\left(\right.\\
&= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}\sqrt{3}bc^{5/3}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc
\end{aligned}
$$

Mathematica [A] time = 0.0451492, size = 183, normalized size = 1.59

$$
-\frac{a}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(c^{2/3}x^2+1\right)-\frac{1}{20}bc^{5/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)-\frac{1}{20}bc^{5/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{10}\sqrt{3}bc^{5/3}\tan
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^6,x]
```

```
[Out] -a/(5*x^5) - (3*b*c)/(10*x^2) - (b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(5
/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[Sqrt[3] +
 2*c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 - (b*c^(5/3)*Log[1
- Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/20 - (b*c^(5/3)*Log[1 + Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/20
```

Maple [A] time = 0.03, size = 105, normalized size = 0.9

$$-\frac{a}{5\,x^5}-\frac{b\arctan\left(cx^3\right)}{5\,x^5}+\frac{bc}{10}\ln\left(x^2+\sqrt[3]{c^{-2}}\right)\frac{1}{\sqrt[3]{c^{-2}}}-\frac{bc}{20}\ln\left(x^4-\sqrt[3]{c^{-2}}x^2+\left(c^{-2}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^{-2}}}-\frac{bc\sqrt{3}}{10}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,\frac{x^2}{\sqrt[3]{c^{-}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctan(c*x^3)+1/10*b*c/(1/c^2)^(1/3)*ln(x^2+(1/c^2)^(1
/3))-1/20*b*c/(1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))-1/10*b*
c*3^(1/2)/(1/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2-1))-3/10*b*
c/x^2
```

Maxima [A] time = 1.54076, size = 144, normalized size = 1.25

$$-\frac{1}{20}\left(\left(2\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\left(2\,x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)\right)+\left(c^2\right)^{\frac{1}{3}}\log\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)-2\left(c^2\right)^{\frac{1}{3}}\log\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)+\frac{6}{x^2}\right)c+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*sqrt(3)*(c^2)^(1/3)*arctan(1/3*sqrt(3)*(c^2)^(1/3)*(2*x^2 - (c^(-
2))^(1/3))) + (c^2)^(1/3)*log(x^4 - (c^(-2))^(1/3)*x^2 + (c^(-2))^(2/3)) -
2*(c^2)^(1/3)*log(x^2 + (c^(-2))^(1/3)) + 6/x^2)*c + 4*arctan(c*x^3)/x^5)*b
 - 1/5*a/x^5
```

Fricas [A] time = 2.73759, size = 336, normalized size = 2.92

$$-\frac{2\sqrt{3}b\left(c^2\right)^{\frac{1}{3}}cx^5\arctan\left(\frac{2}{3}\sqrt{3}\left(c^2\right)^{\frac{1}{3}}x^2-\frac{1}{3}\sqrt{3}\right)+b\left(c^2\right)^{\frac{1}{3}}cx^5\log\left(c^2x^4-\left(c^2\right)^{\frac{2}{3}}x^2+\left(c^2\right)^{\frac{1}{3}}\right)-2\,b\left(c^2\right)^{\frac{1}{3}}cx^5\log\left(c^2x^2+\left(c^{\cdots}\right.\right.}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(3)*b*(c^2)^(1/3)*c*x^5*arctan(2/3*sqrt(3)*(c^2)^(1/3)*x^2 - 1
/3*sqrt(3)) + b*(c^2)^(1/3)*c*x^5*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/
3)) - 2*b*(c^2)^(1/3)*c*x^5*log(c^2*x^2 + (c^2)^(2/3)) + 6*b*c*x^3 + 4*b*ar
ctan(c*x^3) + 4*a)/x^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20456, size = 146, normalized size = 1.27

$$-\frac{1}{20}\,bc^3\left(\frac{2\,\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}+\frac{|c|^{\frac{2}{3}}\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}-\frac{2\,\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right)-\frac{3\,bcx^3+2\,b\arctan\left(c\right.}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2
/3))*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)
^(4/3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 1/10*(3*b*c*x^3 +
 2*b*arctan(c*x^3) + 2*a)/x^5
```

3.108 $\int x^7 \left(a+b\tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=176

$$\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}-\frac{b\tan^{-1}}{}$$

```
[Out] (-3*b*x^5)/(40*c) + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (x^8*(a + b*ArcTan[
c*x^3]))/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x +
c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)
*x^2])/(32*c^(8/3))
```

Rubi [A] time = 0.430877, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 321, 295, 634, 618, 204, 628, 203}

$$\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}-\frac{b\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^5)/(40*c) + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (x^8*(a + b*ArcTan[
c*x^3]))/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x +
c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)
*x^2])/(32*c^(8/3))
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^7\left(a+b\tan^{-1}\left(cx^3\right)\right)dx=\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{10}}{1+c^2x^6}\,dx$$

$$=-\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{(3b)\int\frac{x^4}{1+c^2x^6}\,dx}{8c}$$

$$=-\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{8c^{7/3}}+\frac{b\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{7/3}}+\frac{b\int\frac{-\frac{1}{2}-\frac{1}{2}\sqrt{3}}{1+\sqrt{3}\sqrt[3]{c}x+}}{8c^{7/3}}$$

$$=-\frac{3bx^5}{40c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\left(\sqrt{3}b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{32c^{8/3}}-\frac{\left(\sqrt{3}b\right)\int}{}$$

$$=-\frac{3bx^5}{40c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}-\frac{\sqrt{3}}{}$$

$$=-\frac{3bx^5}{40c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{b\tan^{-1}\left(\sqrt{3}\right.}{16c^{8}}$$

Mathematica [A] time = 0.0748782, size = 181, normalized size = 1.03

$$\frac{ax^8}{8}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}+\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^5)/(40*c) + (a*x^8)/8 + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (b*x^8*
ArcTan[c*x^3])/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*ArcT
an[Sqrt[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3
)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c
^(2/3)*x^2])/(32*c^(8/3))
```

Maple [A] time = 0.069, size = 170, normalized size = 1.

$$\frac{x^8 a}{8} + \frac{b x^8 \arctan\left(c x^3\right)}{8} - \frac{3\, b x^5}{40\, c} - \frac{b\sqrt{3}}{32\, c}\left(c^{-2}\right)^{\frac{5}{6}} \ln\left(x^2 + \sqrt{3}\sqrt[6]{c^{-2}}x + \sqrt[3]{c^{-2}}\right) + \frac{b}{16\, c^3}\arctan\left(2\frac{x}{\sqrt[6]{c^{-2}}} + \sqrt{3}\right)\frac{1}{\sqrt[6]{c^{-2}}} + \frac{b\sqrt{}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctan(c*x^3)-3/40*b*x^5/c-1/32*b/c*3^(1/2)*(1/c^2)^(5/
6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/16*b/c^3/(1/c^2)^(1/6)*a
rctan(2*x/(1/c^2)^(1/6)+3^(1/2))+1/32*b/c*3^(1/2)*(1/c^2)^(5/6)*ln(3^(1/2)*
(1/c^2)^(1/6)*x-x^2-(1/c^2)^(1/3))+1/16*b/c^3/(1/c^2)^(1/6)*arctan(2*x/(1/c
^2)^(1/6)-3^(1/2))+1/8*b/c^3/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))
```

Maxima [B] time = 1.50256, size = 432, normalized size = 2.45

$$\frac{1}{8}\, a x^8 + \frac{1}{160}\left(20\, x^8 \arctan\left(c x^3\right) - \left(\frac{12\, x^5}{c^2} + \frac{5\left(\frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2 + \sqrt{3}\left(c^2\right)^{\frac{1}{6}}x + 1\right)}{\left(c^2\right)^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2 - \sqrt{3}\left(c^2\right)^{\frac{1}{6}}x + 1\right)}{\left(c^2\right)^{\frac{5}{6}}} - \frac{2\log\left(\frac{\left(c^2\right)^{\frac{1}{3}}x - \sqrt{-\left(c^2\right)^{\frac{1}{3}}}}{\left(c^2\right)^{\frac{1}{3}}x + \sqrt{-\left(c^2\right)^{\frac{1}{3}}}}\right)}{\left(c^2\right)^{\frac{2}{3}}\sqrt{-\left(c^2\right)^{\frac{1}{3}}}} \cdots\right)}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/160*(20*x^8*arctan(c*x^3) - (12*x^5/c^2 + 5*(sqrt(3)*log((c^2
)^(1/3)*x^2 + sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(5/6) - sqrt(3)*log((c^2)^(1
/3)*x^2 - sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(5/6) - 2*log(((c^2)^(1/3)*x - s
qrt(-(c^2)^(1/3)))/((c^2)^(1/3)*x + sqrt(-(c^2)^(1/3))))/((c^2)^(2/3)*sqrt(
-(c^2)^(1/3))) - (c^2)^(1/3)*log((2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1/6) - s
qrt(-(c^2)^(1/3)))/(2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1/6) + sqrt(-(c^2)^(1/
3))))/(c^2*sqrt(-(c^2)^(1/3))) - (c^2)^(1/3)*log((2*(c^2)^(1/3)*x - sqrt(3)
*(c^2)^(1/6) - sqrt(-(c^2)^(1/3)))/(2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6) +
 sqrt(-(c^2)^(1/3))))/(c^2*sqrt(-(c^2)^(1/3))))/c^2)*c)*b
```

Fricas [B] time = 3.25014, size = 1069, normalized size = 6.07

$$20\,bcx^8\arctan\left(cx^3\right)+20\,acx^8-12\,bx^5-5\,\sqrt{3}c\left(\frac{b^6}{c^{16}}\right)^{\frac{1}{6}}\log\left(\sqrt{3}b^5c^{13}x\left(\frac{b^6}{c^{16}}\right)^{\frac{5}{6}}+b^6c^{10}\left(\frac{b^6}{c^{16}}\right)^{\frac{2}{3}}+b^{10}x^2\right)+5\,\sqrt{3}c\left(\frac{b^6}{c^{16}}\right)^{\frac{1}{6}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/160*(20*b*c*x^8*arctan(c*x^3) + 20*a*c*x^8 - 12*b*x^5 - 5*sqrt(3)*c*(b^6/
c^16)^(1/6)*log(sqrt(3)*b^5*c^13*x*(b^6/c^16)^(5/6) + b^6*c^10*(b^6/c^16)^(
2/3) + b^10*x^2) + 5*sqrt(3)*c*(b^6/c^16)^(1/6)*log(-sqrt(3)*b^5*c^13*x*(b^
6/c^16)^(5/6) + b^6*c^10*(b^6/c^16)^(2/3) + b^10*x^2) - 20*c*(b^6/c^16)^(1/
6)*arctan(-(2*b^5*c^3*x*(b^6/c^16)^(1/6) + sqrt(3)*b^6 - 2*sqrt(sqrt(3)*b^5
*c^13*x*(b^6/c^16)^(5/6) + b^6*c^10*(b^6/c^16)^(2/3) + b^10*x^2)*c^3*(b^6/c
^16)^(1/6))/b^6) - 20*c*(b^6/c^16)^(1/6)*arctan(-(2*b^5*c^3*x*(b^6/c^16)^(1
/6) - sqrt(3)*b^6 - 2*sqrt(-sqrt(3)*b^5*c^13*x*(b^6/c^16)^(5/6) + b^6*c^10*
(b^6/c^16)^(2/3) + b^10*x^2)*c^3*(b^6/c^16)^(1/6))/b^6) - 40*c*(b^6/c^16)^(
1/6)*arctan(-(b^5*c^3*x*(b^6/c^16)^(1/6) - sqrt(b^6*c^10*(b^6/c^16)^(2/3) +
 b^10*x^2)*c^3*(b^6/c^16)^(1/6))/b^6))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atan(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39375, size = 231, normalized size = 1.31

$$-\frac{1}{32}\,bc^{15}\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{18}}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{18}}-\frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{18}}-\frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(\right.\right.}{c^{18}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] -1/32*b*c^15*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs
(c)^(2/3))/c^18 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1
/abs(c)^(2/3))/c^18 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*ab
s(c)^(1/3))/c^18 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c
)^(1/3))/c^18 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^18) + 1/40*(5*b*c*x
^8*arctan(c*x^3) + 5*a*c*x^8 - 3*b*x^5)/c
```

3.109 $\int x^4 \left(a+b\tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=117

$$\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2+1\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{20c^{5/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}-\frac{3bx^2}{10c}$$

```
[Out] (-3*b*x^2)/(10*c) + (x^5*(a + b*ArcTan[c*x^3]))/5 - (Sqrt[3]*b*ArcTan[(1 -
2*c^(2/3)*x^2)/Sqrt[3]])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3
)) - (b*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(20*c^(5/3))
```

Rubi [A] time = 0.0955969, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5033, 275, 321, 200, 31, 634, 617, 204, 628}

$$\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2+1\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{20c^{5/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}-\frac{3bx^2}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^2)/(10*c) + (x^5*(a + b*ArcTan[c*x^3]))/5 - (Sqrt[3]*b*ArcTan[(1 -
2*c^(2/3)*x^2)/Sqrt[3]])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3
)) - (b*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(20*c^(5/3))
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4 \left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= \frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{5}(3bc)\int\frac{x^7}{1+c^2x^6}dx\\
&= \frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{10}(3bc)\,\mathrm{Subst}\left(\int\frac{x^3}{1+c^2x^3}dx,x,x^2\right)\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{(3b)\,\mathrm{Subst}\left(\int\frac{1}{1+c^2x^3}dx,x,x^2\right)}{10c}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\,\mathrm{Subst}\left(\int\frac{1}{1+c^{2/3}x}dx,x,x^2\right)}{10c}+\frac{b\,\mathrm{Subst}\left(\int\frac{2-c^{2/3}x}{1-c^{2/3}x+c^{4/3}x^2}\right.}{10c}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\,\mathrm{Subst}\left(\int\frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}dx,x,x^2\right)}{20c^{5/3}}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{20c^{5/3}}+\frac{(3b)\,\mathrm{S}}{}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.0266782, size = 185, normalized size = 1.58

$$\frac{ax^5}{5}+\frac{b\log\left(c^{2/3}x^2+1\right)}{10c^{5/3}}-\frac{b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{20c^{5/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{20c^{5/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{10c^{5/3}}-\frac{\sqrt{3}b\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^2)/(10*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^3])/5 - (Sqrt[3]*b*ArcTan
[Sqrt[3] - 2*c^(1/3)*x])/(10*c^(5/3)) - (Sqrt[3]*b*ArcTan[Sqrt[3] + 2*c^(1/
3)*x])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3)) - (b*Log[1 - Sq
rt[3]*c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3)) - (b*Log[1 + Sqrt[3]*c^(1/3)*x
 + c^(2/3)*x^2])/(20*c^(5/3))
```

Maple [A] time = 0.028, size = 113, normalized size = 1.

$$\frac{ax^5}{5}+\frac{bx^5\arctan\left(cx^3\right)}{5}-\frac{3\,bx^2}{10\,c}+\frac{b}{10\,c^3}\ln\left(x^2+\sqrt[3]{c^{-2}}\right)\left(c^{-2}\right)^{-\frac{2}{3}}-\frac{b}{20\,c^3}\ln\left(x^4-\sqrt[3]{c^{-2}}x^2+\left(c^{-2}\right)^{\frac{2}{3}}\right)\left(c^{-2}\right)^{-\frac{2}{3}}+\frac{b\sqrt{3}}{10\,c^3}\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctan(c*x^3)-3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*ln(
x^2+(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2
)^(2/3))+1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/
3)*x^2-1))
```

Maxima [A] time = 1.52126, size = 162, normalized size = 1.38

$$\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\arctan\left(cx^3\right)-c\left(\frac{6\,x^2}{c^2}-\frac{2\,\sqrt{3}\left(c^2\right)^{\frac{2}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\left(2\,x^2-\frac{1}{c^2}^{\frac{1}{3}}\right)\right)}{c^4}+\frac{\left(c^2\right)^{\frac{2}{3}}\log\left(x^4-\frac{1}{c^2}^{\frac{1}{3}}x^2+\frac{1}{c^2}^{\frac{2}{3}}\right)}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x^3) - c*(6*x^2/c^2 - 2*sqrt(3)*(c^2)^(2/3
)*arctan(1/3*sqrt(3)*(c^2)^(1/3)*(2*x^2 - (c^(-2))^(1/3)))/c^4 + (c^2)^(2/3
)*log(x^4 - (c^(-2))^(1/3)*x^2 + (c^(-2))^(2/3))/c^4 - 2*(c^2)^(2/3)*log(x^
2 + (c^(-2))^(1/3))/c^4))*b
```

Fricas [A] time = 3.2461, size = 362, normalized size = 3.09

$$\frac{4\,bc^3x^5\arctan\left(cx^3\right)+4\,ac^3x^5-6\,bc^2x^2+2\,\sqrt{3}b\left(c^2\right)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\left(2\left(c^2\right)^{\frac{2}{3}}x^2-\left(c^2\right)^{\frac{1}{3}}\right)\left(c^2\right)^{\frac{1}{6}}}{3\,c}\right)-b\left(c^2\right)^{\frac{2}{3}}\log\left(c^2x^4-\left(c^2\right)^{\frac{2}{3}}x^2+\right.}{20\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^3*x^5*arctan(c*x^3) + 4*a*c^3*x^5 - 6*b*c^2*x^2 + 2*sqrt(3)*b*(
c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(2*(c^2)^(2/3)*x^2 - (c^2)^(1/3))*(c^2)^(1/
```

```
6)/c) - b*(c^2)^(2/3)*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b*(c
^2)^(2/3)*log(c^2*x^2 + (c^2)^(2/3)))/c^3
```

Sympy [A] time = 160.661, size = 1640, normalized size = 14.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**5/5, Eq(c, 0)), (a*x**5/5 - I*b*x**5*atanh(x**3*sqrt(x**(-6
)))/5, Eq(c, -sqrt(-1/x**6)) | Eq(c, -sqrt(1/(x**6*(1/2 - sqrt(3)*I/2)**3))
) | Eq(c, -sqrt(1/(x**6*(1/2 + sqrt(3)*I/2)**3)))), (4*(-1)**(2/3)*a*c**5*x
**11*(c**(-2))**(2/3)/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)*
*(2/3)*c**3*(c**(-2))**(2/3)) + 4*(-1)**(2/3)*a*c**3*x**5*(c**(-2))**(2/3)/
(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2))*
*(2/3)) - 4*I*b*c**15*x**6*(c**(-2))**(13/2)*atan(c*x**3)/(20*(-1)**(2/3)*c
**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)) - 4*I*b*c
**7*(c**(-2))**(7/2)*atan(c*x**3)/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3
) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)) + 4*(-1)**(2/3)*b*c**5*x**11*(c**
(-2))**(2/3)*atan(c*x**3)/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(
-1)**(2/3)*c**3*(c**(-2))**(2/3)) - 6*(-1)**(2/3)*b*c**4*x**8*(c**(-2))**(2
/3)/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-
2))**(2/3)) + 4*(-1)**(2/3)*b*c**3*x**5*(c**(-2))**(2/3)*atan(c*x**3)/(20*(
-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3
)) + 4*b*c**2*x**6*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(20*(-1)**(2/3)*c*
*5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)) - 3*b*c**2
*x**6*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2
))**(1/3))/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3
*(c**(-2))**(2/3)) + b*c**2*x**6*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1
/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**
(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)) - 2*sqrt(3)*b*c**2*x**6*atan(
2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(20*(-1)**(2/3)*c
**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)) + 2*sqrt(
3)*b*c**2*x**6*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/
3)/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2
))**(2/3)) + 4*b*c**2*x**6*log(2)/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3
) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)) - 6*(-1)**(2/3)*b*c**2*x**2*(c**(
-2))**(2/3)/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**
3*(c**(-2))**(2/3)) + 4*b*log(x - (-1)**(1/6)*(c**(-2))**(1/6))/(20*(-1)**(
2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)) - 3
*b*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))*
*(1/3))/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c
**(-2))**(2/3)) + b*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)*
*(1/3)*(c**(-2))**(1/3))/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-
1)**(2/3)*c**3*(c**(-2))**(2/3)) - 2*sqrt(3)*b*atan(2*(-1)**(5/6)*sqrt(3)*x
/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(20*(-1)**(2/3)*c**5*x**6*(c**(-2))**(2/
3) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)) + 2*sqrt(3)*b*atan(2*(-1)**(5/6)
*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/(20*(-1)**(2/3)*c**5*x**6*(c**
(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)) + 4*b*log(2)/(20*(-1)*
*(2/3)*c**5*x**6*(c**(-2))**(2/3) + 20*(-1)**(2/3)*c**3*(c**(-2))**(2/3)),
True))
```

Giac [A] time = 1.2201, size = 161, normalized size = 1.38

$$\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}+\frac{2\,\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}\right)+\frac{2\,bcx^5\arctan\left(cx^3\right)+2\,acx}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2
/3))/(c^10*abs(c)^(2/3)) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(c^
10*abs(c)^(2/3)) + 2*log(x^2 + 1/abs(c)^(2/3))/(c^10*abs(c)^(2/3))) + 1/10*
(2*b*c*x^5*arctan(c*x^3) + 2*a*c*x^5 - 3*b*x^2)/c
```

3.110 $\int x\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=165

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{b\tan^{-1}\left(\sqrt{\right.}{4c}$$

```
[Out] -(b*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + (x^2*(a + b*ArcTan[c*x^3]))/2 + (b*Arc
Tan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1/3)*x])
/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2
/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3))
```

Rubi [A] time = 0.393489, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5033, 295, 634, 618, 204, 628, 203}

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{b\tan^{-1}\left(\sqrt{\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -(b*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + (x^2*(a + b*ArcTan[c*x^3]))/2 + (b*Arc
Tan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1/3)*x])
/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2
/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3))
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(cx^3\right)\right)dx=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}(3bc)\int\frac{x^4}{1+c^2x^6}\,dx$$
$$=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}$$
$$=-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\left(\sqrt{3}b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{2/3}}+\frac{\left(\sqrt{3}b\right)\int\frac{\sqrt{3}\ldots}{1+\sqrt{3}\ldots}}{8c^{2\ldots}}$$
$$=-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log(\ldots}{\ldots}$$
$$=-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{\ldots}\right)}{4c^{2/3}}$$

Mathematica [A] time = 0.0254029, size = 170, normalized size = 1.03

$$\frac{ax^2}{2}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{b\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\,t\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (a*x^2)/2 - (b*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + (b*x^2*ArcTan[c*x^3])/2 + (
b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1/3
)*x])/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8
*c^(2/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)
)
```

Maple [A] time = 0.056, size = 154, normalized size = 0.9

$$\frac{ax^2}{2}+\frac{bx^2\arctan\left(cx^3\right)}{2}+\frac{bc\sqrt{3}}{8}\left(c^{-2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\sqrt[6]{c^{-2}}x+\sqrt[3]{c^{-2}}\right)-\frac{b}{4c}\arctan\left(2\frac{x}{\sqrt[6]{c^{-2}}}+\sqrt{3}\right)\frac{1}{\sqrt[6]{c^{-2}}}-\frac{bc\sqrt{3}}{8}\left(c^{-2}\right)^{\frac{5}{6}}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctan(c*x^3)+1/8*b*c*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+3^(1
/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))-1/4*b/c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)
^(1/6)+3^(1/2))-1/8*b*c*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*
x+(1/c^2)^(1/3))-1/4*b/c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))-1/
2*b/c/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))
```

Maxima [B] time = 1.5567, size = 412, normalized size = 2.5

$$\frac{1}{2}ax^2 + \frac{1}{8}\left(4x^2\arctan\left(cx^3\right) + c\left(\frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2 + \sqrt{3}\left(c^2\right)^{\frac{1}{6}}x + 1\right)}{\left(c^2\right)^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2 - \sqrt{3}\left(c^2\right)^{\frac{1}{6}}x + 1\right)}{\left(c^2\right)^{\frac{5}{6}}} - \frac{2\log\left(\frac{\left(c^2\right)^{\frac{1}{3}}x-}{\left(c^2\right)^{\frac{1}{3}}x+}\right.}{\left(c^2\right)^{\frac{2}{3}}\sqrt{-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/8*(4*x^2*arctan(c*x^3) + c*(sqrt(3)*log((c^2)^(1/3)*x^2 + sqr
t(3)*(c^2)^(1/6)*x + 1)/(c^2)^(5/6) - sqrt(3)*log((c^2)^(1/3)*x^2 - sqrt(3)
*(c^2)^(1/6)*x + 1)/(c^2)^(5/6) - 2*log(((c^2)^(1/3)*x - sqrt(-(c^2)^(1/3))
)/((c^2)^(1/3)*x + sqrt(-(c^2)^(1/3))))/((c^2)^(2/3)*sqrt(-(c^2)^(1/3))) -
(c^2)^(1/3)*log((2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1/6) - sqrt(-(c^2)^(1/3))
)/(2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1/6) + sqrt(-(c^2)^(1/3))))/(c^2*sqrt(-
(c^2)^(1/3))) - (c^2)^(1/3)*log((2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6) - sq
rt(-(c^2)^(1/3)))/(2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6) + sqrt(-(c^2)^(1/3
))))/(c^2*sqrt(-(c^2)^(1/3)))))*b
```

Fricas [B] time = 2.53996, size = 971, normalized size = 5.88

$$\frac{1}{2}bx^2\arctan\left(cx^3\right) + \frac{1}{2}ax^2 + \frac{1}{8}\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^{10}x^2 + \sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{5}{6}}b^5c^3x + \left(\frac{b^6}{c^4}\right)^{\frac{2}{3}}b^6c^2\right) - \frac{1}{8}\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^{10}x^2 - \sqrt{3}\left(\frac{b^6}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2*arctan(c*x^3) + 1/2*a*x^2 + 1/8*sqrt(3)*(b^6/c^4)^(1/6)*log(b^10*
x^2 + sqrt(3)*(b^6/c^4)^(5/6)*b^5*c^3*x + (b^6/c^4)^(2/3)*b^6*c^2) - 1/8*sq
rt(3)*(b^6/c^4)^(1/6)*log(b^10*x^2 - sqrt(3)*(b^6/c^4)^(5/6)*b^5*c^3*x + (b
^6/c^4)^(2/3)*b^6*c^2) + 1/2*(b^6/c^4)^(1/6)*arctan(-(2*(b^6/c^4)^(1/6)*b^5
*c*x + sqrt(3)*b^6 - 2*sqrt(b^10*x^2 + sqrt(3)*(b^6/c^4)^(5/6)*b^5*c^3*x +
(b^6/c^4)^(2/3)*b^6*c^2)*(b^6/c^4)^(1/6)*c)/b^6) + 1/2*(b^6/c^4)^(1/6)*arct
an(-(2*(b^6/c^4)^(1/6)*b^5*c*x - sqrt(3)*b^6 - 2*sqrt(b^10*x^2 - sqrt(3)*(b
^6/c^4)^(5/6)*b^5*c^3*x + (b^6/c^4)^(2/3)*b^6*c^2)*(b^6/c^4)^(1/6)*c)/b^6)
+ (b^6/c^4)^(1/6)*arctan(-((b^6/c^4)^(1/6)*b^5*c*x - sqrt(b^10*x^2 + (b^6/c
```

```
^4)^(2/3)*b^6*c^2)*(b^6/c^4)^(1/6)*c)/b^6)
```

Sympy [A] time = 67.1106, size = 1620, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**2/2 - I*b*x**2*atanh(x**3*sqrt(x**(-6)))/2, Eq(c, -sqrt(-1/
x**6)) | Eq(c, -sqrt(1/(x**6*(1/2 - sqrt(3)*I/2)**3))) | Eq(c, -sqrt(1/(x**
6*(1/2 + sqrt(3)*I/2)**3)))), (a*x**2/2, Eq(c, 0)), (-4*(-1)**(2/3)*a*c**28
*x**8*(c**(-2))**(41/3)/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-
1)**(2/3)*c**26*(c**(-2))**(41/3)) - 4*(-1)**(2/3)*a*c**26*x**2*(c**(-2))**
(41/3)/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1)**(2/3)*c**26*(
c**(-2))**(41/3)) - 2*sqrt(3)*I*b*c**37*x**6*(c**(-2))**(37/2)*atan(2*(-1)*
*(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(-8*(-1)**(2/3)*c**28*x*
*6*(c**(-2))**(41/3) - 8*(-1)**(2/3)*c**26*(c**(-2))**(41/3)) - 2*sqrt(3)*I
*b*c**37*x**6*(c**(-2))**(37/2)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**
(1/6)) + sqrt(3)/3)/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1)**
(2/3)*c**26*(c**(-2))**(41/3)) - 2*sqrt(3)*I*b*c**35*(c**(-2))**(37/2)*atan
(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(-8*(-1)**(2/3)*
c**28*x**6*(c**(-2))**(41/3) - 8*(-1)**(2/3)*c**26*(c**(-2))**(41/3)) - 2*s
qrt(3)*I*b*c**35*(c**(-2))**(37/2)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2)
)**(1/6)) + sqrt(3)/3)/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1
)**(2/3)*c**26*(c**(-2))**(41/3)) - 4*(-1)**(2/3)*b*c**28*x**8*(c**(-2))**(
41/3)*atan(c*x**3)/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1)**(
2/3)*c**26*(c**(-2))**(41/3)) - 4*(-1)**(2/3)*b*c**26*x**2*(c**(-2))**(41/3
)*atan(c*x**3)/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1)**(2/3)
*c**26*(c**(-2))**(41/3)) - 3*I*b*c**21*x**6*(c**(-2))**(21/2)*log(4*x**2 +
 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(-8*(-1
)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1)**(2/3)*c**26*(c**(-2))**(41/
3)) + 6*I*b*c**21*x**6*(c**(-2))**(21/2)*log(2)/(-8*(-1)**(2/3)*c**28*x**6*
(c**(-2))**(41/3) - 8*(-1)**(2/3)*c**26*(c**(-2))**(41/3)) - 3*I*b*c**19*(c
**(-2))**(21/2)*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/
3)*(c**(-2))**(1/3))/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1)*
*(2/3)*c**26*(c**(-2))**(41/3)) + 6*I*b*c**19*(c**(-2))**(21/2)*log(2)/(-8*
(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1)**(2/3)*c**26*(c**(-2))**(
41/3)) + 3*I*b*c**15*x**6*(c**(-2))**(15/2)*log(4*x**2 - 4*(-1)**(1/6)*x*(c
**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(-8*(-1)**(2/3)*c**28*x**6
*(c**(-2))**(41/3) - 8*(-1)**(2/3)*c**26*(c**(-2))**(41/3)) - 6*I*b*c**15*x
**6*(c**(-2))**(15/2)*log(2)/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) -
 8*(-1)**(2/3)*c**26*(c**(-2))**(41/3)) + 3*I*b*c**13*(c**(-2))**(15/2)*log
(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3)
)/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1)**(2/3)*c**26*(c**(-
2))**(41/3)) - 6*I*b*c**13*(c**(-2))**(15/2)*log(2)/(-8*(-1)**(2/3)*c**28*x
**6*(c**(-2))**(41/3) - 8*(-1)**(2/3)*c**26*(c**(-2))**(41/3)) - 4*b*x**6*a
tan(c*x**3)/(-8*(-1)**(2/3)*c**28*x**6*(c**(-2))**(41/3) - 8*(-1)**(2/3)*c*
*26*(c**(-2))**(41/3)) - 4*b*atan(c*x**3)/(-8*(-1)**(2/3)*c**30*x**6*(c**(-
2))**(41/3) - 8*(-1)**(2/3)*c**28*(c**(-2))**(41/3)), True))
```

Giac [A] time = 1.30739, size = 212, normalized size = 1.28

$$\frac{1}{8}bc^5\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{2|c|^{\frac{1}{3}}\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^6}-\frac{2|c|^{\frac{1}{3}}\arctan\left(\left(2x\right.\right.}{c^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/8*b*c^5*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)
^(2/3))/c^6 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs
(c)^(2/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^
(1/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3
))/c^6 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^6) + 1/2*b*x^2*arctan(c*x^
3) + 1/2*a*x^2
```

3.111 $\int \frac{a+b\tan^{-1}(cx^3)}{x^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}b\sqrt[3]{c}\log\left(c^{2/3}x^2+1\right)-\frac{1}{4}b\sqrt[3]{c}\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)-\frac{1}{2}\sqrt{3}b\sqrt[3]{c}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)$$

```
[Out] -((a + b*ArcTan[c*x^3])/x) - (Sqrt[3]*b*c^(1/3)*ArcTan[(1 - 2*c^(2/3)*x^2)/
Sqrt[3]])/2 + (b*c^(1/3)*Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - c^(2/
3)*x^2 + c^(4/3)*x^4])/4
```

Rubi [A] time = 0.0851211, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 275, 200, 31, 634, 617, 204, 628}

$$-\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}b\sqrt[3]{c}\log\left(c^{2/3}x^2+1\right)-\frac{1}{4}b\sqrt[3]{c}\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)-\frac{1}{2}\sqrt{3}b\sqrt[3]{c}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x^3])/x) - (Sqrt[3]*b*c^(1/3)*ArcTan[(1 - 2*c^(2/3)*x^2)/
Sqrt[3]])/2 + (b*c^(1/3)*Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - c^(2/
3)*x^2 + c^(4/3)*x^4])/4
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^2}\,dx &= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+(3bc)\int \frac{x}{1+c^2x^6}\,dx\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{1}{1+c^2x^3}\,dx,x,x^2\right)\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)+\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{2-c^{2/3}x}{1-c^{2/3}x+c^{4/3}x^2}\right.\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)+\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}+\frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)+\frac{1}{2}\left(3b\sqrt[3]{c}\right)\text{Subs}\\
&= -\frac{a+b\tan^{-1}\left(cx^3\right)}{x}-\frac{1}{2}\sqrt{3}b\sqrt[3]{c}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}b\sqrt[3]{c}\log\left(1-c^{2}\right.
\end{aligned}$$

Mathematica [A] time = 0.0369877, size = 170, normalized size = 1.63

$$-\frac{a}{x}+\frac{1}{2}b\sqrt[3]{c}\log\left(c^{2/3}x^2+1\right)-\frac{1}{4}b\sqrt[3]{c}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)-\frac{1}{4}b\sqrt[3]{c}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)-\frac{b\tan^{-1}\left(cx^3\right)}{x}-\frac{1}{2}\sqrt{3}b$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c*x^3])/x - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] - 2*c^(1/3
)*x]])/2 - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x]])/2 + (b*c^(1/3)*
Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^
2])/4 - (b*c^(1/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/4
```

Maple [A] time = 0.026, size = 104, normalized size = 1.

$$-\frac{a}{x}-\frac{b\arctan\left(cx^{3}\right)}{x}+\frac{b}{2c}\ln\left(x^{2}+\sqrt[3]{c^{-2}}\right)\left(c^{-2}\right)^{-\frac{2}{3}}-\frac{b}{4c}\ln\left(x^{4}-\sqrt[3]{c^{-2}}x^{2}+\left(c^{-2}\right)^{\frac{2}{3}}\right)\left(c^{-2}\right)^{-\frac{2}{3}}+\frac{b\sqrt{3}}{2c}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{x^{2}}{\sqrt[3]{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^2,x)
```

```
[Out] -a/x-b/x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(2/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/
(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2-1))
```

Maxima [A] time = 1.54006, size = 151, normalized size = 1.45

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\left(c^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(c^{2}\right)^{\frac{1}{3}}\left(2x^{2}-\frac{1}{c^{2}}^{\frac{1}{3}}\right)\right)}{c^{2}}-\frac{\left(c^{2}\right)^{\frac{2}{3}}\log\left(x^{4}-\frac{1}{c^{2}}^{\frac{1}{3}}x^{2}+\frac{1}{c^{2}}^{\frac{2}{3}}\right)}{c^{2}}+\frac{2\left(c^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+\frac{1}{c^{2}}^{\frac{1}{3}}\right)}{c^{2}}\right)-\frac{4\,\text{arcta}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*(c^2)^(2/3)*arctan(1/3*sqrt(3)*(c^2)^(1/3)*(2*x^2 - (c^(-
2))^(1/3)))/c^2 - (c^2)^(2/3)*log(x^4 - (c^(-2))^(1/3)*x^2 + (c^(-2))^(2/3)
)/c^2 + 2*(c^2)^(2/3)*log(x^2 + (c^(-2))^(1/3))/c^2) - 4*arctan(c*x^3)/x)*b
 - a/x
```

Fricas [A] time = 2.65988, size = 258, normalized size = 2.48

$$\frac{2\sqrt{3}bc^{\frac{1}{3}}x\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{2}{3}}x^{2}-\frac{1}{3}\sqrt{3}\right)-bc^{\frac{1}{3}}x\log\left(c^{2}x^{4}-c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right)+2bc^{\frac{1}{3}}x\log\left(cx^{2}+c^{\frac{1}{3}}\right)-4b\arctan\left(cx^{3}\right)-4a}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*b*c^(1/3)*x*arctan(2/3*sqrt(3)*c^(2/3)*x^2 - 1/3*sqrt(3)) -
b*c^(1/3)*x*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(1/3)*x*log(c*x^2
+ c^(1/3)) - 4*b*arctan(c*x^3) - 4*a)/x
```

Sympy [A] time = 92.5375, size = 1975, normalized size = 18.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**2,x)
```

```
[Out] Piecewise((-(a - oo*I*b)/x, Eq(c, -I/x**3)), (-(a + oo*I*b)/x, Eq(c, I/x**3
)), (-(a - b*atan((-sqrt(3)/2 - I/2)**(-3)))/x, Eq(c, -1/(x**3*(-sqrt(3)/2
- I/2)**3))), (-(a - b*atan((-sqrt(3)/2 + I/2)**(-3)))/x, Eq(c, -1/(x**3*(-
sqrt(3)/2 + I/2)**3))), (-(a - b*atan((sqrt(3)/2 - I/2)**(-3)))/x, Eq(c, -1
/(x**3*(sqrt(3)/2 - I/2)**3))), (-(a - b*atan((sqrt(3)/2 + I/2)**(-3)))/x,
Eq(c, -1/(x**3*(sqrt(3)/2 + I/2)**3))), (-a/x, Eq(c, 0)), (-28*(-1)**(5/6)*
a*c**42*x**6*(c**(-2))**(161/6)/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/
6) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) - 28*(-1)**(5/6)*a*c**40*(c
**(-2))**(161/6)/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/6) + 28*(-1)**(
5/6)*c**40*x*(c**(-2))**(161/6)) - 72*(-1)**(1/3)*b*c**47*(c**(-2))**(91/3)
/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*x*(c*
*(-2))**(161/6)) - 21*(-1)**(1/6)*b*c**45*x**7*(c**(-2))**(169/6)*log(4*x**
2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(28*
(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*x*(c**(-2)
)**(161/6)) + 7*(-1)**(1/6)*b*c**45*x**7*(c**(-2))**(169/6)*log(4*x**2 + 4*
(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(c**(-2))**(1/3))/(28*(-1)**
(5/6)*c**42*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(16
1/6)) - 14*(-1)**(1/6)*sqrt(3)*b*c**45*x**7*(c**(-2))**(169/6)*atan(2*(-1)*
*(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/(28*(-1)**(5/6)*c**42*x*
*7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) + 14*(-1
)**(1/6)*sqrt(3)*b*c**45*x**7*(c**(-2))**(169/6)*atan(2*(-1)**(5/6)*sqrt(3)
*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**
(161/6) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) + 28*(-1)**(1/6)*b*c**
45*x**7*(c**(-2))**(169/6)*log(2)/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(16
1/6) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) - 21*(-1)**(1/6)*b*c**43*
x*(c**(-2))**(169/6)*log(4*x**2 - 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)
**(1/3)*(c**(-2))**(1/3))/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/6) + 2
8*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) + 7*(-1)**(1/6)*b*c**43*x*(c**(-2
))**(169/6)*log(4*x**2 + 4*(-1)**(1/6)*x*(c**(-2))**(1/6) + 4*(-1)**(1/3)*(
c**(-2))**(1/3))/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/6) + 28*(-1)**(
5/6)*c**40*x*(c**(-2))**(161/6)) - 14*(-1)**(1/6)*sqrt(3)*b*c**43*x*(c**(-2
))**(169/6)*atan(2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) - sqrt(3)/3)/
(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*x*(c**
(-2))**(161/6)) + 14*(-1)**(1/6)*sqrt(3)*b*c**43*x*(c**(-2))**(169/6)*atan(
2*(-1)**(5/6)*sqrt(3)*x/(3*(c**(-2))**(1/6)) + sqrt(3)/3)/(28*(-1)**(5/6)*c
**42*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) +
 28*(-1)**(1/6)*b*c**43*x*(c**(-2))**(169/6)*log(2)/(28*(-1)**(5/6)*c**42*x
**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) - 28*(-
1)**(5/6)*b*c**42*x**6*(c**(-2))**(161/6)*atan(c*x**3)/(28*(-1)**(5/6)*c**4
2*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) - 28
*(-1)**(5/6)*b*c**40*(c**(-2))**(161/6)*atan(c*x**3)/(28*(-1)**(5/6)*c**42*
x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) + 28*(
-1)**(1/6)*b*c**37*x**7*(c**(-2))**(145/6)*log(x - (-1)**(1/6)*(c**(-2))**(
1/6))/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**40*
x*(c**(-2))**(161/6)) + 28*(-1)**(1/6)*b*c**35*x*(c**(-2))**(145/6)*log(x -
 (-1)**(1/6)*(c**(-2))**(1/6))/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/6
) + 28*(-1)**(5/6)*c**40*x*(c**(-2))**(161/6)) + 72*(-1)**(1/3)*b*c**35*(c*
*(-2))**(73/3)/(28*(-1)**(5/6)*c**42*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/
6)*c**40*x*(c**(-2))**(161/6)) - 28*(-1)**(2/3)*b*x**7*(c**(-2))**(8/3)*ata
n(c*x**3)/(28*(-1)**(5/6)*c**48*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c*
*46*x*(c**(-2))**(161/6)) - 28*(-1)**(2/3)*b*x*(c**(-2))**(8/3)*atan(c*x**3
)/(28*(-1)**(5/6)*c**50*x**7*(c**(-2))**(161/6) + 28*(-1)**(5/6)*c**48*x*(c
**(-2))**(161/6)), True))
```

Giac [A] time = 1.23004, size = 123, normalized size = 1.18

$$\frac{1}{4}\,bc\left(\frac{2\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}+\frac{2\,\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{2}{3}}}\right)-\frac{b\arctan\left(cx^3\right)+a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2/3)
)/abs(c)^(2/3) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3)
+ 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(2/3)) - (b*arctan(c*x^3) + a)/x
```

3.112 $\int \frac{a+b\tan^{-1}(cx^3)}{x^5} \, dx$

Optimal. Leaf size=174

$$-\frac{a+b\tan^{-1}(cx^3)}{4x^4}-\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}x\right)+$$

```
[Out] (-3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (a + b*ArcTan[c*x^3])/(4
*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(
2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/16
```

Rubi [A] time = 0.412866, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5033, 325, 295, 634, 618, 204, 628, 203}

$$-\frac{a+b\tan^{-1}(cx^3)}{4x^4}-\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}x\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^5,x]
```


```
[Out] (-3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (a + b*ArcTan[c*x^3])/(4
*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(
2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/16
```


Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^5}\,dx = -\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{4}(3bc)\int \frac{1}{x^2\left(1+c^2x^6\right)}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{4}\left(3bc^3\right)\int \frac{x^4}{1+c^2x^6}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{4}\left(bc^{5/3}\right)\int \frac{1}{1+c^{2/3}x^2}\,dx-\frac{1}{4}\left(bc^{5/3}\right)\int \frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}\left(\sqrt{3}bc^{4/3}\right)\int \frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx+$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{16}$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{8}bc^{4/3}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}x\right)-\frac{1}{8}bc^{4/3}\tan^{-}$$

Mathematica [A] time = 0.0495834, size = 179, normalized size = 1.03

$$-\frac{a}{4x^4}-\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)-\frac{1}{4}bc^{4/3}\tan^{-1}\left(\sqrt[3]{c}x\right)+\frac{1}{8}bc^{4/3}\text{ta}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^5,x]
```

```
[Out] -a/(4*x^4) - (3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (b*ArcTan[c*
x^3])/(4*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*Ar
cTan[Sqrt[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2
/3)*x^2])/16
```

Maple [A] time = 0.059, size = 159, normalized size = 0.9

$$-\frac{a}{4x^4}-\frac{b\arctan\left(cx^3\right)}{4x^4}+\frac{bc^3\sqrt{3}}{16}\left(c^{-2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\sqrt[6]{c^{-2}}x+\sqrt[3]{c^{-2}}\right)-\frac{bc}{8}\arctan\left(2\frac{x}{\sqrt[6]{c^{-2}}}+\sqrt{3}\right)\frac{1}{\sqrt[6]{c^{-2}}}-\frac{bc^3\sqrt{3}}{16}\left(c^{-2}\right)^{\frac{5}{6}}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctan(c*x^3)+1/16*b*c^3*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+
3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))-1/8*b*c/(1/c^2)^(1/6)*arctan(2*x/(1/
c^2)^(1/6)+3^(1/2))-1/16*b*c^3*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2)*(1/c^2)
^(1/6)*x+(1/c^2)^(1/3))-1/8*b*c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1
/2))-1/4*b*c/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))-3/4*b*c/x
```

Maxima [B] time = 1.5348, size = 425, normalized size = 2.44

$$\frac{1}{16}\left(\left(\left(c^2\left(\frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2+\sqrt{3}\left(c^2\right)^{\frac{1}{6}}x+1\right)}{\left(c^2\right)^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(\left(c^2\right)^{\frac{1}{3}}x^2-\sqrt{3}\left(c^2\right)^{\frac{1}{6}}x+1\right)}{\left(c^2\right)^{\frac{5}{6}}}-\frac{2\log\left(\frac{\left(c^2\right)^{\frac{1}{3}}x-\sqrt{-\left(c^2\right)^{\frac{1}{3}}}}{\left(c^2\right)^{\frac{1}{3}}x+\sqrt{-\left(c^2\right)^{\frac{1}{3}}}}\right)}{\left(c^2\right)^{\frac{2}{3}}\sqrt{-\left(c^2\right)^{\frac{1}{3}}}}-\frac{\left(c^2\right)^{\frac{1}{3}}\log\left(\frac{2(}{2(}\right.}{c^2}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*((c^2*(sqrt(3)*log((c^2)^(1/3)*x^2 + sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^
(5/6) - sqrt(3)*log((c^2)^(1/3)*x^2 - sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(5/6
) - 2*log(((c^2)^(1/3)*x - sqrt(-(c^2)^(1/3)))/((c^2)^(1/3)*x + sqrt(-(c^2)
^(1/3))))/((c^2)^(2/3)*sqrt(-(c^2)^(1/3))) - (c^2)^(1/3)*log((2*(c^2)^(1/3)
*x + sqrt(3)*(c^2)^(1/6) - sqrt(-(c^2)^(1/3)))/(2*(c^2)^(1/3)*x + sqrt(3)*(
c^2)^(1/6) + sqrt(-(c^2)^(1/3))))/(c^2*sqrt(-(c^2)^(1/3))) - (c^2)^(1/3)*lo
g((2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6) - sqrt(-(c^2)^(1/3)))/(2*(c^2)^(1/
3)*x - sqrt(3)*(c^2)^(1/6) + sqrt(-(c^2)^(1/3))))/(c^2*sqrt(-(c^2)^(1/3)))
 - 12/x)*c - 4*arctan(c*x^3)/x^4)*b - 1/4*a/x^4
```

Fricas [B] time = 2.96316, size = 1385, normalized size = 7.96

$$\sqrt{3}\left(b^6c^8\right)^{\frac{1}{6}}x^4\log\left(4b^{10}c^{14}x^2+4\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8+4\sqrt{3}\left(b^6c^8\right)^{\frac{5}{6}}b^5c^7x\right)-\sqrt{3}\left(b^6c^8\right)^{\frac{1}{6}}x^4\log\left(4b^{10}c^{14}x^2+4\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(3)*(b^6*c^8)^(1/6)*x^4*log(4*b^10*c^14*x^2 + 4*(b^6*c^8)^(2/3)*b
^6*c^8 + 4*sqrt(3)*(b^6*c^8)^(5/6)*b^5*c^7*x) - sqrt(3)*(b^6*c^8)^(1/6)*x^4
*log(4*b^10*c^14*x^2 + 4*(b^6*c^8)^(2/3)*b^6*c^8 - 4*sqrt(3)*(b^6*c^8)^(5/6
)*b^5*c^7*x) + sqrt(3)*(b^6*c^8)^(1/6)*x^4*log(b^10*c^14*x^2 + (b^6*c^8)^(2
/3)*b^6*c^8 + sqrt(3)*(b^6*c^8)^(5/6)*b^5*c^7*x) - sqrt(3)*(b^6*c^8)^(1/6)*
x^4*log(b^10*c^14*x^2 + (b^6*c^8)^(2/3)*b^6*c^8 - sqrt(3)*(b^6*c^8)^(5/6)*b
^5*c^7*x) + 8*(b^6*c^8)^(1/6)*x^4*arctan(-(sqrt(3)*b^6*c^8 + 2*(b^6*c^8)^(1
/6)*b^5*c^7*x - 2*sqrt(b^10*c^14*x^2 + (b^6*c^8)^(2/3)*b^6*c^8 + sqrt(3)*(b
^6*c^8)^(5/6)*b^5*c^7*x)*(b^6*c^8)^(1/6))/(b^6*c^8)) + 8*(b^6*c^8)^(1/6)*x^
4*arctan((sqrt(3)*b^6*c^8 - 2*(b^6*c^8)^(1/6)*b^5*c^7*x + 2*sqrt(b^10*c^14*
x^2 + (b^6*c^8)^(2/3)*b^6*c^8 - sqrt(3)*(b^6*c^8)^(5/6)*b^5*c^7*x)*(b^6*c^8
)^(1/6))/(b^6*c^8)) + 16*(b^6*c^8)^(1/6)*x^4*arctan(-((b^6*c^8)^(1/6)*b^5*c
^7*x - sqrt(b^10*c^14*x^2 + (b^6*c^8)^(2/3)*b^6*c^8)*(b^6*c^8)^(1/6))/(b^6*
c^8)) - 24*b*c*x^3 - 8*b*arctan(c*x^3) - 8*a)/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.71353, size = 217, normalized size = 1.25

$$\frac{1}{16}bc^3\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}-\frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}-\frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(\right.\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="giac")
```

```
[Out] 1/16*b*c^3*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c
)^(2/3))/c^2 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/ab
s(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)
^(1/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/
3))/c^2 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^2) - 1/4*(3*b*c*x^3 + b*a
rctan(c*x^3) + a)/x^4
```

3.113 $\int x^{11} \left(a + b \tan^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=124

$$\frac{abx^3}{6c^3} - \frac{\left(a + b\tan^{-1}\left(cx^3\right)\right)^2}{12c^4} + \frac{1}{12}x^{12}\left(a + b\tan^{-1}\left(cx^3\right)\right)^2 - \frac{bx^9\left(a + b\tan^{-1}\left(cx^3\right)\right)}{18c} + \frac{b^2x^6}{36c^2} - \frac{b^2\log\left(c^2x^6+1\right)}{9c^4} + \frac{b^2x^3 \text{ta}}{}$$

```
[Out] (a*b*x^3)/(6*c^3) + (b^2*x^6)/(36*c^2) + (b^2*x^3*ArcTan[c*x^3])/(6*c^3) -
(b*x^9*(a + b*ArcTan[c*x^3]))/(18*c) - (a + b*ArcTan[c*x^3])^2/(12*c^4) + (
x^12*(a + b*ArcTan[c*x^3])^2)/12 - (b^2*Log[1 + c^2*x^6])/(9*c^4)
```

Rubi [C] time = 1.65455, antiderivative size = 731, normalized size of antiderivative = 5.9, number of steps used = 62, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {5035, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)}{24c^4} - \frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)}{24c^4} + \frac{abx^3}{12c^3} - \frac{bx^6\left(2ia - b\log\left(1-icx^3\right)\right)}{48c^2} + \frac{1}{288}ib\left(-\frac{3\left(1-icx^3\right.}{c^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (a*b*x^3)/(12*c^3) - (((23*I)/288)*b^2*x^3)/c^3 + (b^2*x^6)/(192*c^2) - (((
7*I)/864)*b^2*x^9)/c + (b^2*x^12)/384 - (b^2*(1 - I*c*x^3)^2)/(16*c^4) + (b
^2*(1 - I*c*x^3)^3)/(54*c^4) - (b^2*(1 - I*c*x^3)^4)/(384*c^4) - (b^2*Log[I
 - c*x^3])/(36*c^4) - (b^2*(1 - I*c*x^3)*Log[1 - I*c*x^3])/(24*c^4) - (b^2*
Log[1 - I*c*x^3]^2)/(48*c^4) - (b*x^6*((2*I)*a - b*Log[1 - I*c*x^3]))/(48*c
^2) + ((I/72)*b*x^9*((2*I)*a - b*Log[1 - I*c*x^3]))/c + (b*x^12*((2*I)*a -
b*Log[1 - I*c*x^3]))/96 + (x^12*(2*a + I*b*Log[1 - I*c*x^3])^2)/48 + (I/288
)*b*(2*a + I*b*Log[1 - I*c*x^3])*((48*(1 - I*c*x^3))/c^4 - (36*(1 - I*c*x^3
)^2)/c^4 + (16*(1 - I*c*x^3)^3)/c^4 - (3*(1 - I*c*x^3)^4)/c^4 - (12*Log[1 -
 I*c*x^3])/c^4) + (b*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/(
24*c^4) + ((I/36)*b^2*x^9*Log[1 + I*c*x^3])/c - (b^2*(1 + I*c*x^3)*Log[1 +
I*c*x^3])/(12*c^4) - (b^2*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3])/(24*c^4) -
 (b*x^12*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3])/24 + (b^2*Log[1 +
 I*c*x^3]^2)/(48*c^4) - (b^2*x^12*Log[1 + I*c*x^3]^2)/48 + (5*b^2*Log[I + c
*x^3])/(288*c^4) - (b^2*PolyLog[2, (1 - I*c*x^3)/2])/(24*c^4) - (b^2*PolyLo
g[2, (1 + I*c*x^3)/2])/(24*c^4)
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
```

```
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^{11}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx = \int \left(\frac{1}{4}x^{11}\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^{11}\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{4}\right.$$

$$=\frac{1}{4}\int x^{11}\left(2a+ib\log\left(1-icx^3\right)\right)^2 dx+\frac{1}{2}b\int x^{11}\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx\right.$$

$$=\frac{1}{12}\text{Subst}\left(\int x^3(2a+ib\log(1-icx))^2\,dx,x,x^3\right)+\frac{1}{6}b\,\text{Subst}\left(\int x^3(-2ia+b\log(1-\right.$$

$$=\frac{1}{48}x^{12}\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{24}bx^{12}\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{48}b$$

$$=\frac{1}{48}x^{12}\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{24}bx^{12}\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{48}b$$

$$=\frac{1}{48}x^{12}\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{288}ib\left(2a+ib\log\left(1-icx^3\right)\right)\left(\frac{48\left(1-icx^3\right)}{c^4}-\frac{36(1}{}\right.$$

$$=\frac{abx^3}{12c^3}-\frac{bx^6\left(2ia-b\log\left(1-icx^3\right)\right)}{48c^2}+\frac{ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)}{72c}+\frac{1}{96}bx^{12}\left(2ia-b\right.$$

$$=\frac{abx^3}{12c^3}-\frac{bx^6\left(2ia-b\log\left(1-icx^3\right)\right)}{48c^2}+\frac{ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)}{72c}+\frac{1}{96}bx^{12}\left(2ia-b\right.$$

$$=\frac{abx^3}{12c^3}-\frac{55ib^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{ib^2x^9}{864c}+\frac{b^2x^{12}}{384}-\frac{b^2\left(1-icx^3\right)^2}{16c^4}+\frac{b^2\left(1-icx^3\right)^3}{54c^4}-\frac{b^2(1-}{384}$$

$$=\frac{abx^3}{12c^3}-\frac{55ib^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{ib^2x^9}{864c}+\frac{b^2x^{12}}{384}-\frac{b^2\left(1-icx^3\right)^2}{16c^4}+\frac{b^2\left(1-icx^3\right)^3}{54c^4}-\frac{b^2(1-}{384}$$

Mathematica [A] time = 0.0954633, size = 121, normalized size = 0.98

$$\frac{cx^3\left(3a^2c^3x^9-2abc^2x^6+6ab+b^2cx^3\right)-2b\tan^{-1}\left(cx^3\right)\left(a\left(3-3c^4x^{12}\right)+bcx^3\left(c^2x^6-3\right)\right)-4b^2\log\left(c^2x^6+1\right)+3b^2(}{36c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (c*x^3*(6*a*b + b^2*c*x^3 - 2*a*b*c^2*x^6 + 3*a^2*c^3*x^9) - 2*b*(b*c*x^3*(
-3 + c^2*x^6) + a*(3 - 3*c^4*x^12))*ArcTan[c*x^3] + 3*b^2*(-1 + c^4*x^12)*A
rcTan[c*x^3]^2 - 4*b^2*Log[1 + c^2*x^6])/(36*c^4)
```

Maple [A] time = 0.037, size = 151, normalized size = 1.2

$$\frac{x^{12}a^2}{12}+\frac{b^2x^{12}\left(\arctan\left(cx^3\right)\right)^2}{12}-\frac{b^2\arctan\left(cx^3\right)x^9}{18c}+\frac{b^2x^3\arctan\left(cx^3\right)}{6c^3}-\frac{b^2\left(\arctan\left(cx^3\right)\right)^2}{12c^4}+\frac{b^2x^6}{36c^2}-\frac{b^2\ln\left(c^2x^6\right.}{9c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] 1/12*x^12*a^2+1/12*b^2*x^12*arctan(c*x^3)^2-1/18*b^2*arctan(c*x^3)/c*x^9+1/
6*b^2*x^3*arctan(c*x^3)/c^3-1/12*b^2/c^4*arctan(c*x^3)^2+1/36*b^2*x^6/c^2-1
/9*b^2*ln(c^2*x^6+1)/c^4+1/6*a*b*x^12*arctan(c*x^3)-1/18*a*b/c*x^9+1/6*a*b*
x^3/c^3-1/6*a*b/c^4*arctan(c*x^3)
```

Maxima [A] time = 1.92842, size = 228, normalized size = 1.84

$$\frac{1}{12}\,b^2x^{12}\arctan\left(cx^3\right)^2+\frac{1}{12}\,a^2x^{12}+\frac{1}{18}\left(3\,x^{12}\arctan\left(cx^3\right)-c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\right)ab-\frac{1}{36}\left(2\,c\left(\frac{c^2x^9-3\,x^3}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^12*arctan(c*x^3)^2 + 1/12*a^2*x^12 + 1/18*(3*x^12*arctan(c*x^3)
- c*((c^2*x^9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5))*a*b - 1/36*(2*c*((c^2*x^
9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5)*arctan(c*x^3) - (c^2*x^6 + 3*arctan(c
*x^3)^2 - 3*log(18*c^7*x^6 + 18*c^5) - log(c^2*x^6 + 1))/c^4)*b^2
```

Fricas [A] time = 2.71134, size = 281, normalized size = 2.27

$$\frac{3\,a^2c^4x^{12}-2\,abc^3x^9+b^2c^2x^6+6\,abcx^3+3\left(b^2c^4x^{12}-b^2\right)\arctan\left(cx^3\right)^2-4\,b^2\log\left(c^2x^6+1\right)+2\left(3\,abc^4x^{12}-b^2c^3x^9+3\right.}{36\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*a^2*c^4*x^12 - 2*a*b*c^3*x^9 + b^2*c^2*x^6 + 6*a*b*c*x^3 + 3*(b^2*c
^4*x^12 - b^2)*arctan(c*x^3)^2 - 4*b^2*log(c^2*x^6 + 1) + 2*(3*a*b*c^4*x^12
 - b^2*c^3*x^9 + 3*b^2*c*x^3 - 3*a*b)*arctan(c*x^3))/c^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14039, size = 196, normalized size = 1.58

$$\frac{3\,a^2cx^{12}+2\left(3\,cx^{12}\arctan\left(cx^3\right)-\frac{3\,\arctan\left(cx^3\right)}{c^3}-\frac{c^9x^9-3\,c^7x^3}{c^9}\right)ab+\left(3\,cx^{12}\arctan\left(cx^3\right)^2-\frac{2\,c^3x^9\arctan\left(cx^3\right)-c^2x^6-6\,cx^3\arctan\left(c\right.}{c^3}\right.}{36\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/36*(3*a^2*c*x^12 + 2*(3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9
*x^9 - 3*c^7*x^3)/c^9)*a*b + (3*c*x^12*arctan(c*x^3)^2 - (2*c^3*x^9*arctan(
c*x^3) - c^2*x^6 - 6*c*x^3*arctan(c*x^3) + 3*arctan(c*x^3)^2 + 4*log(c^2*x^
6 + 1))/c^3)*b^2)/c
```

3.114 $\int x^8 \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=154

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{9c^3}-\frac{i\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{9c^3}-\frac{2b\log\left(\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)}{9c^3}+\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\cdots$$

```
[Out] (b^2*x^3)/(9*c^2) - (b^2*ArcTan[c*x^3])/(9*c^3) - (b*x^6*(a + b*ArcTan[c*x^
3]))/(9*c) - ((I/9)*(a + b*ArcTan[c*x^3])^2)/c^3 + (x^9*(a + b*ArcTan[c*x^3
])^2)/9 - (2*b*(a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/(9*c^3) - ((I/9)
*b^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c^3
```

Rubi [B] time = 1.39352, antiderivative size = 647, normalized size of antiderivative = 4.2, number of steps used = 53, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {5035, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)}{18c^3}-\frac{ib^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)}{18c^3}-\frac{iabx^3}{9c^2}+\frac{1}{108}ib\left(\frac{2i\left(1-icx^3\right)^3}{c^3}-\frac{9i\left(1-icx^3\right)^2}{c^3}+\frac{18i\left(1-\cdots\right.}{c^3}\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] ((-I/9)*a*b*x^3)/c^2 + (19*b^2*x^3)/(108*c^2) - (((5*I)/216)*b^2*x^6)/c + (
b^2*x^9)/162 - ((I/24)*b^2*(1 - I*c*x^3)^2)/c^3 + ((I/162)*b^2*(1 - I*c*x^3
)^3)/c^3 + ((I/18)*b^2*Log[I - c*x^3])/c^3 + ((I/18)*b^2*(1 - I*c*x^3)*Log[
1 - I*c*x^3])/c^3 - ((I/36)*b^2*Log[1 - I*c*x^3]^2)/c^3 + ((I/36)*b*x^6*((2
*I)*a - b*Log[1 - I*c*x^3]))/c + (b*x^9*((2*I)*a - b*Log[1 - I*c*x^3]))/54
+ (x^9*(2*a + I*b*Log[1 - I*c*x^3])^2)/36 + (I/108)*b*(2*a + I*b*Log[1 - I*
c*x^3])*(((18*I)*(1 - I*c*x^3))/c^3 - ((9*I)*(1 - I*c*x^3)^2)/c^3 + ((2*I)*
(1 - I*c*x^3)^3)/c^3 - ((6*I)*Log[1 - I*c*x^3])/c^3) - ((I/18)*b*((2*I)*a -
 b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/c^3 + ((I/18)*b^2*x^6*Log[1 + I*
c*x^3])/c - ((I/18)*b^2*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3])/c^3 - (b*x^9
*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3])/18 - ((I/36)*b^2*Log[1 +
I*c*x^3]^2)/c^3 - (b^2*x^9*Log[1 + I*c*x^3]^2)/36 - ((I/108)*b^2*Log[I + c*
x^3])/c^3 + ((I/18)*b^2*PolyLog[2, (1 - I*c*x^3)/2])/c^3 - ((I/18)*b^2*Poly
Log[2, (1 + I*c*x^3)/2])/c^3
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
```

```
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\,dx=\int\left(\frac{1}{4}x^8\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^8\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{4}b^2\right.$$

$$=\frac{1}{4}\int x^8\left(2a+ib\log\left(1-icx^3\right)\right)^2\,dx+\frac{1}{2}b\int x^8\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)$$

$$=\frac{1}{12}\text{Subst}\left(\int x^2(2a+ib\log(1-icx))^2\,dx,x,x^3\right)+\frac{1}{6}b\,\text{Subst}\left(\int x^2(-2ia+b\log(1-i\right.$$

$$=\frac{1}{36}x^9\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{18}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{36}b^2x^9$$

$$=\frac{1}{36}x^9\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{18}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{36}b^2x^9$$

$$=\frac{1}{36}x^9\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{108}ib\left(2a+ib\log\left(1-icx^3\right)\right)\left(\frac{18i\left(1-icx^3\right)}{c^3}-\frac{9i\left(1-\right.}{}\right.$$

$$=-\frac{iabx^3}{9c^2}+\frac{ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)}{36c}+\frac{1}{54}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{1}{36}x^9\left(2a+i\right.$$

$$=-\frac{iabx^3}{9c^2}+\frac{ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)}{36c}+\frac{1}{54}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{1}{36}x^9\left(2a+i\right.$$

$$=-\frac{iabx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{ib^2x^6}{216c}+\frac{b^2x^9}{162}-\frac{ib^2\left(1-icx^3\right)^2}{24c^3}+\frac{ib^2\left(1-icx^3\right)^3}{162c^3}+\frac{ib^2\left(1-icx^3\right)\log}{18c}$$

$$=-\frac{iabx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{ib^2x^6}{216c}+\frac{b^2x^9}{162}-\frac{ib^2\left(1-icx^3\right)^2}{24c^3}+\frac{ib^2\left(1-icx^3\right)^3}{162c^3}+\frac{ib^2\left(1-icx^3\right)\log}{18c}$$

Mathematica [A] time = 0.292919, size = 141, normalized size = 0.92

$$\frac{ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}\left(cx^3\right)}\right)+a^2c^3x^9-abc^2x^6+ab\log\left(c^2x^6+1\right)-b\tan^{-1}\left(cx^3\right)\left(-2ac^3x^9+bc^2x^6+2b\log\left(1+e^{2}\right.\right.}{9c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*c*x^3 - a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(I + c^3*x^9)*ArcTan[c*x^3]^2
- b*ArcTan[c*x^3]*(b + b*c^2*x^6 - 2*a*c^3*x^9 + 2*b*Log[1 + E^((2*I)*ArcTa
n[c*x^3])]) + a*b*Log[1 + c^2*x^6] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^
3])])/(9*c^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] int(x^8*(a+b*arctan(c*x^3))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{9}a^2x^9 + \frac{1}{9}\left(2x^9\arctan\left(cx^3\right) - \left(\frac{x^6}{c^2} - \frac{\log\left(c^2x^6+1\right)}{c^4}\right)c\right)ab + \frac{1}{144}\left(4x^9\arctan\left(cx^3\right)^2 - x^9\log\left(c^2x^6+1\right)^2 + 144\int \frac{4c^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/9*a^2*x^9 + 1/9*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c
)*a*b + 1/144*(4*x^9*arctan(c*x^3)^2 - x^9*log(c^2*x^6 + 1)^2 + 144*integra
te(1/48*(4*c^2*x^14*log(c^2*x^6 + 1) - 8*c*x^11*arctan(c*x^3) + 36*(c^2*x^1
4 + x^8)*arctan(c*x^3)^2 + 3*(c^2*x^14 + x^8)*log(c^2*x^6 + 1)^2)/(c^2*x^6
+ 1), x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^8\arctan\left(cx^3\right)^2 + 2\,abx^8\arctan\left(cx^3\right) + a^2x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^8*arctan(c*x^3)^2 + 2*a*b*x^8*arctan(c*x^3) + a^2*x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(cx^3\right) + a\right)^2 x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2*x^8, x)
```

3.115 $\int x^5 \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=90

$$\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{6c^2}-\frac{abx^3}{3c}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)^2+\frac{b^2\log\left(c^2x^6+1\right)}{6c^2}-\frac{b^2x^3\tan^{-1}\left(cx^3\right)}{3c}$$

```
[Out] -(a*b*x^3)/(3*c) - (b^2*x^3*ArcTan[c*x^3])/(3*c) + (a + b*ArcTan[c*x^3])^2/
(6*c^2) + (x^6*(a + b*ArcTan[c*x^3])^2)/6 + (b^2*Log[1 + c^2*x^6])/(6*c^2)
```

Rubi [C] time = 1.05174, antiderivative size = 612, normalized size of antiderivative = 6.8, number of steps used = 44, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)}{12c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)}{12c^2}-\frac{\left(1-icx^3\right)^2\left(2a+ib\log\left(1-icx^3\right)\right)^2}{24c^2}+\frac{\left(1-icx^3\right)\left(2a+i\right.}{12}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] -(a*b*x^3)/(2*c) + (b^2*x^6)/24 + (b^2*(1 - I*c*x^3)^2)/(48*c^2) + (b^2*(1
+ I*c*x^3)^2)/(48*c^2) - (b^2*Log[I - c*x^3])/(24*c^2) + (b^2*(1 - I*c*x^3)
*Log[1 - I*c*x^3])/(4*c^2) + (b*x^6*((2*I)*a - b*Log[1 - I*c*x^3]))/24 + ((
I/24)*b*(1 - I*c*x^3)^2*(2*a + I*b*Log[1 - I*c*x^3]))/c^2 + ((1 - I*c*x^3)*
(2*a + I*b*Log[1 - I*c*x^3])^2)/(12*c^2) - ((1 - I*c*x^3)^2*(2*a + I*b*Log[
1 - I*c*x^3])^2)/(24*c^2) - (b*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*
x^3)/2])/(12*c^2) - (b^2*x^6*Log[1 + I*c*x^3])/24 + (b^2*(1 + I*c*x^3)*Log[
1 + I*c*x^3])/(4*c^2) - (b^2*(1 + I*c*x^3)^2*Log[1 + I*c*x^3])/(24*c^2) + (
b^2*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3])/(12*c^2) - (b*x^6*((2*I)*a - b*L
og[1 - I*c*x^3])*Log[1 + I*c*x^3])/12 - (b^2*(1 + I*c*x^3)*Log[1 + I*c*x^3]
^2)/(12*c^2) + (b^2*(1 + I*c*x^3)^2*Log[1 + I*c*x^3]^2)/(24*c^2) - (b^2*Log
[I + c*x^3])/(24*c^2) + (b^2*PolyLog[2, (1 - I*c*x^3)/2])/(12*c^2) + (b^2*P
olyLog[2, (1 + I*c*x^3)/2])/(12*c^2)
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
```

```
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)^2dx=\int\left(\frac{1}{4}x^5\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^5\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{4}b^2x^5\right.$$
$$=\frac{1}{4}\int x^5\left(2a+ib\log\left(1-icx^3\right)\right)^2dx+\frac{1}{2}b\int x^5\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)dx$$
$$=\frac{1}{12}\text{Subst}\left(\int x(2a+ib\log(1-icx))^2dx,x,x^3\right)+\frac{1}{6}b\,\text{Subst}\left(\int x(-2ia+b\log(1-icx))\right.$$
$$=-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)+\frac{1}{12}\text{Subst}\left(\int\left(-\frac{i(2a+ib\log(1-icx))^2}{c}\right.\right.$$
$$=-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{i\,\text{Subst}\left(\int(2a+ib\log(1-icx))^2dx,x,x^3\right.}{12c}$$
$$=-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{12}b\,\text{Subst}\left(\int x(-2ia+b\log(1-icx))\right.$$
$$=-\frac{abx^3}{6c}+\frac{1}{24}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c^2}-\frac{\left(1-icx^3\right)}{}$$
$$=-\frac{abx^3}{2c}-\frac{ib^2x^3}{4c}+\frac{b^2\left(1-icx^3\right)^2}{48c^2}+\frac{b^2\left(1+icx^3\right)^2}{48c^2}+\frac{1}{24}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{ib\left(1\right.}{}$$
$$=-\frac{abx^3}{2c}+\frac{b^2x^6}{24}+\frac{b^2\left(1-icx^3\right)^2}{48c^2}+\frac{b^2\left(1+icx^3\right)^2}{48c^2}-\frac{b^2\log\left(i-cx^3\right)}{24c^2}+\frac{b^2\left(1-icx^3\right)\log\left(1-\right.}{4c^2}$$

Mathematica [A] time = 0.0649459, size = 85, normalized size = 0.94

$$\frac{2b\tan^{-1}\left(cx^3\right)\left(ac^2x^6+a-bcx^3\right)+acx^3\left(acx^3-2b\right)+b^2\log\left(c^2x^6+1\right)+b^2\left(c^2x^6+1\right)\tan^{-1}\left(cx^3\right)^2}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (a*c*x^3*(-2*b + a*c*x^3) + 2*b*(a - b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b
^2*(1 + c^2*x^6)*ArcTan[c*x^3]^2 + b^2*Log[1 + c^2*x^6])/(6*c^2)
```

Maple [A] time = 0.037, size = 113, normalized size = 1.3

$$\frac{x^6a^2}{6}+\frac{b^2x^6\left(\arctan\left(cx^3\right)\right)^2}{6}-\frac{b^2x^3\arctan\left(cx^3\right)}{3c}+\frac{b^2\left(\arctan\left(cx^3\right)\right)^2}{6c^2}+\frac{b^2\ln\left(c^2x^6+1\right)}{6c^2}+\frac{abx^6\arctan\left(cx^3\right)}{3}-\frac{abx^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] 1/6*x^6*a^2+1/6*b^2*x^6*arctan(c*x^3)^2-1/3*b^2*x^3*arctan(c*x^3)/c+1/6*b^2
/c^2*arctan(c*x^3)^2+1/6*b^2*ln(c^2*x^6+1)/c^2+1/3*a*b*x^6*arctan(c*x^3)-1/
3*a*b*x^3/c+1/3*a*b/c^2*arctan(c*x^3)
```

Maxima [A] time = 1.87051, size = 170, normalized size = 1.89

$$\frac{1}{6}\,b^2x^6\arctan\left(cx^3\right)^2+\frac{1}{6}\,a^2x^6+\frac{1}{3}\left(x^6\arctan\left(cx^3\right)-c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\right)ab-\frac{1}{6}\left(2\,c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^2*x^6 + 1/3*(x^6*arctan(c*x^3) - c*(x^3
/c^2 - arctan(c*x^3)/c^3))*a*b - 1/6*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)*arc
tan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*b^2
```

Fricas [A] time = 2.61903, size = 200, normalized size = 2.22

$$\frac{a^2c^2x^6-2\,abcx^3+\left(b^2c^2x^6+b^2\right)\arctan\left(cx^3\right)^2+b^2\log\left(c^2x^6+1\right)+2\left(abc^2x^6-b^2cx^3+ab\right)\arctan\left(cx^3\right)}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*c^2*x^6 - 2*a*b*c*x^3 + (b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + b^2*
log(c^2*x^6 + 1) + 2*(a*b*c^2*x^6 - b^2*c*x^3 + a*b)*arctan(c*x^3))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15752, size = 135, normalized size = 1.5

$$\frac{a^2cx^6+\frac{2\left(c^2x^6\arctan\left(cx^3\right)-cx^3+\arctan\left(cx^3\right)\right)ab}{c}+\frac{\left(c^2x^6\arctan\left(cx^3\right)^2-2\,cx^3\arctan\left(cx^3\right)+\arctan\left(cx^3\right)^2+\log\left(c^2x^6+1\right)\right)b^2}{c}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/6*(a^2*c*x^6 + 2*(c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*a*b/c +
(c^2*x^6*arctan(c*x^3)^2 - 2*c*x^3*arctan(c*x^3) + arctan(c*x^3)^2 + log(c^
2*x^6 + 1))*b^2/c)/c
```

3.116 $\int x^2 \left(a + b \tan^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=104

$$\frac{ib^2 \text{PolyLog}\left(2, 1 - \frac{2}{1+icx^3}\right)}{3c} + \frac{1}{3}x^3 \left(a + b \tan^{-1}\left(cx^3\right)\right)^2 + \frac{i\left(a + b \tan^{-1}\left(cx^3\right)\right)^2}{3c} + \frac{2b \log\left(\frac{2}{1+icx^3}\right)\left(a + b \tan^{-1}\left(cx^3\right)\right)}{3c}$$

```
[Out] ((I/3)*(a + b*ArcTan[c*x^3])^2)/c + (x^3*(a + b*ArcTan[c*x^3])^2)/3 + (2*b*
(a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/(3*c) + ((I/3)*b^2*PolyLog[2, 1
 - 2/(1 + I*c*x^3)])/c
```

Rubi [B] time = 0.644147, antiderivative size = 255, normalized size of antiderivative = 2.45, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{ib^2 \text{PolyLog}\left(2, \frac{1}{2}\left(1 - icx^3\right)\right)}{6c} + \frac{ib^2 \text{PolyLog}\left(2, \frac{1}{2}\left(1 + icx^3\right)\right)}{6c} - \frac{1}{6}bx^3 \log\left(1 + icx^3\right)\left(2ia - b \log\left(1 - icx^3\right)\right) + \frac{i\left(1 - icx^3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] ((I/12)*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^2)/c + ((I/6)*b*((2*I)*a
 - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/c + ((I/6)*b^2*Log[(1 - I*c*x^
3)/2]*Log[1 + I*c*x^3])/c - (b*x^3*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I
*c*x^3])/6 + ((I/12)*b^2*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/c - ((I/6)*b^2*P
olyLog[2, (1 - I*c*x^3)/2])/c + ((I/6)*b^2*PolyLog[2, (1 + I*c*x^3)/2])/c
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^2dx=\int\left(\frac{1}{4}x^2\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^2\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{4}b^2x^2\right.$$

$$=\frac{1}{4}\int x^2\left(2a+ib\log\left(1-icx^3\right)\right)^2dx+\frac{1}{2}b\int x^2\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)dx$$

$$=\frac{1}{12}\text{Subst}\left(\int(2a+ib\log(1-icx))^2dx,x,x^3\right)+\frac{1}{6}b\,\text{Subst}\left(\int(-2ia+b\log(1-icx))\log(\right.$$

$$=-\frac{1}{6}bx^3\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)+\frac{i\,\text{Subst}\left(\int(2a+ib\log(x))^2dx,x,1-icx^3\right)}{12c}$$

$$=\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}-\frac{1}{6}bx^3\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)+\frac{ib^2(1}{}$$

$$=-\frac{1}{3}iabx^3-\frac{b^2x^3}{6}+\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}-\frac{ib^2\left(1+icx^3\right)\log\left(1+icx^3\right)}{6c}-\frac{1}{6}$$

$$=-\frac{1}{3}b^2x^3+\frac{ib^2\left(1-icx^3\right)\log\left(1-icx^3\right)}{6c}+\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}+\frac{ib\left(2ia-b\right.}{}$$

$$=-\frac{1}{6}b^2x^3+\frac{ib^2\left(1-icx^3\right)\log\left(1-icx^3\right)}{6c}+\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}+\frac{ib\left(2ia-b\right.}{}$$

$$=\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}+\frac{ib\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(\frac{1}{2}\left(1+icx^3\right)\right)}{6c}+\frac{ib^2}{}$$

Mathematica [A] time = 0.0706193, size = 107, normalized size = 1.03

$$\frac{-ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}\left(cx^3\right)}\right)+a\left(acx^3-b\log\left(c^2x^6+1\right)\right)+2b\tan^{-1}\left(cx^3\right)\left(acx^3+b\log\left(1+e^{2i\tan^{-1}\left(cx^3\right)}\right)\right)+b^2\left(cx^3\right.}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*(-I + c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(a*c*x^3 + b*Log[1 +
E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3 - b*Log[1 + c^2*x^6]) - I*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x^3])])/(3*c)
```

Maple [A] time = 0.09, size = 148, normalized size = 1.4

$$\frac{x^3b^2\left(\arctan\left(cx^3\right)\right)^2}{3}+\frac{2x^3ab\arctan\left(cx^3\right)}{3}+\frac{x^3a^2}{3}-\frac{\frac{i}{3}\left(\arctan\left(cx^3\right)\right)^2b^2}{c}+\frac{2\arctan\left(cx^3\right)b^2}{3c}\ln\left(\frac{\left(1+icx^3\right)^2}{c^2x^6+1}+1\right)-\frac{i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] 1/3*x^3*b^2*arctan(c*x^3)^2+2/3*x^3*a*b*arctan(c*x^3)+1/3*x^3*a^2-1/3*I/c*a
rctan(c*x^3)^2*b^2+2/3/c*arctan(c*x^3)*ln((1+I*c*x^3)^2/(c^2*x^6+1)+1)*b^2-
1/3*I/c*polylog(2,-(1+I*c*x^3)^2/(c^2*x^6+1))*b^2-1/3/c*a*b*ln(c^2*x^6+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2x^3+\frac{1}{48}\left(4x^3\arctan\left(cx^3\right)^2-x^3\log\left(c^2x^6+1\right)^2+576c^2\int\frac{x^8\arctan\left(cx^3\right)^2}{16\left(c^2x^6+1\right)}\,dx+48c^2\int\frac{x^8\log\left(c^2x^6+1\right)^2}{16\left(c^2x^6+1\right)}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/48*(4*x^3*arctan(c*x^3)^2 - x^3*log(c^2*x^6 + 1)^2 + 576*c^
2*integrate(1/16*x^8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 48*c^2*integrate(1
/16*x^8*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 192*c^2*integrate(1/16*x^8*l
og(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 4*arctan(c*x^3)^3/c - 384*c*integrate(1
/16*x^5*arctan(c*x^3)/(c^2*x^6 + 1), x) + 48*integrate(1/16*x^2*log(c^2*x^6
 + 1)^2/(c^2*x^6 + 1), x))*b^2 + 1/3*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 +
 1))*a*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2x^2\arctan\left(cx^3\right)^2+2\,abx^2\arctan\left(cx^3\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c*x^3)^2 + 2*a*b*x^2*arctan(c*x^3) + a^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\,\mathrm{atan}\left(cx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x**3))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(cx^3\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2*x^2, x)
```

3.117 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{3}ib\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{1}{3}ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{6}b^2\text{PolyLog}$$

```
[Out] (2*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/3)*b*(a + b
*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/3)*b*(a + b*ArcTan[c*x
^3])*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^3)
])/6 + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^3)])/6
```

Rubi [A] time = 0.316484, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5031, 4850, 4988, 4884, 4994, 6610}

$$-\frac{1}{3}ib\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{1}{3}ib\text{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{6}b^2\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/3)*b*(a + b
*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/3)*b*(a + b*ArcTan[c*x
^3])*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^3)
])/6 + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^3)])/6
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx,x,x^3\right)$$

$$=\frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}(4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\text{tanh}}{1+c^2x^2}\right.$$

$$=\frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)+\frac{1}{3}(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\right.}{1+c^2x^2}\right.$$

$$=\frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)\text{Li}_2\left(1-\frac{2}{1+icx^3}\right.$$

$$=\frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)\text{Li}_2\left(1-\frac{2}{1+icx^3}\right.$$

Mathematica [A] time = 0.115795, size = 167, normalized size = 1.08

$$\frac{1}{6}\left(b\left(2i\text{PolyLog}\left(2,\frac{cx^3+i}{-cx^3+i}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-2i\text{PolyLog}\left(2,\frac{cx^3+i}{cx^3-i}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+b\left(\text{PolyLog}\left(3,\frac{cx}{-c}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x,x]
```

```
[Out] (4*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 + (2*I)/(-I + c*x^3)] + b*((2*I)*(a +
b*ArcTan[c*x^3])*PolyLog[2, (I + c*x^3)/(I - c*x^3)] - (2*I)*(a + b*ArcTan[
c*x^3])*PolyLog[2, (I + c*x^3)/(-I + c*x^3)] + b*(PolyLog[3, (I + c*x^3)/(I
 - c*x^3)] - PolyLog[3, (I + c*x^3)/(-I + c*x^3)])))/6
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x,x)
```

```
[Out] int((a+b*arctan(c*x^3))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\log(x) + \frac{1}{16}\int \frac{12\,b^2\arctan\left(cx^3\right)^2 + b^2\log\left(c^2x^6+1\right)^2 + 32\,ab\arctan\left(cx^3\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^3)^2 + b^2*log(c^2*x^6 + 1)^
2 + 32*a*b*arctan(c*x^3))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\arctan\left(cx^3\right)^2 + 2\,ab\arctan\left(cx^3\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\mathrm{atan}\left(cx^3\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arctan\left(cx^3\right) + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x, x)
```

3.118 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=100

$$-\frac{1}{3}ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)-\frac{1}{3}ic\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{3x^3}+\frac{2}{3}bc\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)$$

```
[Out] (-I/3)*c*(a + b*ArcTan[c*x^3])^2 - (a + b*ArcTan[c*x^3])^2/(3*x^3) + (2*b*c
*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)])/3 - (I/3)*b^2*c*PolyLog[2,
 -1 + 2/(1 - I*c*x^3)]
```

Rubi [B] time = 0.660795, antiderivative size = 290, normalized size of antiderivative = 2.9, number of steps used = 24, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5035, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$\frac{1}{3}ib^2c\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{3}ib^2c\text{PolyLog}\left(2,icx^3\right)-\frac{1}{6}ib^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)+\frac{1}{6}ib^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

```
[Out] 2*a*b*c*Log[x] - ((1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^2)/(12*x^3) +
(I/6)*b*c*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2] + (I/6)*b^2*c
*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3] + (b*((2*I)*a - b*Log[1 - I*c*x^3])*
Log[1 + I*c*x^3])/(6*x^3) + (b^2*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/(12*x^3)
 + (I/3)*b^2*c*PolyLog[2, (-I)*c*x^3] - (I/3)*b^2*c*PolyLog[2, I*c*x^3] - (
I/6)*b^2*c*PolyLog[2, (1 - I*c*x^3)/2] + (I/6)*b^2*c*PolyLog[2, (1 + I*c*x^
3)/2]
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{4x^4}+\frac{b\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{2x^4}-\frac{b^2\log^2\left(1+ic\right.}{4x^4}$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{x^4}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{x^4}\,dx-\frac{1}{4}$$

$$=\frac{1}{12}\,\mathrm{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^2}\,dx,x,x^3\right)+\frac{1}{6}b\,\mathrm{Subst}\left(\int \frac{(-2ia+b\log(1-icx))\,\mathrm{lo}}{x^2}\right.$$

$$=-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{6x^3}+\frac{b^2\left(1+\right.}{}$$

$$=abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{6x^3}$$

$$=abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{6x^3}$$

$$=2abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{1}{6}ibc\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(\frac{1}{2}\right.$$

$$=2abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{1}{6}ibc\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(\frac{1}{2}\right.$$

$$=2abc\log(x)-\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12x^3}+\frac{1}{6}ibc\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(\frac{1}{2}\right.$$

Mathematica [A] time = 0.154743, size = 125, normalized size = 1.25

$$\frac{-ib^2cx^3\mathrm{PolyLog}\left(2,e^{2i\tan^{-1}\left(cx^3\right)}\right)-a\left(a+bcx^3\log\left(c^2x^6+1\right)-2bcx^3\log\left(cx^3\right)\right)+2b\tan^{-1}\left(cx^3\right)\left(-a+bcx^3\log\left(1-\right.\right.}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

```
[Out] (b^2*(-1 - I*c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(-a + b*c*x^3*Log[1
 - E^((2*I)*ArcTan[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[1
+ c^2*x^6]) - I*b^2*c*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/(3*x^3)
```

Maple [F] time = 0.408, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x^4,x)
```

```
[Out] int((a+b*arctan(c*x^3))^2/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\left(c\left(\log\left(c^2x^6+1\right)-\log\left(x^6\right)\right)+\frac{2\arctan\left(cx^3\right)}{x^3}\right)ab+\frac{\frac{1}{4}\left(12\,x^3\int-\frac{12\,c^2x^6\log\left(c^2x^6+1\right)-56\,cx^3\arctan\left(cx^3\right)-36\left(c^2x^6+1\right)\arctan\left(cx^3\right)^2}{4\left(c^2x^{10}+x^4\right)}\right.}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a*b + 1/48*(48
*x^3*integrate(-1/16*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3*arctan(c*x^3) -
12*(c^2*x^6 + 1)*arctan(c*x^3)^2 - (c^2*x^6 + 1)*log(c^2*x^6 + 1)^2)/(c^2*x
^10 + x^4), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2)*b^2/x^3 - 1/3*a^2/
x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\arctan\left(cx^3\right)^2+2\,ab\arctan\left(cx^3\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(cx^3\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="giac")

[Out] integrate((b*arctan(c*x^3) + a)^2/x^4, x)
```

3.119 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^7}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{6}c^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{bc\left(a+b\tan^{-1}\left(cx^3\right)\right)}{3x^3}-\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{6x^6}-\frac{1}{6}b^2c^2\log\left(c^2x^6+1\right)+b^2c^2\log(x)$$

```
[Out] -(b*c*(a + b*ArcTan[c*x^3]))/(3*x^3) - (c^2*(a + b*ArcTan[c*x^3])^2)/6 - (a
 + b*ArcTan[c*x^3])^2/(6*x^6) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^6])
/6
```

Rubi [C] time = 1.12245, antiderivative size = 419, normalized size of antiderivative = 4.82, number of steps used = 46, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{12}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)-\frac{1}{12}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1+icx^3\right)\right)+\frac{1}{12}bc^2\log\left(\frac{1}{2}\left(1+icx^3\right)\right)\left(2ia-b\log\left(1-icx^3\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

```
[Out] b^2*c^2*Log[x] - (b^2*c^2*Log[I - c*x^3])/6 + ((I/12)*b*c*((2*I)*a - b*Log[
1 - I*c*x^3]))/x^3 - (b*c*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3]))/(12*x
^3) - (c^2*(2*a + I*b*Log[1 - I*c*x^3])^2)/24 - (2*a + I*b*Log[1 - I*c*x^3]
)^2/(24*x^6) + (b*c^2*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/
12 + ((I/6)*b^2*c*Log[1 + I*c*x^3])/x^3 - (b^2*c^2*Log[(1 - I*c*x^3)/2]*Log
[1 + I*c*x^3])/12 + (b*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3])/(12
*x^6) + (b^2*c^2*Log[1 + I*c*x^3]^2)/24 + (b^2*Log[1 + I*c*x^3]^2)/(24*x^6)
 - (b^2*c^2*Log[I + c*x^3])/12 - (b^2*c^2*PolyLog[2, (1 - I*c*x^3)/2])/12 -
 (b^2*c^2*PolyLog[2, (1 + I*c*x^3)/2])/12
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
```

```
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

eQ[q, -1]

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^7}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{4x^7}+\frac{b\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{2x^7}-\frac{b^2\log^2\left(1+ic\right.}{4x^7}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{x^7}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{x^7}\,dx-\frac{1}{4}$$

$$=\frac{1}{12}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^3}\,dx,x,x^3\right)+\frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx))\log}{x^3}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{24x^6}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{12x^6}+\frac{b^2\log^2\left(1+icx^3\right)}{24x^6}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{24x^6}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{12x^6}+\frac{b^2\log^2\left(1+icx^3\right)}{24x^6}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{24x^6}+\frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{12x^6}+\frac{b^2\log^2\left(1+icx^3\right)}{24x^6}$$

$$=-\frac{1}{2}iabc^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{12x^3}-\frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)}{12x^3}-(2$$

$$=\frac{1}{4}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{12x^3}-\frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)}{12x^3}-\frac{1}{24}c^2$$

$$=\frac{1}{2}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{12x^3}-\frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)}{12x^3}-\frac{1}{24}c^2$$

Mathematica [A] time = 0.0774895, size = 98, normalized size = 1.13

$$-\frac{a^2+2b\tan^{-1}\left(cx^3\right)\left(ac^2x^6+a+bcx^3\right)+2abcx^3-6b^2c^2x^6\log(x)+b^2c^2x^6\log\left(c^2x^6+1\right)+b^2\left(c^2x^6+1\right)\tan^{-1}\left(cx^3\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

```
[Out] -(a^2 + 2*a*b*c*x^3 + 2*b*(a + b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b^2*(1
+ c^2*x^6)*ArcTan[c*x^3]^2 - 6*b^2*c^2*x^6*Log[x] + b^2*c^2*x^6*Log[1 + c^2
*x^6])/(6*x^6)
```

Maple [A] time = 0.04, size = 118, normalized size = 1.4

$$-\frac{a^2}{6\,x^6}-\frac{b^2\left(\arctan\left(cx^3\right)\right)^2}{6\,x^6}-\frac{b^2c^2\left(\arctan\left(cx^3\right)\right)^2}{6}-\frac{b^2c\arctan\left(cx^3\right)}{3\,x^3}-\frac{b^2c^2\ln\left(c^2x^6+1\right)}{6}+b^2c^2\ln\left(x\right)-\frac{ab\arctan\left(cx\right.}{3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x^7,x)
```

```
[Out] -1/6*a^2/x^6-1/6*b^2/x^6*arctan(c*x^3)^2-1/6*b^2*c^2*arctan(c*x^3)^2-1/3*b^
2*c*arctan(c*x^3)/x^3-1/6*b^2*c^2*ln(c^2*x^6+1)+b^2*c^2*ln(x)-1/3*a*b/x^6*a
rctan(c*x^3)-1/3*a*b*c^2*arctan(c*x^3)-1/3*a*b*c/x^3
```

Maxima [A] time = 1.94716, size = 149, normalized size = 1.71

$$-\frac{1}{3}\left(\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c+\frac{\arctan\left(cx^3\right)}{x^6}\right)ab+\frac{1}{6}\left(\left(\arctan\left(cx^3\right)^2-\log\left(c^2x^6+1\right)+6\,\log\left(x\right)\right)c^2-2\left(c\arctan\left(cx^3\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a*b + 1/6*((arctan(c
*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*c*
arctan(c*x^3))*b^2 - 1/6*b^2*arctan(c*x^3)^2/x^6 - 1/6*a^2/x^6
```

Fricas [A] time = 2.84334, size = 232, normalized size = 2.67

$$-\frac{b^2c^2x^6\log\left(c^2x^6+1\right)-6\,b^2c^2x^6\log\left(x\right)+2\,abcx^3+\left(b^2c^2x^6+b^2\right)\arctan\left(cx^3\right)^2+a^2+2\left(abc^2x^6+b^2cx^3+ab\right)\arctan\left(\right.}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(b^2*c^2*x^6*log(c^2*x^6 + 1) - 6*b^2*c^2*x^6*log(x) + 2*a*b*c*x^3 + (
b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + a^2 + 2*(a*b*c^2*x^6 + b^2*c*x^3 + a*b
)*arctan(c*x^3))/x^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(cx^3\right) + a\right)^2}{x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x^7, x)
```

3.120 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=154

$$\frac{1}{9}ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)+\frac{1}{9}ic^3\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{2}{9}bc^3\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{bc\left(a+b\tan\right.}{9}$$

```
[Out] -(b^2*c^2)/(9*x^3) - (b^2*c^3*ArcTan[c*x^3])/9 - (b*c*(a + b*ArcTan[c*x^3])
)/(9*x^6) + (I/9)*c^3*(a + b*ArcTan[c*x^3])^2 - (a + b*ArcTan[c*x^3])^2/(9*
x^9) - (2*b*c^3*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)])/9 + (I/9)*b
^2*c^3*PolyLog[2, -1 + 2/(1 - I*c*x^3)]
```

Rubi [B] time = 1.44996, antiderivative size = 536, normalized size of antiderivative = 3.48, number of steps used = 59, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.5, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2395, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{9}ib^2c^3\text{PolyLog}\left(2,-icx^3\right)+\frac{1}{9}ib^2c^3\text{PolyLog}\left(2,icx^3\right)+\frac{1}{18}ib^2c^3\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)-\frac{1}{18}ib^2c^3\text{PolyLog}\left(2,\frac{1}{2}\left(1-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

```
[Out] -(b^2*c^2)/(9*x^3) - (2*a*b*c^3*Log[x])/3 + (I/18)*b^2*c^3*Log[I - c*x^3] +
 ((I/36)*b*c*((2*I)*a - b*Log[1 - I*c*x^3]))/x^6 + (b*c^2*((2*I)*a - b*Log[
1 - I*c*x^3]))/(18*x^3) - (b*c*(2*a + I*b*Log[1 - I*c*x^3]))/(36*x^6) - ((I
/18)*b*c^2*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3]))/x^3 - (I/36)*c^3*(2*
a + I*b*Log[1 - I*c*x^3])^2 - (2*a + I*b*Log[1 - I*c*x^3])^2/(36*x^9) - (I/
18)*b*c^3*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2] + ((I/18)*b^2
*c*Log[1 + I*c*x^3])/x^6 - (I/18)*b^2*c^3*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*
x^3] + (b*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3])/(18*x^9) - (I/36
)*b^2*c^3*Log[1 + I*c*x^3]^2 + (b^2*Log[1 + I*c*x^3]^2)/(36*x^9) - (I/9)*b^
2*c^3*PolyLog[2, (-I)*c*x^3] + (I/9)*b^2*c^3*PolyLog[2, I*c*x^3] + (I/18)*b
^2*c^3*PolyLog[2, (1 - I*c*x^3)/2] - (I/18)*b^2*c^3*PolyLog[2, (1 + I*c*x^3
)/2]
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x^{10}}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{4x^{10}} + \frac{b\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{2x^{10}} - \frac{b^2\log^2\left(1+icx^3\right)}{4x^{10}}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{x^{10}}\,dx + \frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{x^{10}}\,dx - \frac{1}{4}b^2$$

$$= \frac{1}{12}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^4}\,dx, x, x^3\right) + \frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx))\log(1}{x^4}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{36x^9} + \frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{18x^9} + \frac{b^2\log^2\left(1+icx^3\right)}{36x^9} +$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{36x^9} + \frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{18x^9} + \frac{b^2\log^2\left(1+icx^3\right)}{36x^9} +$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^2}{36x^9} + \frac{b\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)}{18x^9} + \frac{b^2\log^2\left(1+icx^3\right)}{36x^9} +$$

$$= -\frac{1}{3}abc^3\log(x) + \frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{36x^6} + \frac{bc^2\left(2ia-b\log\left(1-icx^3\right)\right)}{18x^3} - \frac{bc\left(2a+ib\log\right.}{36x^6}$$

$$= -\frac{1}{3}abc^3\log(x) + \frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{36x^6} + \frac{bc^2\left(2ia-b\log\left(1-icx^3\right)\right)}{18x^3} - \frac{bc\left(2a+ib\log\right.}{36x^6}$$

$$= -\frac{b^2c^2}{18x^3} - \frac{2}{3}abc^3\log(x) + \frac{1}{6}ib^2c^3\log(x) + \frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{36x^6} + \frac{bc^2\left(2ia-b\log\left(1-\right.\right.}{18x^3}$$

$$= -\frac{b^2c^2}{18x^3} - \frac{2}{3}abc^3\log(x) + \frac{1}{6}ib^2c^3\log(x) + \frac{ibc\left(2ia-b\log\left(1-icx^3\right)\right)}{36x^6} + \frac{bc^2\left(2ia-b\log\left(1-\right.\right.}{18x^3}$$

Mathematica [A] time = 0.374299, size = 167, normalized size = 1.08

$$-\frac{-ib^2c^3x^9\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(cx^3\right)}\right) + a^2 + 2abc^3x^9\log\left(cx^3\right) - abc^3x^9\log\left(c^2x^6+1\right) + b\tan^{-1}\left(cx^3\right)\left(2a+bc^3x^9+2bc^3x\right.}{9x^9}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

```
[Out] -(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - I*c^3*x^9)*ArcTan[c*x^3]^2 + b*A
rcTan[c*x^3]*(2*a + b*c*x^3 + b*c^3*x^9 + 2*b*c^3*x^9*Log[1 - E^((2*I)*ArcT
an[c*x^3])]) + 2*a*b*c^3*x^9*Log[c*x^3] - a*b*c^3*x^9*Log[1 + c^2*x^6] - I*
b^2*c^3*x^9*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/(9*x^9)
```

Maple [F] time = 0.288, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x^10,x)
```

```
[Out] int((a+b*arctan(c*x^3))^2/x^10,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{9}\left(\left(c^{2}\log\left(c^{2}x^{6}+1\right)-c^{2}\log\left(x^{6}\right)-\frac{1}{x^{6}}\right)c-\frac{2\arctan\left(cx^{3}\right)}{x^{9}}\right)ab+\frac{\frac{1}{4}\left(12\,x^{9}\int-\frac{12\,c^{2}x^{6}\log\left(c^{2}x^{6}+1\right)-56\,cx^{3}\arctan\left(cx^{3}\right)-108\,\left(c^{2}x^{6}+\right.}{4\left(c^{2}x^{16}+x^{1}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="maxima")
```

```
[Out] 1/9*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9)
*a*b + 1/144*(144*x^9*integrate(-1/48*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3
*arctan(c*x^3) - 36*(c^2*x^6 + 1)*arctan(c*x^3)^2 - 3*(c^2*x^6 + 1)*log(c^2
*x^6 + 1)^2)/(c^2*x^16 + x^10), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2
)*b^2/x^9 - 1/9*a^2/x^9
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{2}\arctan\left(cx^{3}\right)^{2}+2\,ab\arctan\left(cx^{3}\right)+a^{2}}{x^{10}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^10, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x**10,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(cx^{3}\right)+a\right)^{2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="giac")

[Out] integrate((b*arctan(c*x^3) + a)^2/x^10, x)
```

3.121 $\int x^8 \left(a + b\tan^{-1}\left(cx^3\right)\right)^3 dx$

Optimal. Leaf size=240

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)}{3c^3}-\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)}{6c^3}+\frac{ab^2x^3}{3c^2}-\frac{i\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{9c^3}-\frac{b\left(a\right.}{}$$

```
[Out] (a*b^2*x^3)/(3*c^2) + (b^3*x^3*ArcTan[c*x^3])/(3*c^2) - (b*(a + b*ArcTan[c*
x^3])^2)/(6*c^3) - (b*x^6*(a + b*ArcTan[c*x^3])^2)/(6*c) - ((I/9)*(a + b*Ar
cTan[c*x^3])^3)/c^3 + (x^9*(a + b*ArcTan[c*x^3])^3)/9 - (b*(a + b*ArcTan[c*
x^3])^2*Log[2/(1 + I*c*x^3)])/(3*c^3) - (b^3*Log[1 + c^2*x^6])/(6*c^3) - ((
I/3)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c^3 - (b^3*
PolyLog[3, 1 - 2/(1 + I*c*x^3)])/(6*c^3)
```

Rubi [B] time = 7.0337, antiderivative size = 1867, normalized size of antiderivative = 7.78, number of steps used = 239, number of rules used = 32, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2., Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2398, 2411, 43, 2334, 12, 14, 2301, 2439, 2416, 2396, 2433, 2374, 6589, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2410, 2425}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (2*a*b^2*x^3)/(3*c^2) + (((7*I)/216)*b^3*x^3)/c^2 - (23*b^3*x^6)/(432*c) +
(I/324)*b^3*x^9 - (b^3*(1 - I*c*x^3)^2)/(48*c^3) - (b^3*(1 + I*c*x^3)^2)/(2
4*c^3) + (b^3*(1 + I*c*x^3)^3)/(324*c^3) + (7*b^3*Log[I - c*x^3])/(108*c^3)
 - (b^3*(1 - I*c*x^3)*Log[1 - I*c*x^3])/(3*c^3) + (b^3*Log[1 - I*c*x^3]^2)/
(72*c^3) - (b^2*x^6*((2*I)*a - b*Log[1 - I*c*x^3]))/(24*c) - (b^2*(1 - I*c*
x^3)^2*((2*I)*a - b*Log[1 - I*c*x^3]))/(48*c^3) - (I/72)*b*x^9*((2*I)*a - b
*Log[1 - I*c*x^3])^2 - (b*(1 - I*c*x^3)^2*((2*I)*a - b*Log[1 - I*c*x^3])^2)
/(48*c^3) - ((I/16)*b^2*(1 - I*c*x^3)^2*(2*a + I*b*Log[1 - I*c*x^3]))/c^3 +
 ((I/108)*b^2*(1 - I*c*x^3)^3*(2*a + I*b*Log[1 - I*c*x^3]))/c^3 - (b*(1 - I
*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^2)/(8*c^3) + (b*(1 - I*c*x^3)^2*(2*a +
 I*b*Log[1 - I*c*x^3])^2)/(16*c^3) - (b*(1 - I*c*x^3)^3*(2*a + I*b*Log[1 -
I*c*x^3])^2)/(72*c^3) - ((I/24)*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^
3)/c^3 + ((I/24)*(1 - I*c*x^3)^2*(2*a + I*b*Log[1 - I*c*x^3])^3)/c^3 - ((I/
72)*(1 - I*c*x^3)^3*(2*a + I*b*Log[1 - I*c*x^3])^3)/c^3 + (I/216)*b^2*((2*I
)*a - b*Log[1 - I*c*x^3])*(((18*I)*(1 - I*c*x^3))/c^3 - ((9*I)*(1 - I*c*x^3
)^2)/c^3 + ((2*I)*(1 - I*c*x^3)^3)/c^3 - ((6*I)*Log[1 - I*c*x^3])/c^3) + (b
^2*((2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/(12*c^3) - (b*((2*I
)*a - b*Log[1 - I*c*x^3])^2*Log[(1 + I*c*x^3)/2])/(24*c^3) + (b*(2*a + I*b*
Log[1 - I*c*x^3])^2*Log[(1 + I*c*x^3)/2])/(24*c^3) + (b^3*x^6*Log[1 + I*c*x
^3])/(18*c) - (I/108)*b^3*x^9*Log[1 + I*c*x^3] - (11*b^3*(1 + I*c*x^3)*Log[
1 + I*c*x^3])/(36*c^3) + (b^3*(1 + I*c*x^3)^2*Log[1 + I*c*x^3])/(12*c^3) -
(b^3*(1 + I*c*x^3)^3*Log[1 + I*c*x^3])/(108*c^3) - (b^3*Log[(1 - I*c*x^3)/2
]*Log[1 + I*c*x^3])/(12*c^3) + (b^2*x^6*((2*I)*a - b*Log[1 - I*c*x^3])*Log[
1 + I*c*x^3])/(12*c) + (I/24)*b*x^9*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[1
+ I*c*x^3] - (b*(2*a + I*b*Log[1 - I*c*x^3])^2*Log[1 + I*c*x^3])/(24*c^3) -
 (b^3*Log[1 + I*c*x^3]^2)/(72*c^3) + (I/72)*b^3*x^9*Log[1 + I*c*x^3]^2 + (b
^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/(8*c^3) - (b^3*(1 + I*c*x^3)^2*Log[1 +
 I*c*x^3]^2)/(12*c^3) + (b^3*(1 + I*c*x^3)^3*Log[1 + I*c*x^3]^2)/(72*c^3) -
 (b^3*Log[(1 - I*c*x^3)/2]*Log[1 + I*c*x^3]^2)/(12*c^3) + (I/24)*b^2*x^9*((
2*I)*a - b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3]^2 - ((I/24)*b^2*(2*a + I*b*Lo
```

```
g[1 - I*c*x^3])*Log[1 + I*c*x^3]^2)/c^3 - (b^3*(1 + I*c*x^3)*Log[1 + I*c*x^
3]^3)/(24*c^3) + (b^3*(1 + I*c*x^3)^2*Log[1 + I*c*x^3]^3)/(24*c^3) - (b^3*(
1 + I*c*x^3)^3*Log[1 + I*c*x^3]^3)/(72*c^3) + (b^3*Log[I + c*x^3])/(24*c^3)
 - (b^3*PolyLog[2, (1 - I*c*x^3)/2])/(12*c^3) + (b^2*((2*I)*a - b*Log[1 - I
*c*x^3])*PolyLog[2, (1 - I*c*x^3)/2])/(12*c^3) + ((I/12)*b^2*(2*a + I*b*Log
[1 - I*c*x^3])*PolyLog[2, (1 - I*c*x^3)/2])/c^3 - (b^3*PolyLog[2, (1 + I*c*
x^3)/2])/(12*c^3) - (b^3*Log[1 + I*c*x^3]*PolyLog[2, (1 + I*c*x^3)/2])/(6*c
^3) + (b^3*PolyLog[3, (1 - I*c*x^3)/2])/(6*c^3) + (b^3*PolyLog[3, (1 + I*c*
x^3)/2])/(6*c^3)
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
```

```
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^8 \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 dx = \int \left(\frac{1}{8}x^8\left(2a+ib\log\left(1-icx^3\right)\right)^3+\frac{3}{8}ibx^8\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)-\frac{3}{8}ib^2x\right.$$

$$=\frac{1}{8}\int x^8\left(2a+ib\log\left(1-icx^3\right)\right)^3 dx+\frac{1}{8}(3ib)\int x^8\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right.$$

$$=\frac{1}{24}\operatorname{Subst}\left(\int x^2(2a+ib\log(1-icx))^3\,dx,x,x^3\right)+\frac{1}{8}(ib)\operatorname{Subst}\left(\int x^2(-2ia+b\log(1-ic\right.$$

$$=\frac{1}{24}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{24}ib^2x^9\left(2ia-b\log\left(1-icx^3\right)\right)\log^2\left(1+\right.$$

$$=\frac{1}{24}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{24}ib^2x^9\left(2ia-b\log\left(1-icx^3\right)\right)\log^2\left(1+\right.$$

$$=\frac{1}{24}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{24}ib^2x^9\left(2ia-b\log\left(1-icx^3\right)\right)\log^2\left(1+\right.$$

$$=-\frac{1}{72}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2-\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c^3}+\frac{i\left(1-icx^3\right)^2\left(2\right.}{}$$

$$=-\frac{1}{72}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2-\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{24c^3}-\frac{b\left(1-icx^3\right)\left(2a\right.}{}$$

$$=\frac{ab^2x^3}{3c^2}+\frac{ib^3x^3}{6c^2}-\frac{b^3\left(1-icx^3\right)^2}{32c^3}-\frac{b^3\left(1+icx^3\right)^2}{32c^3}+\frac{b^3\left(1+icx^3\right)^3}{324c^3}+\frac{ib^3\left(i+cx^3\right)^3}{324c^3}-\frac{1}{72}ibx$$

$$=\frac{ab^2x^3}{3c^2}-\frac{b^3\left(1-icx^3\right)^2}{32c^3}-\frac{b^3\left(1+icx^3\right)^2}{32c^3}+\frac{b^3\left(1+icx^3\right)^3}{324c^3}+\frac{ib^3\left(i+cx^3\right)^3}{324c^3}-\frac{b^3\left(1-icx^3\right)\mathrm{l}}{6c}$$

$$=\frac{ab^2x^3}{2c^2}+\frac{ib^3x^3}{18c^2}-\frac{b^3\left(1-icx^3\right)^2}{24c^3}-\frac{b^3\left(1+icx^3\right)^2}{24c^3}+\frac{b^3\left(1+icx^3\right)^3}{324c^3}+\frac{ib^3\left(i+cx^3\right)^3}{324c^3}-\frac{b^3(1}{}$$

$$=\frac{ab^2x^3}{2c^2}+\frac{7ib^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{1}{324}ib^3x^9-\frac{b^3\left(1-icx^3\right)^2}{48c^3}-\frac{b^3\left(1-icx^3\right)^3}{324c^3}-\frac{b^3\left(1+icx^3\right)^2}{24c^3}+$$

$$=\frac{ab^2x^3}{2c^2}+\frac{7ib^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{1}{324}ib^3x^9-\frac{b^3\left(1-icx^3\right)^2}{48c^3}-\frac{b^3\left(1-icx^3\right)^3}{324c^3}-\frac{b^3\left(1+icx^3\right)^2}{24c^3}+$$

Mathematica [A] time = 0.504979, size = 346, normalized size = 1.44

$$\frac{6ib^2\operatorname{PolyLog}\left(2,-e^{2i\tan^{-1}\left(cx^3\right)}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-3b^3\operatorname{PolyLog}\left(3,-e^{2i\tan^{-1}\left(cx^3\right)}\right)-3a^2bc^2x^6+3a^2b\log\left(c^2x^6+1\right)+6}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (6*a*b^2*c*x^3 - 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTan[c*x^3] +
6*b^3*c*x^3*ArcTan[c*x^3] - 6*a*b^2*c^2*x^6*ArcTan[c*x^3] + 6*a^2*b*c^3*x^9
*ArcTan[c*x^3] + (6*I)*a*b^2*ArcTan[c*x^3]^2 - 3*b^3*ArcTan[c*x^3]^2 - 3*b^
```

```
3*c^2*x^6*ArcTan[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTan[c*x^3]^2 + (2*I)*b^3*Arc
Tan[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTan[c*x^3]^3 - 12*a*b^2*ArcTan[c*x^3]*Log[1
 + E^((2*I)*ArcTan[c*x^3])] - 6*b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan
[c*x^3])] + 3*a^2*b*Log[1 + c^2*x^6] - 3*b^3*Log[1 + c^2*x^6] + (6*I)*b^2*(
a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] - 3*b^3*PolyLog[3
, -E^((2*I)*ArcTan[c*x^3])])/(18*c^3)
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] int(x^8*(a+b*arctan(c*x^3))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{72}b^3x^9\arctan\left(cx^3\right)^3-\frac{1}{96}b^3x^9\arctan\left(cx^3\right)\log\left(c^2x^6+1\right)^2+\frac{1}{9}a^3x^9+\frac{1}{6}\left(2x^9\arctan\left(cx^3\right)-\left(\frac{x^6}{c^2}-\frac{\log\left(c^2x^6+1\right)}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/72*b^3*x^9*arctan(c*x^3)^3 - 1/96*b^3*x^9*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 + 1/9*a^3*x^9 + 1/6*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^
4)*c)*a^2*b + integrate(1/32*(4*b^3*c^2*x^14*arctan(c*x^3)*log(c^2*x^6 + 1)
 + 28*(b^3*c^2*x^14 + b^3*x^8)*arctan(c*x^3)^3 + 4*(24*a*b^2*c^2*x^14 - b^3
*c*x^11 + 24*a*b^2*x^8)*arctan(c*x^3)^2 + (b^3*c*x^11 + 3*(b^3*c^2*x^14 + b
^3*x^8)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^8\arctan\left(cx^3\right)^3+3\,ab^2x^8\arctan\left(cx^3\right)^2+3\,a^2bx^8\arctan\left(cx^3\right)+a^3x^8,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^8*arctan(c*x^3)^3 + 3*a*b^2*x^8*arctan(c*x^3)^2 + 3*a^2*b*x^
8*arctan(c*x^3) + a^3*x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atan(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arctan \left(c x^{3}\right)+a\right)^{3} x^{8} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*x^8, x)
```

3.122 $\int x^5 \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 dx$

Optimal. Leaf size=147

$$-\frac{ib^3\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{2c^2}-\frac{b^2\log\left(\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)}{c^2}+\frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{6c^2}-\frac{ib\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{2c^2}+\ldots$$

```
[Out] ((-I/2)*b*(a + b*ArcTan[c*x^3])^2)/c^2 - (b*x^3*(a + b*ArcTan[c*x^3])^2)/(2
*c) + (a + b*ArcTan[c*x^3])^3/(6*c^2) + (x^6*(a + b*ArcTan[c*x^3])^3)/6 - (
b^2*(a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/c^2 - ((I/2)*b^3*PolyLog[2,
 1 - 2/(1 + I*c*x^3)])/c^2
```

Rubi [B] time = 4.73315, antiderivative size = 951, normalized size of antiderivative = 6.47, number of steps used = 155, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2395, 2394, 2393, 2391, 2375, 2317, 2430, 2425}

$$\frac{1}{16}ib^2\left(2ia-b\log\left(1-icx^3\right)\right)\log^2\left(icx^3+1\right)x^6+\frac{1}{16}ib\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(icx^3+1\right)x^6+\frac{b^2\left(2ia-b\log\left(1-\ldots\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] ((I/32)*b^2*(1 - I*c*x^3)^2*((2*I)*a - b*Log[1 - I*c*x^3]))/c^2 + ((I/32)*b
*(1 - I*c*x^3)^2*((2*I)*a - b*Log[1 - I*c*x^3])^2)/c^2 + (b^2*(1 - I*c*x^3)
^2*(2*a + I*b*Log[1 - I*c*x^3]))/(32*c^2) - ((I/8)*b*(1 - I*c*x^3)*(2*a + I
*b*Log[1 - I*c*x^3])^2)/c^2 + ((I/32)*b*(1 - I*c*x^3)^2*(2*a + I*b*Log[1 -
I*c*x^3])^2)/c^2 + ((1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^3)/(24*c^2)
- ((1 - I*c*x^3)^2*(2*a + I*b*Log[1 - I*c*x^3])^3)/(48*c^2) - ((I/4)*b^2*((
2*I)*a - b*Log[1 - I*c*x^3])*Log[(1 + I*c*x^3)/2])/c^2 + ((I/16)*b*((2*I)*a
 - b*Log[1 - I*c*x^3])^2*Log[(1 + I*c*x^3)/2])/c^2 + ((I/16)*b*(2*a + I*b*L
og[1 - I*c*x^3])^2*Log[(1 + I*c*x^3)/2])/c^2 - ((I/4)*b^3*Log[(1 - I*c*x^3)
/2]*Log[1 + I*c*x^3])/c^2 + (b^2*x^3*((2*I)*a - b*Log[1 - I*c*x^3])*Log[1 +
 I*c*x^3])/(4*c) + (I/16)*b*x^6*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[1 + I*
c*x^3] - ((I/16)*b*(2*a + I*b*Log[1 - I*c*x^3])^2*Log[1 + I*c*x^3])/c^2 - (
(I/8)*b^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/c^2 + (I/16)*b^2*x^6*((2*I)*a -
 b*Log[1 - I*c*x^3])*Log[1 + I*c*x^3]^2 - (b^2*(2*a + I*b*Log[1 - I*c*x^3])
*Log[1 + I*c*x^3]^2)/(16*c^2) + ((I/24)*b^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^
3)/c^2 - ((I/48)*b^3*(1 + I*c*x^3)^2*Log[1 + I*c*x^3]^3)/c^2 + ((I/4)*b^3*P
olyLog[2, (1 - I*c*x^3)/2])/c^2 - ((I/8)*b^2*((2*I)*a - b*Log[1 - I*c*x^3])
*PolyLog[2, (1 - I*c*x^3)/2])/c^2 - (b^2*(2*a + I*b*Log[1 - I*c*x^3])*PolyL
og[2, (1 - I*c*x^3)/2])/(8*c^2) - ((I/4)*b^3*PolyLog[2, (1 + I*c*x^3)/2])/c
^2
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
```

```
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
```

```
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^5 \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 dx = \int \left(\frac{1}{8}x^5 \left(2a+ib\log\left(1-icx^3\right)\right)^3 + \frac{3}{8}ibx^5 \left(-2ia+b\log\left(1-icx^3\right)\right)^2 \log\left(1+icx^3\right) - \frac{3}{8}ib^2x$$

$$= \frac{1}{8}\int x^5 \left(2a+ib\log\left(1-icx^3\right)\right)^3 dx + \frac{1}{8}(3ib)\int x^5 \left(-2ia+b\log\left(1-icx^3\right)\right)^2 \log\left(1+icx^3\right.$$

$$= \frac{1}{24}\text{Subst}\left(\int x(2a+ib\log(1-icx))^3\,dx, x, x^3\right) + \frac{1}{8}(ib)\,\text{Subst}\left(\int x(-2ia+b\log(1-icx)\right.$$

$$= \frac{1}{16}ibx^6 \left(2ia-b\log\left(1-icx^3\right)\right)^2 \log\left(1+icx^3\right) + \frac{1}{16}ib^2x^6 \left(2ia-b\log\left(1-icx^3\right)\right)\log^2\left(1+\right.$$

$$= \frac{1}{16}ibx^6 \left(2ia-b\log\left(1-icx^3\right)\right)^2 \log\left(1+icx^3\right) + \frac{1}{16}ib^2x^6 \left(2ia-b\log\left(1-icx^3\right)\right)\log^2\left(1+\right.$$

$$= \frac{1}{16}ibx^6 \left(2ia-b\log\left(1-icx^3\right)\right)^2 \log\left(1+icx^3\right) + \frac{1}{16}ib^2x^6 \left(2ia-b\log\left(1-icx^3\right)\right)\log^2\left(1+\right.$$

$$= \frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c^2} - \frac{\left(1-icx^3\right)^2\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48c^2} + \frac{ib\left(2ia-b\log\right.}{}$$

$$= \frac{ib\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{16c^2} - \frac{ib\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c^2} + \frac{ib\left(1-icx\right.}{}$$

$$= \frac{3iab^2x^3}{4c} + \frac{3b^3x^3}{8c} - \frac{ib^3\left(1-icx^3\right)^2}{64c^2} + \frac{ib^3\left(1+icx^3\right)^2}{64c^2} + \frac{ib\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right.}{16c^2}$$

$$= \frac{3iab^2x^3}{4c} + \frac{3b^3x^3}{4c} - \frac{ib^3\left(1-icx^3\right)^2}{64c^2} + \frac{ib^3\left(1+icx^3\right)^2}{64c^2} - \frac{3ib^3\left(1-icx^3\right)\log\left(1-icx^3\right)}{8c^2} + \frac{ib\left(\right.}{}$$

$$= \frac{iab^2x^3}{2c} + \frac{5b^3x^3}{8c} - \frac{3ib^3\left(1-icx^3\right)\log\left(1-icx^3\right)}{8c^2} + \frac{ib^2\left(1-icx^3\right)^2\left(2ia-b\log\left(1-icx^3\right)\right)}{32c^2} -$$

$$= \frac{iab^2x^3}{2c} + \frac{b^3x^3}{2c} - \frac{ib^3\left(1-icx^3\right)\log\left(1-icx^3\right)}{4c^2} + \frac{ib^2\left(1-icx^3\right)^2\left(2ia-b\log\left(1-icx^3\right)\right)}{32c^2} + \frac{i}{}$$

Mathematica [A] time = 0.165836, size = 170, normalized size = 1.16

$$\frac{3ib^3\text{PolyLog}\left(2,-e^{2i\tan^{-1}\left(cx^3\right)}\right) + a\left(acx^3\left(acx^3-3b\right)+3b^2\log\left(c^2x^6+1\right)\right) + 3b^2\tan^{-1}\left(cx^3\right)^2\left(ac^2x^6+a+b\left(-cx^3+i\right)\right)}{6c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^6 + b*(I - c*x^3))*ArcTan[c*x^3]^2 + b^3*(1 + c^2*x^6)*
ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a - 2*b*c*x^3 + a*c^2*x^6) - 2*b^2*
Log[1 + E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3*(-3*b + a*c*x^3) + 3*b^2*Log
[1 + c^2*x^6]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])])/(6*c^2)
```

Maple [C] time = 0.917, size = 867, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] 1/4*I*a^2*b*x^6*ln(1-I*c*x^3)+1/48*I*b^3*(c^2*x^6+1)/c^2*ln(1+I*c*x^3)^3+1/
2*a^2*b/c^2*arctan(c*x^3)+1/8*b^3/c*x^3*ln(1-I*c*x^3)^2-1/2*I/c*a*b^2*x^3*l
n(1-I*c*x^3)-1/48*I*b^3*x^6*ln(1-I*c*x^3)^3+1/6*a^3*x^6+(1/16*I*b^3*(c^2*x^
6+1)/c^2*ln(1-I*c*x^3)^2+1/4*b^2*x^3*(a*c*x^3-b)/c*ln(1-I*c*x^3)-1/4*I*b*(a
^2*c^2*x^6-2*a*b*c*x^3+b^2*ln(1-I*c*x^3)+I*ln(1-I*c*x^3)*a*b)/c^2)*ln(1+I*c
*x^3)-1/8*a*b^2*x^6*ln(1-I*c*x^3)^2+1/2/c^2*a*b^2*ln(c^2*x^6+1)-1/8/c^2*a*b
^2*ln(1-I*c*x^3)^2+1/8*I*b^3/c^2*ln(1-I*c*x^3)^2+3/4*I/c*b^2*Sum(2/3*(ln(x-
_alpha)*ln(1-I*c*x^3)+3*c*(-1/3*ln(x-_alpha)*(ln((RootOf(_Z^2+_Z*RootOf(c*_
Z^3-I)+RootOf(c*_Z^3-I)^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I
)+RootOf(c*_Z^3-I)^2,index=1))+ln((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c
*_Z^3-I)^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3
-I)^2,index=2))+ln(1/2*(2*(I/c)^(1/3)+x-_alpha)/(I/c)^(1/3)))/c-1/3*(dilog(
(RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1)-x+_alpha)/Root
Of(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1))+dilog((RootOf(_Z^2
+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*R
ootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2))+dilog(1/2*(2*(I/c)^(1/3)+x-_al
pha)/(I/c)^(1/3)))/c))*b/c,_alpha=RootOf(c*_Z^3-RootOf(_Z^2+1,index=1)))-1/
2*a^2*b/c*x^3-1/16*b^2*(I*x^6*b*ln(1-I*c*x^3)*c^2+2*a*c^2*x^6-2*b*c*x^3+I*b
*ln(1-I*c*x^3)+2*I*b+2*a)/c^2*ln(1+I*c*x^3)^2-1/48*I*b^3/c^2*ln(1-I*c*x^3)^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}ab^2x^6\arctan\left(cx^3\right)^2 + \frac{1}{6}a^3x^6 + \frac{1}{2}\left(x^6\arctan\left(cx^3\right) - c\left(\frac{x^3}{c^2} - \frac{\arctan\left(cx^3\right)}{c^3}\right)\right)a^2b - \frac{1}{2}\left(2c\left(\frac{x^3}{c^2} - \frac{\arctan\left(cx^3\right)}{c^3}\right)\text{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^3*x^6 + 1/2*(x^6*arctan(c*x^3) - c*(x
^3/c^2 - arctan(c*x^3)/c^3))*a^2*b - 1/2*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)
*arctan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*a*b^2 + 1/
192*(4*x^6*arctan(c*x^3)^3 - 3*x^6*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 192*i
ntegrate(1/64*(12*c^2*x^11*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^8*arctan
(c*x^3)^2 + 56*(c^2*x^11 + x^5)*arctan(c*x^3)^3 + 3*(c*x^8 + 2*(c^2*x^11 +
x^5)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^5\arctan\left(cx^3\right)^3 + 3\,ab^2x^5\arctan\left(cx^3\right)^2 + 3\,a^2bx^5\arctan\left(cx^3\right) + a^3x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctan(c*x^3)^3 + 3*a*b^2*x^5*arctan(c*x^3)^2 + 3*a^2*b*x^
5*arctan(c*x^3) + a^3*x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arctan\left(cx^3\right) + a\right)^3 x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*x^5, x)
```

3.123 $\int x^2 \left(a+b\tan^{-1}\left(cx^3\right)\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)}{c}+\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)}{2c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)^3+\frac{i\left(a+b\tan^{-1}\right.}{3c}$$

```
[Out] ((I/3)*(a + b*ArcTan[c*x^3])^3)/c + (x^3*(a + b*ArcTan[c*x^3])^3)/3 + (b*(a
 + b*ArcTan[c*x^3])^2*Log[2/(1 + I*c*x^3)])/c + (I*b^2*(a + b*ArcTan[c*x^3]
)*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c + (b^3*PolyLog[3, 1 - 2/(1 + I*c*x^3)]
)/(2*c)
```

Rubi [B] time = 2.70185, antiderivative size = 545, normalized size of antiderivative = 3.92, number of steps used = 82, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-icx^3\right)\right)\left(2ia-b\log\left(1-icx^3\right)\right)}{2c}-\frac{b^3\text{PolyLog}\left(3,\frac{1}{2}\left(1-icx^3\right)\right)}{2c}-\frac{b^3\text{PolyLog}\left(3,\frac{1}{2}\left(1+icx^3\right)\right)}{2c}+$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (b*(1 - I*c*x^3)*((2*I)*a - b*Log[1 - I*c*x^3])^2)/(8*c) + (b*(1 - I*c*x^3)
*(2*a + I*b*Log[1 - I*c*x^3])^2)/(8*c) + ((I/24)*(1 - I*c*x^3)*(2*a + I*b*L
og[1 - I*c*x^3])^3)/c + (b*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[(1 + I*c*x^
3)/2])/(4*c) - (b*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[1 + I*c*x^3])/(8*c)
+ (I/8)*b*x^3*((2*I)*a - b*Log[1 - I*c*x^3])^2*Log[1 + I*c*x^3] + (b^3*Log[
(1 - I*c*x^3)/2]*Log[1 + I*c*x^3]^2)/(4*c) + (b^2*((2*I)*a - b*Log[1 - I*c*
x^3])*Log[1 + I*c*x^3]^2)/(8*c) + (I/8)*b^2*x^3*((2*I)*a - b*Log[1 - I*c*x^
3])*Log[1 + I*c*x^3]^2 + (b^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^3)/(24*c) - (b
^2*((2*I)*a - b*Log[1 - I*c*x^3])*PolyLog[2, (1 - I*c*x^3)/2])/(2*c) + (b^3
*Log[1 + I*c*x^3]*PolyLog[2, (1 + I*c*x^3)/2])/(2*c) - (b^3*PolyLog[3, (1 -
 I*c*x^3)/2])/(2*c) - (b^3*PolyLog[3, (1 + I*c*x^3)/2])/(2*c)
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\,dx=\int\left(\frac{1}{8}x^2\left(2a+ib\log\left(1-icx^3\right)\right)^3+\frac{3}{8}ibx^2\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)-\frac{3}{8}ib^2x$$

$$=\frac{1}{8}\int x^2\left(2a+ib\log\left(1-icx^3\right)\right)^3\,dx+\frac{1}{8}(3ib)\int x^2\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3$$

$$=\frac{1}{24}\text{Subst}\left(\int(2a+ib\log(1-icx))^3\,dx,x,x^3\right)+\frac{1}{8}(ib)\,\text{Subst}\left(\int(-2ia+b\log(1-icx))^2\,\text{l}$$

$$=\frac{1}{8}ibx^3\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{8}ib^2x^3\left(2ia-b\log\left(1-icx^3\right)\right)\log^2\left(1+ic$$

$$=\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c}+\frac{1}{8}ibx^3\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{8}ib$$

$$=\frac{b\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c}+\frac{1}{8}ibx^3\left(2ia\right.$$

$$=-\frac{1}{2}ab^2x^3-\frac{1}{4}ib^3x^3+\frac{b\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-\right.\right.}{24c}$$

$$=-\frac{1}{2}ab^2x^3+\frac{b^3\left(1-icx^3\right)\log\left(1-icx^3\right)}{4c}+\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+\frac{b\left(1-icx\right.}{}$$

$$=\frac{1}{4}ib^3x^3+\frac{b^3\left(1-icx^3\right)\log\left(1-icx^3\right)}{4c}+\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+\frac{b\left(1-icx^3\right.}{}$$

$$=\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+\frac{b\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+\frac{i\left(1-icx^3\right.}{}$$

$$=\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+\frac{b\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+\frac{i\left(1-icx^3\right.}{}$$

Mathematica [A] time = 0.100396, size = 224, normalized size = 1.61

$$\frac{-6ib^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}\left(cx^3\right)}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+3b^3\text{PolyLog}\left(3,-e^{2i\tan^{-1}\left(cx^3\right)}\right)-3a^2b\log\left(c^2x^6+1\right)+6a^2bcx^3\,\text{ta}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (2*a^3*c*x^3 + 6*a^2*b*c*x^3*ArcTan[c*x^3] - (6*I)*a*b^2*ArcTan[c*x^3]^2 +
6*a*b^2*c*x^3*ArcTan[c*x^3]^2 - (2*I)*b^3*ArcTan[c*x^3]^3 + 2*b^3*c*x^3*Arc
Tan[c*x^3]^3 + 12*a*b^2*ArcTan[c*x^3]*Log[1 + E^((2*I)*ArcTan[c*x^3])] + 6*
b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])] - 3*a^2*b*Log[1 + c^2*
x^6] - (6*I)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])]
 + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])])/(6*c)
```

Maple [B] time = 0.129, size = 303, normalized size = 2.2

$$\frac{a^3x^3}{3} - \frac{\frac{i}{3}b^3\left(\arctan\left(cx^3\right)\right)^3}{c} + \frac{b^3\left(\arctan\left(cx^3\right)\right)^3x^3}{3} + \frac{b^3\left(\arctan\left(cx^3\right)\right)^2}{c}\ln\left(\frac{\left(1+icx^3\right)^2}{c^2x^6+1}+1\right) - \frac{ib^3\arctan\left(cx^3\right)}{c}\text{pol}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] 1/3*a^3*x^3-1/3*I/c*b^3*arctan(c*x^3)^3+1/3*b^3*arctan(c*x^3)^3*x^3+1/c*b^3
*arctan(c*x^3)^2*ln((1+I*c*x^3)^2/(c^2*x^6+1)+1)-I/c*b^3*arctan(c*x^3)*poly
log(2,-(1+I*c*x^3)^2/(c^2*x^6+1))+1/2/c*b^3*polylog(3,-(1+I*c*x^3)^2/(c^2*x
^6+1))-I/c*arctan(c*x^3)^2*a*b^2+arctan(c*x^3)^2*x^3*a*b^2+2/c*arctan(c*x^3
)*ln((1+I*c*x^3)^2/(c^2*x^6+1)+1)*a*b^2-I/c*polylog(2,-(1+I*c*x^3)^2/(c^2*x
^6+1))*a*b^2+a^2*b*x^3*arctan(c*x^3)-1/2/c*a^2*b*ln(c^2*x^6+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}b^3x^3\arctan\left(cx^3\right)^3 - \frac{1}{32}b^3x^3\arctan\left(cx^3\right)\log\left(c^2x^6+1\right)^2 + \frac{1}{3}a^3x^3 + \frac{7\,b^3\arctan\left(cx^3\right)^4}{96\,c} + 28\,b^3c^2\int\frac{x^8\arctan\left(\right.}{32\left(c^2x^6+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan(c*x^3)^3 - 1/32*b^3*x^3*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 + 1/3*a^3*x^3 + 7/96*b^3*arctan(c*x^3)^4/c + 28*b^3*c^2*integrate(1/32*x^
8*arctan(c*x^3)^3/(c^2*x^6 + 1), x) + 3*b^3*c^2*integrate(1/32*x^8*arctan(c
*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^
8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 12*b^3*c^2*integrate(1/32*x^8*arctan(
c*x^3)*log(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 1/3*a*b^2*arctan(c*x^3)^3/c - 1
2*b^3*c*integrate(1/32*x^5*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 3*b^3*c*inte
grate(1/32*x^5*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 3*b^3*integrate(1/32*
x^2*arctan(c*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 1/2*(2*c*x^3*arcta
n(c*x^3) - log(c^2*x^6 + 1))*a^2*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^2\arctan\left(cx^3\right)^3 + 3\,ab^2x^2\arctan\left(cx^3\right)^2 + 3\,a^2bx^2\arctan\left(cx^3\right) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctan(c*x^3)^3 + 3*a*b^2*x^2*arctan(c*x^3)^2 + 3*a^2*b*x^
2*arctan(c*x^3) + a^3*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arctan\left(cx^3\right) + a\right)^3 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*x^2, x)
```

3.124 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x}\,dx$

Optimal. Leaf size=232

$$-\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}ib\text{PolyI}$$

```
[Out] (2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/2)*b*(a + b
*ArcTan[c*x^3])^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/2)*b*(a + b*ArcTan[c
*x^3])^2*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*(a + b*ArcTan[c*x^3])*Poly
Log[3, 1 - 2/(1 + I*c*x^3)])/2 + (b^2*(a + b*ArcTan[c*x^3])*PolyLog[3, -1 +
 2/(1 + I*c*x^3)])/2 + (I/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^3)] - (I/4)*b^
3*PolyLog[4, -1 + 2/(1 + I*c*x^3)]
```

Rubi [A] time = 0.523477, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5031, 4850, 4988, 4884, 4994, 4998, 6610}

$$-\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}ib\text{PolyI}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/2)*b*(a + b
*ArcTan[c*x^3])^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/2)*b*(a + b*ArcTan[c
*x^3])^2*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*(a + b*ArcTan[c*x^3])*Poly
Log[3, 1 - 2/(1 + I*c*x^3)])/2 + (b^2*(a + b*ArcTan[c*x^3])*PolyLog[3, -1 +
 2/(1 + I*c*x^3)])/2 + (I/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^3)] - (I/4)*b^
3*PolyLog[4, -1 + 2/(1 + I*c*x^3)]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx, x, x^3\right)$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) - (2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(\right.}{1+c^2x^2}\right.$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) + (bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right.}{1+c^2x^2}\right.$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) - \frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx^3}\right) -$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) - \frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx^3}\right) -$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) - \frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+icx^3}\right) -$$

Mathematica [A] time = 0.194612, size = 248, normalized size = 1.07

$$\frac{1}{4}ib\left(2\text{PolyLog}\left(2,\frac{cx^3+i}{-cx^3+i}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)^2 - 2\text{PolyLog}\left(2,\frac{cx^3+i}{cx^3-i}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)^2 + b\left(-2i\text{PolyLog}\left(3,\frac{cx}{-c}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 + (2*I)/(-I + c*x^3)])/3 + (I/4)*b*(2*
(a + b*ArcTan[c*x^3])^2*PolyLog[2, (I + c*x^3)/(I - c*x^3)] - 2*(a + b*ArcT
an[c*x^3])^2*PolyLog[2, (I + c*x^3)/(-I + c*x^3)] + b*((-2*I)*(a + b*ArcTan
[c*x^3])*PolyLog[3, (I + c*x^3)/(I - c*x^3)] + (2*I)*(a + b*ArcTan[c*x^3])*
PolyLog[3, (I + c*x^3)/(-I + c*x^3)] + b*(-PolyLog[4, (I + c*x^3)/(I - c*x^
3)] + PolyLog[4, (I + c*x^3)/(-I + c*x^3)])))
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(c\,x^{3}\right)\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^3/x,x)
```

```
[Out] int((a+b*arctan(c*x^3))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\log\left(x\right)+\frac{1}{32}\int \frac{28\,b^{3}\arctan\left(cx^{3}\right)^{3}+3\,b^{3}\arctan\left(cx^{3}\right)\log\left(c^{2}x^{6}+1\right)^{2}+96\,ab^{2}\arctan\left(cx^{3}\right)^{2}+96\,a^{2}b\arctan\left(cx\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^3)^3 + 3*b^3*arctan(c*x^3)*l
og(c^2*x^6 + 1)^2 + 96*a*b^2*arctan(c*x^3)^2 + 96*a^2*b*arctan(c*x^3))/x, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{3}\arctan\left(cx^{3}\right)^{3}+3\,ab^{2}\arctan\left(cx^{3}\right)^{2}+3\,a^{2}b\arctan\left(cx^{3}\right)+a^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arctan\left(cx^3\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3/x, x)
```

3.125 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=133

$$-ib^2c\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{1}{2}b^3c\text{PolyLog}\left(3,-1+\frac{2}{1-icx^3}\right)-\frac{1}{3}ic\left(a+b\tan^{-1}\left(cx^3\right)\right)^3-$$

```
[Out] (-I/3)*c*(a + b*ArcTan[c*x^3])^3 - (a + b*ArcTan[c*x^3])^3/(3*x^3) + b*c*(a
 + b*ArcTan[c*x^3])^2*Log[2 - 2/(1 - I*c*x^3)] - I*b^2*c*(a + b*ArcTan[c*x^
3])*PolyLog[2, -1 + 2/(1 - I*c*x^3)] + (b^3*c*PolyLog[3, -1 + 2/(1 - I*c*x^
3)])/2
```

Rubi [F] time = 0.860001, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

```
[Out] (b*c*Log[I*c*x^3]*(2*a + I*b*Log[1 - I*c*x^3])^2)/8 - ((1 - I*c*x^3)*(2*a +
 I*b*Log[1 - I*c*x^3])^3)/(24*x^3) - (b^3*c*Log[(-I)*c*x^3]*Log[1 + I*c*x^3
]^2)/8 - ((I/24)*b^3*(1 + I*c*x^3)*Log[1 + I*c*x^3]^3)/x^3 + (I/4)*b^2*c*(2
*a + I*b*Log[1 - I*c*x^3])*PolyLog[2, 1 - I*c*x^3] - (b^3*c*Log[1 + I*c*x^3
]*PolyLog[2, 1 + I*c*x^3])/4 + (b^3*c*PolyLog[3, 1 - I*c*x^3])/4 + (b^3*c*P
olyLog[3, 1 + I*c*x^3])/4 + (I/8)*b*Defer[Subst][Defer[Int][(((-2*I)*a + b*
Log[1 - I*c*x])^2*Log[1 + I*c*x])/x^2, x], x, x^3] - (I/8)*b^2*Defer[Subst]
[Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x])*Log[1 + I*c*x]^2)/x^2, x], x, x^
3]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^4}dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{8x^4} + \frac{3ib\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)}{8x^4} - \frac{3ib^2\left(-2ia+b\right.}{}\right.$$
$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{x^4}dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)}{x^4}dx - \frac{1}{8}$$
$$= \frac{1}{24}\operatorname{Subst}\left(\int \frac{(2a+ib\log(1-icx))^3}{x^2}dx, x, x^3\right) + \frac{1}{8}(ib)\operatorname{Subst}\left(\int \frac{(-2ia+b\log(1-icx))^2\log}{x^2}\right.$$
$$= -\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3} - \frac{ib^3\left(1+icx^3\right)\log^3\left(1+icx^3\right)}{24x^3} + \frac{1}{8}(ib)\operatorname{Subst}\left(\int \frac{(-2i}{}\right.$$
$$= \frac{1}{8}bc\log\left(icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2 - \frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3} - \frac{1}{8}b^3c\log\left(-ic\right.$$
$$= \frac{1}{8}bc\log\left(icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2 - \frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3} - \frac{1}{8}b^3c\log\left(-ic\right.$$
$$= \frac{1}{8}bc\log\left(icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2 - \frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3} - \frac{1}{8}b^3c\log\left(-ic\right.$$
$$= \frac{1}{8}bc\log\left(icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2 - \frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24x^3} - \frac{1}{8}b^3c\log\left(-ic\right.$$

Mathematica [A] time = 0.397328, size = 240, normalized size = 1.8

$$ab^2c\left(\tan^{-1}\left(cx^3\right)\left(\left(-\frac{1}{cx^3}-i\right)\tan^{-1}\left(cx^3\right)+2\log\left(1-e^{2i\tan^{-1}\left(cx^3\right)}\right)\right)-i\operatorname{PolyLog}\left(2,e^{2i\tan^{-1}\left(cx^3\right)}\right)\right)+\frac{1}{3}b^3c\left(3i\tan^{-1}\left(cx^3\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (a^2*b*ArcTan[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Log[
1 + c^2*x^6])/2 + a*b^2*c*(ArcTan[c*x^3]*((-I - 1/(c*x^3))*ArcTan[c*x^3] +
2*Log[1 - E^((2*I)*ArcTan[c*x^3])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^3])]
) + (b^3*c*((-I/8)*Pi^3 + I*ArcTan[c*x^3]^3 - ArcTan[c*x^3]^3/(c*x^3) + 3*A
rcTan[c*x^3]^2*Log[1 - E^((-2*I)*ArcTan[c*x^3])] + (3*I)*ArcTan[c*x^3]*Poly
Log[2, E^((-2*I)*ArcTan[c*x^3])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x^3])])
/2))/3
```

Maple [F] time = 0.381, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^4}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^3/x^4,x)
```

```
[Out] int((a+b*arctan(c*x^3))^3/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^6+1\right)-\log\left(x^6\right)\right)+\frac{2\arctan\left(cx^3\right)}{x^3}\right)a^2b-\frac{a^3}{3x^3}-\frac{\frac{15}{2}b^3\arctan\left(cx^3\right)^3-\frac{21}{8}b^3\arctan\left(cx^3\right)\log\left(c^2x^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a^2*b - 1/3*a^
3/x^3 - 1/96*(4*b^3*arctan(c*x^3)^3 - 3*b^3*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 - 96*x^3*integrate(-1/32*(12*b^3*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) -
 28*(b^3*c^2*x^6 + b^3)*arctan(c*x^3)^3 - 12*(8*a*b^2*c^2*x^6 + b^3*c*x^3 +
 8*a*b^2)*arctan(c*x^3)^2 + 3*(b^3*c*x^3 - (b^3*c^2*x^6 + b^3)*arctan(c*x^3
))*log(c^2*x^6 + 1)^2)/(c^2*x^10 + x^4), x))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\arctan\left(cx^3\right)^3+3\,ab^2\arctan\left(cx^3\right)^2+3\,a^2b\arctan\left(cx^3\right)+a^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**3/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\arctan\left(cx^3\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3/x^4, x)
```

3.126 $\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx$

Optimal. Leaf size=146

$$-\frac{1}{2}ib^3c^2\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)+b^2c^2\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}ibc^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^2-\frac{1}{6}c^2\left(a+\right.$$

```
[Out] (-I/2)*b*c^2*(a + b*ArcTan[c*x^3])^2 - (b*c*(a + b*ArcTan[c*x^3])^2)/(2*x^3
) - (c^2*(a + b*ArcTan[c*x^3])^3)/6 - (a + b*ArcTan[c*x^3])^3/(6*x^6) + b^2
*c^2*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)] - (I/2)*b^3*c^2*PolyLog
[2, -1 + 2/(1 - I*c*x^3)]
```

Rubi [F] time = 1.66992, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

```
[Out] (3*a*b^2*c^2*Log[x])/4 - (b*c*(1 - I*c*x^3)*(2*a + I*b*Log[1 - I*c*x^3])^2)
/(16*x^3) + (I/16)*b*c^2*Log[I*c*x^3]*(2*a + I*b*Log[1 - I*c*x^3])^2 - (c^2
*(2*a + I*b*Log[1 - I*c*x^3])^3)/48 - (2*a + I*b*Log[1 - I*c*x^3])^3/(48*x^
6) + (b^3*c*(1 + I*c*x^3)*Log[1 + I*c*x^3]^2)/(16*x^3) + (I/16)*b^3*c^2*Log
[(-I)*c*x^3]*Log[1 + I*c*x^3]^2 - (I/48)*b^3*c^2*Log[1 + I*c*x^3]^3 - ((I/4
8)*b^3*Log[1 + I*c*x^3]^3)/x^6 + (I/8)*b^3*c^2*PolyLog[2, (-I)*c*x^3] - (I/
8)*b^3*c^2*PolyLog[2, I*c*x^3] - (b^2*c^2*(2*a + I*b*Log[1 - I*c*x^3])*Poly
Log[2, 1 - I*c*x^3])/8 + (I/8)*b^3*c^2*Log[1 + I*c*x^3]*PolyLog[2, 1 + I*c*
x^3] + (I/8)*b^3*c^2*PolyLog[3, 1 - I*c*x^3] - (I/8)*b^3*c^2*PolyLog[3, 1 +
 I*c*x^3] + (I/8)*b*Defer[Subst][Defer[Int][(((-2*I)*a + b*Log[1 - I*c*x])^
2*Log[1 + I*c*x])/x^3, x], x, x^3] - (I/8)*b^2*Defer[Subst][Defer[Int][(((-
2*I)*a + b*Log[1 - I*c*x])*Log[1 + I*c*x]^2)/x^3, x], x, x^3]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{8x^7} + \frac{3ib\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)}{8x^7} - \frac{3ib^2\left(-2ia\right.}{}\right.$$
$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{x^7}\,dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)}{x^7}\,dx$$
$$= \frac{1}{24}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^3}{x^3}\,dx, x, x^3\right) + \frac{1}{8}(ib)\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx))}{x^3}\right.$$
$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} - \frac{ib^3\log^3\left(1+icx^3\right)}{48x^6} + \frac{1}{8}(ib)\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-icx)}{x^3}\right.$$
$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} - \frac{ib^3\log^3\left(1+icx^3\right)}{48x^6} + \frac{1}{16}(ib)\,\text{Subst}\left(\int \frac{(2a+ib\log(x))^2}{x\left(-\frac{i}{c}+\frac{ix}{c}\right)^2}\,dx,\right.$$
$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} - \frac{ib^3\log^3\left(1+icx^3\right)}{48x^6} + \frac{1}{16}(ib)\,\text{Subst}\left(\int \frac{(2a+ib\log(x))^2}{\left(-\frac{i}{c}+\frac{ix}{c}\right)^2}\,dx,\right.$$
$$= -\frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} - \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} + \frac{b^3c\left(1+icx^3\right)\log^2\left(\right.}{16x^3}$$
$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} + \frac{1}{16}ibc^2\log\left(icx^3\right)\left(2a+ib\log\left(1\right.\right.$$
$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} + \frac{1}{16}ibc^2\log\left(icx^3\right)\left(2a+ib\log\left(1\right.\right.$$
$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} + \frac{1}{16}ibc^2\log\left(icx^3\right)\left(2a+ib\log\left(1\right.\right.$$

Mathematica [A] time = 0.329347, size = 196, normalized size = 1.34

$$-\frac{3ib^3c^2x^6\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(cx^3\right)}\right) + a\left(a\left(a+3bcx^3\right) - 6b^2c^2x^6\log\left(\frac{cx^3}{\sqrt{c^2x^6+1}}\right)\right) + 3b^2\tan^{-1}\left(cx^3\right)^2\left(ac^2x^6+a+bcx^3\left(\right.\right.}{6x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

```
[Out] -(3*b^2*(a + a*c^2*x^6 + b*c*x^3*(1 + I*c*x^3))*ArcTan[c*x^3]^2 + b^3*(1 +
c^2*x^6)*ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a + 2*b*c*x^3 + a*c^2*x^6)
 - 2*b^2*c^2*x^6*Log[1 - E^((2*I)*ArcTan[c*x^3])]) + a*(a*(a + 3*b*c*x^3) -
 6*b^2*c^2*x^6*Log[(c*x^3)/Sqrt[1 + c^2*x^6]]) + (3*I)*b^3*c^2*x^6*PolyLog[
2, E^((2*I)*ArcTan[c*x^3])])/(6*x^6)
```

Maple [F] time = 0.557, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^3/x^7,x)
```

```
[Out] int((a+b*arctan(c*x^3))^3/x^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c+\frac{\arctan\left(cx^3\right)}{x^6}\right)a^2b+\frac{1}{2}\left(\left(\arctan\left(cx^3\right)^2-\log\left(c^2x^6+1\right)+6\log\left(x\right)\right)c^2-2\left(c\arctan\left(cx^3\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a^2*b + 1/2*((arctan
(c*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*
c*arctan(c*x^3))*a*b^2 - 1/2*a*b^2*arctan(c*x^3)^2/x^6 + 1/192*(192*x^6*int
egrate(-1/64*(12*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^3*arctan(c
*x^3)^2 - 56*(c^2*x^6 + 1)*arctan(c*x^3)^3 + 3*(c*x^3 - 2*(c^2*x^6 + 1)*arc
tan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^13 + x^7), x) - 4*arctan(c*x^3)^3 +
3*arctan(c*x^3)*log(c^2*x^6 + 1)^2)*b^3/x^6 - 1/6*a^3/x^6
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\arctan\left(cx^3\right)^3+3\,ab^2\arctan\left(cx^3\right)^2+3\,a^2b\arctan\left(cx^3\right)+a^3}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**3/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(cx^3\right)+a\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3/x^7, x)
```

3.127 $\int (dx)^m \left(a + b\tan^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a + b\tan^{-1}\left(cx^3\right)\right)^3, x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

Rubi [A] time = 0.0236325, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a + b\tan^{-1}\left(cx^3\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a + b\tan^{-1}\left(cx^3\right)\right)^3 \, dx = \int (dx)^m \left(a + b\tan^{-1}\left(cx^3\right)\right)^3 \, dx$$

Mathematica [A] time = 1.79824, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b\tan^{-1}\left(cx^3\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

Maple [A] time = 0.261, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b\arctan\left(cx^3\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \arctan\left(cx^3\right)^3 + 3\,ab^2 \arctan\left(cx^3\right)^2 + 3\,a^2b \arctan\left(cx^3\right) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arctan\left(cx^3\right) + a\right)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*(d*x)^m, x)
```

3.128 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

Rubi [A] time = 0.0240861, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]

[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 \, dx=\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 \, dx$$

Mathematica [A] time = 1.19173, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]

[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

Maple [A] time = 0.23, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^3))^2,x)

[Out] int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\arctan\left(cx^3\right)^2 + 2\,ab\arctan\left(cx^3\right) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(cx^3\right) + a\right)^2 (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2*(d*x)^m, x)
```

3.129 $\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=75

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{6},\frac{m+10}{6},-c^2x^6\right)}{d^4(m+1)(m+4)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*Hy
pergeometric2F1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)])/(d^4*(1 + m)*(4 + m)
)
```

Rubi [A] time = 0.0424709, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5033, 16, 364}

$$\frac{(dx)^{m+1}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{6};\frac{m+10}{6};-c^2x^6\right)}{d^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*Hy
pergeometric2F1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)])/(d^4*(1 + m)*(4 + m)
)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right) dx = \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(1+m)} - \frac{(3bc)\int \frac{x^2(dx)^{1+m}}{1+c^2x^6} dx}{d(1+m)}$$
$$= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(1+m)} - \frac{(3bc)\int \frac{(dx)^{3+m}}{1+c^2x^6} dx}{d^3(1+m)}$$
$$= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(1+m)} - \frac{3bc(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{6};\frac{10+m}{6};-c^2x^6\right)}{d^4(1+m)(4+m)}$$

Mathematica [A] time = 0.0611742, size = 65, normalized size = 0.87

$$-\frac{x(dx)^m\left(3bcx^3\text{Hypergeometric2F1}\left(1,\frac{m+4}{6},\frac{m+10}{6},-c^2x^6\right)-(m+4)\left(a+b\tan^{-1}\left(cx^3\right)\right)\right)}{(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -((x*(d*x)^m*(-((4 + m)*(a + b*ArcTan[c*x^3])) + 3*b*c*x^3*Hypergeometric2F
1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)]))/((1 + m)*(4 + m)))
```

Maple [F] time = 0.239, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\arctan\left(cx^3\right)+a\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \arctan\left(cx^3\right) + a\right)(dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

3.130 $\int \frac{(dx)^m}{a+b\tan^{-1}(cx^3)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\tan^{-1}\left(cx^3\right)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

Rubi [A] time = 0.0266075, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^3\right)}\,dx=\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^3\right)}\,dx$$

Mathematica [A] time = 0.314289, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

Maple [A] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\arctan\left(cx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^3)),x)
```

```
[Out] int((d*x)^m/(a+b*arctan(c*x^3)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \arctan(cx^3) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b \arctan(cx^3) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \arctan(cx^3) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

3.131 $\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

Rubi [A] time = 0.0258208, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx$$

Mathematica [A] time = 0.331078, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

Maple [A] time = 0.205, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\arctan\left(cx^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2d^mx^6+d^m\right)x^m-\left(b^2cx^2\arctan\left(cx^3\right)+abcx^2\right)\int\frac{\left(\left(c^2d^mm+4\,c^2d^m\right)x^6+d^mm-2\,d^m\right)x^m}{b^2cx^3\arctan\left(cx^3\right)+abcx^3}\,dx}{3\left(b^2cx^2\arctan\left(cx^3\right)+abcx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] -1/3*((c^2*d^m*x^6 + d^m)*x^m - 3*(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)*int
egrate(1/3*((c^2*d^m*m + 4*c^2*d^m)*x^6 + d^m*m - 2*d^m)*x^m/(b^2*c*x^3*arc
tan(c*x^3) + a*b*c*x^3), x))/(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{b^2\arctan\left(cx^3\right)^2+2\,ab\arctan\left(cx^3\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^m}{\left(b\arctan\left(cx^3\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a)^2, x)
```

3.132 $\int x^3 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right) dx$

Optimal. Leaf size=50

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^3x+\frac{1}{4}bc^4\tan^{-1}\left(\frac{x}{c}\right)+\frac{1}{12}bcx^3$$

```
[Out] -(b*c^3*x)/4 + (b*c*x^3)/12 + (x^4*(a + b*ArcTan[c/x]))/4 + (b*c^4*ArcTan[x
/c])/4
```

Rubi [A] time = 0.0309079, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 263, 302, 203}

$$\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^3x+\frac{1}{4}bc^4\tan^{-1}\left(\frac{x}{c}\right)+\frac{1}{12}bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c/x]),x]
```

```
[Out] -(b*c^3*x)/4 + (b*c*x^3)/12 + (x^4*(a + b*ArcTan[c/x]))/4 + (b*c^4*ArcTan[x
/c])/4
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)dx=\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^2}{1+\frac{c^2}{x^2}}\,dx$$
$$=\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^4}{c^2+x^2}\,dx$$
$$=\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\left(-c^2+x^2+\frac{c^4}{c^2+x^2}\right)dx$$
$$=-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}\left(bc^5\right)\int\frac{1}{c^2+x^2}\,dx$$
$$=-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\tan^{-1}\left(\frac{x}{c}\right)$$

Mathematica [A] time = 0.0103591, size = 55, normalized size = 1.1

$$\frac{ax^4}{4}-\frac{1}{4}bc^3x-\frac{1}{4}bc^4\tan^{-1}\left(\frac{c}{x}\right)+\frac{1}{12}bcx^3+\frac{1}{4}bx^4\tan^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c/x]),x]
```

```
[Out] -(b*c^3*x)/4 + (b*c*x^3)/12 + (a*x^4)/4 - (b*c^4*ArcTan[c/x])/4 + (b*x^4*Ar
cTan[c/x])/4
```

Maple [A] time = 0.032, size = 46, normalized size = 0.9

$$\frac{x^4a}{4}+\frac{bx^4}{4}\arctan\left(\frac{c}{x}\right)+\frac{bc^4}{4}\arctan\left(\frac{x}{c}\right)+\frac{bcx^3}{12}-\frac{bc^3x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c/x)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctan(c/x)+1/4*b*c^4*arctan(x/c)+1/12*b*c*x^3-1/4*b*c^
3*x
```

Maxima [A] time = 1.47929, size = 61, normalized size = 1.22

$$\frac{1}{4}ax^4+\frac{1}{12}\left(3x^4\arctan\left(\frac{c}{x}\right)+\left(3c^3\arctan\left(\frac{x}{c}\right)-3c^2x+x^3\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arctan(c/x) + (3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c
)*b
```

Fricas [A] time = 2.20965, size = 101, normalized size = 2.02

$$-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}ax^4-\frac{1}{4}\left(bc^4-bx^4\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="fricas")
```

```
[Out] -1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/4*(b*c^4 - b*x^4)*arctan(c/x)
```

Sympy [A] time = 0.827307, size = 46, normalized size = 0.92

$$\frac{ax^4}{4} - \frac{bc^4 \operatorname{atan}\left(\frac{c}{x}\right)}{4} - \frac{bc^3x}{4} + \frac{bcx^3}{12} + \frac{bx^4 \operatorname{atan}\left(\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c/x)),x)
```

```
[Out] a*x**4/4 - b*c**4*atan(c/x)/4 - b*c**3*x/4 + b*c*x**3/12 + b*x**4*atan(c/x)
/4
```

Giac [A] time = 1.16115, size = 81, normalized size = 1.62

$$-\frac{1}{8}\, bc^4 i \log\left(ix + c\right) + \frac{1}{8}\, bc^4 i \log\left(-ix + c\right) + \frac{1}{4}\, bx^4 \arctan\left(\frac{c}{x}\right) - \frac{1}{4}\, bc^3 x + \frac{1}{12}\, bcx^3 + \frac{1}{4}\, ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="giac")
```

```
[Out] -1/8*b*c^4*i*log(i*x + c) + 1/8*b*c^4*i*log(-i*x + c) + 1/4*b*x^4*arctan(c/
x) - 1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4
```

3.133 $\int x^2 \left(a + b\tan^{-1}\left(\frac{c}{x}\right)\right) dx$

Optimal. Leaf size=43

$$\frac{1}{3}x^3\left(a + b\tan^{-1}\left(\frac{c}{x}\right)\right) - \frac{1}{6}bc^3\log\left(c^2 + x^2\right) + \frac{1}{6}bcx^2$$

```
[Out] (b*c*x^2)/6 + (x^3*(a + b*ArcTan[c/x]))/3 - (b*c^3*Log[c^2 + x^2])/6
```

Rubi [A] time = 0.029785, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 263, 266, 43}

$$\frac{1}{3}x^3\left(a + b\tan^{-1}\left(\frac{c}{x}\right)\right) - \frac{1}{6}bc^3\log\left(c^2 + x^2\right) + \frac{1}{6}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (x^3*(a + b*ArcTan[c/x]))/3 - (b*c^3*Log[c^2 + x^2])/6
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x}{1+\frac{c^2}{x^2}}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x^3}{c^2+x^2}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int\frac{x}{c^2+x}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int\left(1-\frac{c^2}{c^2+x}\right)dx,x,x^2\right)\\ &= \frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0073186, size = 48, normalized size = 1.12

$$\frac{ax^3}{3}-\frac{1}{6}bc^3\log\left(c^2+x^2\right)+\frac{1}{6}bcx^2+\frac{1}{3}bx^3\tan^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTan[c/x])/3 - (b*c^3*Log[c^2 + x^2])/6
```

Maple [A] time = 0.033, size = 55, normalized size = 1.3

$$\frac{x^3a}{3}+\frac{bx^3}{3}\arctan\left(\frac{c}{x}\right)-\frac{c^3b}{6}\ln\left(1+\frac{c^2}{x^2}\right)+\frac{bcx^2}{6}+\frac{c^3b}{3}\ln\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c/x)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctan(c/x)-1/6*c^3*b*ln(1+c^2/x^2)+1/6*b*c*x^2+1/3*c^3
*b*ln(c/x)
```

Maxima [A] time = 0.988883, size = 58, normalized size = 1.35

$$\frac{1}{3}ax^3+\frac{1}{6}\left(2x^3\arctan\left(\frac{c}{x}\right)-\left(c^2\log\left(c^2+x^2\right)-x^2\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*b
```

Fricas [A] time = 2.1818, size = 103, normalized size = 2.4

$$\frac{1}{3}bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)+\frac{1}{6}bcx^2+\frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3*arctan(c/x) - 1/6*b*c^3*log(c^2 + x^2) + 1/6*b*c*x^2 + 1/3*a*x^3
```

Sympy [A] time = 0.5446, size = 41, normalized size = 0.95

$$\frac{ax^3}{3} - \frac{bc^3\log\left(c^2+x^2\right)}{6} + \frac{bcx^2}{6} + \frac{bx^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c/x)),x)
```

```
[Out] a*x**3/3 - b*c**3*log(c**2 + x**2)/6 + b*c*x**2/6 + b*x**3*atan(c/x)/3
```

Giac [A] time = 1.12529, size = 54, normalized size = 1.26

$$\frac{1}{3}\,bx^3\arctan\left(\frac{c}{x}\right) - \frac{1}{6}\,bc^3\log\left(c^2+x^2\right) + \frac{1}{6}\,bcx^2 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*arctan(c/x) - 1/6*b*c^3*log(c^2 + x^2) + 1/6*b*c*x^2 + 1/3*a*x^3
```

3.134 $\int x \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tan^{-1}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

```
[Out] (b*c*x)/2 + (x^2*(a + b*ArcTan[c/x]))/2 - (b*c^2*ArcTan[x/c])/2
```

Rubi [A] time = 0.0173373, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5033, 193, 321, 203}

$$\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tan^{-1}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x)/2 + (x^2*(a + b*ArcTan[c/x]))/2 - (b*c^2*ArcTan[x/c])/2
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)dx=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{1}{1+\frac{c^2}{x^2}}dx$$
$$=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{x^2}{c^2+x^2}dx$$
$$=\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}\left(bc^3\right)\int\frac{1}{c^2+x^2}dx$$
$$=\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tan^{-1}\left(\frac{x}{c}\right)$$

Mathematica [A] time = 0.0080284, size = 44, normalized size = 1.13

$$\frac{ax^2}{2}+\frac{1}{2}bc^2\tan^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bx^2\tan^{-1}\left(\frac{c}{x}\right)+\frac{bcx}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x)/2 + (a*x^2)/2 + (b*c^2*ArcTan[c/x])/2 + (b*x^2*ArcTan[c/x])/2
```

Maple [A] time = 0.03, size = 37, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^2}{2}\arctan\left(\frac{c}{x}\right)-\frac{bc^2}{2}\arctan\left(\frac{x}{c}\right)+\frac{xbc}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c/x)),x)
```

```
[Out] 1/2*a*x^2+1/2*arctan(c/x)*b*x^2-1/2*b*c^2*arctan(x/c)+1/2*x*b*c
```

Maxima [A] time = 1.48726, size = 49, normalized size = 1.26

$$\frac{1}{2}ax^2+\frac{1}{2}\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/2*(x^2*arctan(c/x) - (c*arctan(x/c) - x)*c)*b
```

Fricas [A] time = 2.33068, size = 77, normalized size = 1.97

$$\frac{1}{2}bcx+\frac{1}{2}ax^2+\frac{1}{2}\left(bc^2+bx^2\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="fricas")
```

```
[Out] 1/2*b*c*x + 1/2*a*x^2 + 1/2*(b*c^2 + b*x^2)*arctan(c/x)
```

Sympy [A] time = 0.384635, size = 36, normalized size = 0.92

$$\frac{ax^2}{2} + \frac{bc^2 \operatorname{atan}\left(\frac{c}{x}\right)}{2} + \frac{bcx}{2} + \frac{bx^2 \operatorname{atan}\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c/x)),x)
```

```
[Out] a*x**2/2 + b*c**2*atan(c/x)/2 + b*c*x/2 + b*x**2*atan(c/x)/2
```

Giac [A] time = 1.1642, size = 69, normalized size = 1.77

$$\frac{1}{4} bc^2 i \log(ix + c) - \frac{1}{4} bc^2 i \log(-ix + c) + \frac{1}{2} bx^2 \arctan\left(\frac{c}{x}\right) + \frac{1}{2} bcx + \frac{1}{2} ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="giac")
```

```
[Out] 1/4*b*c^2*i*log(i*x + c) - 1/4*b*c^2*i*log(-i*x + c) + 1/2*b*x^2*arctan(c/x
) + 1/2*b*c*x + 1/2*a*x^2
```

3.135 $\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=27

$$ax+\frac{1}{2}bc\log\left(c^2+x^2\right)+bx\tan^{-1}\left(\frac{c}{x}\right)$$

```
[Out] a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Rubi [A] time = 0.0116355, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5027, 263, 260}

$$ax+\frac{1}{2}bc\log\left(c^2+x^2\right)+bx\tan^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c/x],x]
```

```
[Out] a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx &= ax+b\int \tan^{-1}\left(\frac{c}{x}\right)\,dx\\ &= ax+bx\tan^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x}\,dx\\ &= ax+bx\tan^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{x}{c^2+x^2}\,dx\\ &= ax+bx\tan^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.002662, size = 27, normalized size = 1.

$$ax+\frac{1}{2}bc\log\left(c^2+x^2\right)+bx\tan^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c/x],x]

[Out] a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Maple [A] time = 0.029, size = 38, normalized size = 1.4

$$ax + bx\arctan\left(\frac{c}{x}\right) + \frac{bc}{2}\ln\left(1 + \frac{c^2}{x^2}\right) - bc\ln\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c/x),x)

[Out] a*x+b*x*arctan(c/x)+1/2*b*c*ln(1+c^2/x^2)-b*c*ln(c/x)
```

Maxima [A] time = 0.987169, size = 36, normalized size = 1.33

$$\frac{1}{2}\left(2\,x\arctan\left(\frac{c}{x}\right) + c\log\left(c^2 + x^2\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c/x),x, algorithm="maxima")

[Out] 1/2*(2*x*arctan(c/x) + c*log(c^2 + x^2))*b + a*x
```

Fricas [A] time = 2.12627, size = 65, normalized size = 2.41

$$bx\arctan\left(\frac{c}{x}\right) + \frac{1}{2}\,bc\log\left(c^2 + x^2\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c/x),x, algorithm="fricas")

[Out] b*x*arctan(c/x) + 1/2*b*c*log(c^2 + x^2) + a*x
```

Sympy [A] time = 0.220325, size = 22, normalized size = 0.81

$$ax + b\left(\frac{c\log\left(c^2 + x^2\right)}{2} + x\,\mathrm{atan}\left(\frac{c}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c/x),x)

[Out] a*x + b*(c*log(c**2 + x**2)/2 + x*atan(c/x))
```

Giac [A] time = 1.12084, size = 36, normalized size = 1.33

$$\frac{1}{2}\left(2\,x\arctan\left(\frac{c}{x}\right)+c\log\left(c^2+x^2\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c/x),x, algorithm="giac")
```

```
[Out] 1/2*(2*x*arctan(c/x) + c*log(c^2 + x^2))*b + a*x
```

3.136 $\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}ib\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\text{PolyLog}\left(2,\frac{ic}{x}\right)+a\log(x)$$

```
[Out] a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

Rubi [A] time = 0.0448919, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5031, 4848, 2391}

$$-\frac{1}{2}ib\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\text{PolyLog}\left(2,\frac{ic}{x}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x,x]
```

```
[Out] a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}\,dx &= -\text{Subst}\left(\int \frac{a+b\tan^{-1}(cx)}{x}\,dx,x,\frac{1}{x}\right)\\ &= a\log(x)-\frac{1}{2}(ib)\,\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,\frac{1}{x}\right)+\frac{1}{2}(ib)\,\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx,x,\frac{1}{x}\right)\\ &= a\log(x)-\frac{1}{2}ib\text{Li}_2\left(-\frac{ic}{x}\right)+\frac{1}{2}ib\text{Li}_2\left(\frac{ic}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0056957, size = 39, normalized size = 1.

$$-\frac{1}{2}ib\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\text{PolyLog}\left(2,\frac{ic}{x}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x,x]
```

```
[Out] a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

Maple [B] time = 0.036, size = 94, normalized size = 2.4

$$-a\ln\left(\frac{c}{x}\right)-b\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{i}{2}b\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)+\frac{i}{2}b\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)-\frac{i}{2}b\mathrm{dilog}\left(1+\frac{ic}{x}\right)+\frac{i}{2}b\mathrm{dilog}\left(1-\frac{ic}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x,x)
```

```
[Out] -a*ln(c/x)-b*ln(c/x)*arctan(c/x)-1/2*I*b*ln(c/x)*ln(1+I*c/x)+1/2*I*b*ln(c/x
)*ln(1-I*c/x)-1/2*I*b*dilog(1+I*c/x)+1/2*I*b*dilog(1-I*c/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\arctan(c,x)}{x}\,dx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(c, x)/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\arctan\left(\frac{c}{x}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c/x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{atan}\left(\frac{c}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x,x)
```

```
[Out] Integral((a + b*atan(c/x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan\left(\frac{c}{x}\right) + a}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)/x, x)
```

3.137 $\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=34

$$\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}$$

```
[Out] -((a + b*ArcTan[c/x])/x) + (b*Log[1 + c^2/x^2])/(2*c)
```

Rubi [A] time = 0.0190498, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5033, 260}

$$\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c/x])/x) + (b*Log[1 + c^2/x^2])/(2*c)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}-(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^3}\,dx\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.0080562, size = 37, normalized size = 1.09

$$-\frac{a}{x}+\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{b\tan^{-1}\left(\frac{c}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c/x])/x + (b*Log[1 + c^2/x^2])/(2*c)
```

Maple [A] time = 0.021, size = 36, normalized size = 1.1

$$-\frac{a}{x}-\frac{b}{x}\arctan\left(\frac{c}{x}\right)+\frac{b}{2c}\ln\left(1+\frac{c^2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x^2,x)
```

```
[Out] -a/x-b/x*arctan(c/x)+1/2*b*ln(1+c^2/x^2)/c
```

Maxima [A] time = 1.02608, size = 51, normalized size = 1.5

$$-\frac{b\left(\frac{2\,c\arctan\left(\frac{c}{x}\right)}{x}-\log\left(\frac{c^2}{x^2}+1\right)\right)}{2\,c}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a/x
```

Fricas [A] time = 2.22687, size = 103, normalized size = 3.03

$$-\frac{2\,bc\arctan\left(\frac{c}{x}\right)-bx\log\left(c^2+x^2\right)+2\,bx\log\left(x\right)+2\,ac}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*arctan(c/x) - b*x*log(c^2 + x^2) + 2*b*x*log(x) + 2*a*c)/(c*x)
```

Sympy [A] time = 0.976839, size = 36, normalized size = 1.06

$$\begin{cases} -\frac{a}{x}-\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(x\right)}{c}+\frac{b\log\left(c^2+x^2\right)}{2c} & \text{for } c\neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x**2,x)
```

```
[Out] Piecewise((-a/x - b*atan(c/x)/x - b*log(x)/c + b*log(c**2 + x**2)/(2*c), Ne
(c, 0)), (-a/x, True))
```

Giac [A] time = 1.11594, size = 54, normalized size = 1.59

$$-\frac{b\left(\frac{2\,c\arctan\left(\frac{c}{x}\right)}{x}-\log\left(\frac{c^2}{x^2}+1\right)\right)+\frac{2\,ac}{x}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1)) + 2*a*c/x)/c
```

3.138 $\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^3} dx$

Optimal. Leaf size=43

$$-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tan^{-1}\left(\frac{x}{c}\right)}{2c^2}+\frac{b}{2cx}$$

```
[Out] b/(2*c*x) - (a + b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

Rubi [A] time = 0.0238361, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 263, 325, 203}

$$-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tan^{-1}\left(\frac{x}{c}\right)}{2c^2}+\frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x^3,x]
```

```
[Out] b/(2*c*x) - (a + b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^3}\,dx = -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{1}{2}(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^4}\,dx$$
$$= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{1}{2}(bc)\int \frac{1}{x^2\left(c^2+x^2\right)}\,dx$$
$$= \frac{b}{2cx} - \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b\int \frac{1}{c^2+x^2}\,dx}{2c}$$
$$= \frac{b}{2cx} - \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b\tan^{-1}\left(\frac{x}{c}\right)}{2c^2}$$

Mathematica [A] time = 0.0098911, size = 48, normalized size = 1.12

$$-\frac{a}{2x^2} + \frac{b\tan^{-1}\left(\frac{x}{c}\right)}{2c^2} - \frac{b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x^3,x]
```

```
[Out] -a/(2*x^2) + b/(2*c*x) - (b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

Maple [A] time = 0.026, size = 41, normalized size = 1.

$$-\frac{a}{2\,x^2} - \frac{b}{2\,x^2}\arctan\left(\frac{c}{x}\right) + \frac{b}{2\,cx} + \frac{b}{2\,c^2}\arctan\left(\frac{x}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c/x)+1/2*b/c/x+1/2*b*arctan(x/c)/c^2
```

Maxima [A] time = 1.52523, size = 57, normalized size = 1.33

$$\frac{1}{2}\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3} + \frac{1}{c^2x}\right) - \frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*b - 1/2*a/x^2
```

Fricas [A] time = 2.06838, size = 84, normalized size = 1.95

$$-\frac{ac^2 - bcx + \left(bc^2 + bx^2\right)\arctan\left(\frac{c}{x}\right)}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^2 - b*c*x + (b*c^2 + b*x^2)*arctan(c/x))/(c^2*x^2)
```

Sympy [A] time = 1.23747, size = 44, normalized size = 1.02

$$\begin{cases} -\frac{a}{2x^2} - \frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2x^2} + \frac{b}{2cx} - \frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2c^2} & \text{for}\ c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atan(c/x)/(2*x**2) + b/(2*c*x) - b*atan(c/x)/(2*
c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

Giac [A] time = 1.14501, size = 84, normalized size = 1.95

$$-\frac{2\,bc^2 i \arctan\left(\frac{c}{x}\right) + 2\,ac^2 i - 2\,bcix - bx^2 \log\left(ix + c\right) + bx^2 \log\left(-ix + c\right)}{4\,c^2 ix^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*c^2*i*arctan(c/x) + 2*a*c^2*i - 2*b*c*i*x - b*x^2*log(i*x + c) +
b*x^2*log(-i*x + c))/(c^2*i*x^2)
```

3.139 $\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^4} dx$

Optimal. Leaf size=55

$$-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}+\frac{b\log(x)}{3c^3}+\frac{b}{6cx^2}$$

```
[Out] b/(6*c*x^2) - (a + b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c^2
 + x^2])/(6*c^3)
```

Rubi [A] time = 0.0362819, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5033, 263, 266, 44}

$$-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}+\frac{b\log(x)}{3c^3}+\frac{b}{6cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x^4,x]
```

```
[Out] b/(6*c*x^2) - (a + b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c^2
 + x^2])/(6*c^3)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^4}\,dx &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^5}\,dx\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{x^3\left(c^2+x^2\right)}\,dx\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(c^2+x\right)}\,dx,x,x^2\right)\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\,\text{Subst}\left(\int \left(\frac{1}{c^2x^2}-\frac{1}{c^4x}+\frac{1}{c^4\left(c^2+x\right)}\right)dx,x,x^2\right)\\ &= \frac{b}{6cx^2}-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}\end{aligned}$$

Mathematica [A] time = 0.0099862, size = 60, normalized size = 1.09

$$-\frac{a}{3x^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}+\frac{b\log(x)}{3c^3}+\frac{b}{6cx^2}-\frac{b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x^4,x]
```

```
[Out] -a/(3*x^3) + b/(6*c*x^2) - (b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (
b*Log[c^2 + x^2])/(6*c^3)
```

Maple [A] time = 0.026, size = 45, normalized size = 0.8

$$-\frac{a}{3\,x^3}-\frac{b}{3\,x^3}\arctan\left(\frac{c}{x}\right)+\frac{b}{6\,cx^2}-\frac{b}{6\,c^3}\ln\left(1+\frac{c^2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c/x)+1/6*b/c/x^2-1/6/c^3*b*ln(1+c^2/x^2)
```

Maxima [A] time = 1.02616, size = 73, normalized size = 1.33

$$-\frac{1}{6}\left(c\left(\frac{\log\left(c^2+x^2\right)}{c^4}-\frac{\log\left(x^2\right)}{c^4}-\frac{1}{c^2x^2}\right)+\frac{2\arctan\left(\frac{c}{x}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(c^2 + x^2)/c^4 - log(x^2)/c^4 - 1/(c^2*x^2)) + 2*arctan(c/x)/x
^3)*b - 1/3*a/x^3
```

Fricas [A] time = 2.23219, size = 132, normalized size = 2.4

$$-\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)+bx^3\log\left(c^2+x^2\right)-2\,bx^3\log\left(x\right)+2\,ac^3-bc^2x}{6\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*c^3*arctan(c/x) + b*x^3*log(c^2 + x^2) - 2*b*x^3*log(x) + 2*a*c^3
 - b*c^2*x)/(c^3*x^3)
```

Sympy [A] time = 1.63721, size = 60, normalized size = 1.09

$$\begin{cases} -\frac{a}{3x^3}-\frac{b\operatorname{atan}\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6cx^2}+\frac{b\log\left(x\right)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3} & \text{for}\: c\neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) - b*atan(c/x)/(3*x**3) + b/(6*c*x**2) + b*log(x)/(3*
c**3) - b*log(c**2 + x**2)/(6*c**3), Ne(c, 0)), (-a/(3*x**3), True))
```

Giac [A] time = 1.13462, size = 74, normalized size = 1.35

$$-\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)+bx^3\log\left(c^2+x^2\right)-2\,bx^3\log\left(x\right)+2\,ac^3-bc^2x}{6\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*c^3*arctan(c/x) + b*x^3*log(c^2 + x^2) - 2*b*x^3*log(x) + 2*a*c^3
 - b*c^2*x)/(c^3*x^3)
```

3.140 $\int x^3 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 \, dx$

Optimal. Leaf size=122

$$-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{12}b^2c^2x^2-$$

```
[Out] (b^2*c^2*x^2)/12 - (b*c^3*x*(a + b*ArcCot[x/c]))/2 + (b*c*x^3*(a + b*ArcCot
[x/c]))/6 - (c^4*(a + b*ArcCot[x/c])^2)/4 + (x^4*(a + b*ArcCot[x/c])^2)/4 -
 (b^2*c^4*Log[1 + c^2/x^2])/3 - (2*b^2*c^4*Log[x])/3
```

Rubi [C] time = 1.79076, antiderivative size = 862, normalized size of antiderivative = 7.07, number of steps used = 88, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.125, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{16}\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2c^4+\frac{1}{16}b^2\log^2\left(\frac{ic}{x}+1\right)c^4-\frac{11}{48}b^2\log\left(i-\frac{c}{x}\right)c^4-\frac{5}{48}b^2\log\left(\frac{c}{x}+i\right)c^4-\frac{5}{48}b^2\log(c-ix)c^4$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] -(a*b*c^3*x)/4 - (I/8)*a*b*c^2*x^2 + (b^2*c^2*x^2)/12 + (a*b*c*x^3)/12 - (1
1*b^2*c^4*Log[I - c/x])/48 - (I/8)*b^2*c^3*x*Log[1 - (I*c)/x] + (b^2*c^2*x^
2*Log[1 - (I*c)/x])/16 + (I/24)*b^2*c*x^3*Log[1 - (I*c)/x] - (b*c^3*(1 - (I
*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x]))/8 + (I/16)*b*c^2*x^2*(2*a + I*b*Log[
1 - (I*c)/x]) + (b*c*x^3*(2*a + I*b*Log[1 - (I*c)/x]))/24 - (c^4*(2*a + I*b
*Log[1 - (I*c)/x])^2)/16 + (x^4*(2*a + I*b*Log[1 - (I*c)/x])^2)/16 + (I/4)*
b^2*c^3*x*Log[1 + (I*c)/x] - (I/12)*b^2*c*x^3*Log[1 + (I*c)/x] - (I/4)*a*b*
x^4*Log[1 + (I*c)/x] + (b^2*x^4*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/8 + (b^2
*c^4*Log[1 + (I*c)/x]^2)/16 - (b^2*x^4*Log[1 + (I*c)/x]^2)/16 - (5*b^2*c^4*
Log[I + c/x])/48 + (I/4)*a*b*c^4*Log[c - I*x] - (5*b^2*c^4*Log[c - I*x])/48
 - (b^2*c^4*Log[1 - (I*c)/x]*Log[c - I*x])/8 - (5*b^2*c^4*Log[c + I*x])/48
- (b^2*c^4*Log[1 + (I*c)/x]*Log[c + I*x])/8 + (b^2*c^4*Log[(c - I*x)/(2*c)]
*Log[c + I*x])/8 + (b^2*c^4*Log[c - I*x]*Log[(c + I*x)/(2*c)])/8 - (I/4)*a*
b*c^4*Log[x] - (11*b^2*c^4*Log[x])/24 - (b^2*c^4*Log[c + I*x]*Log[((-I)*x)/
c])/8 - (b^2*c^4*Log[c - I*x]*Log[(I*x)/c])/8 + (b^2*c^4*PolyLog[2, (c - I*
x)/(2*c)])/8 + (b^2*c^4*PolyLog[2, (c + I*x)/(2*c)])/8 + (b^2*c^4*PolyLog[2
, ((-I)*c)/x])/8 + (b^2*c^4*PolyLog[2, (I*c)/x])/8 - (b^2*c^4*PolyLog[2, 1
- (I*x)/c])/8 - (b^2*c^4*PolyLog[2, 1 + (I*x)/c])/8
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int\left(\frac{1}{4}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{2}bx^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{4}b^2x^3\log^2\left(\right.\right.$$

$$=\frac{1}{4}\int x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2 dx+\frac{1}{2}b\int x^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)dx-\frac{1}{4}b$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^2}{x^5}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(-2iax^3\log\left(1+\frac{ic}{x}\right)+bx^3\log\left(\right.\right.$$

$$=\frac{1}{16}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{16}b^2x^4\log^2\left(1+\frac{ic}{x}\right)-(iab)\int x^3\log\left(1+\frac{ic}{x}\right)dx+\frac{1}{2}b^2\int$$

$$=\frac{1}{16}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{4}iabx^4\log\left(1+\frac{ic}{x}\right)+\frac{1}{8}b^2x^4\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{16}$$

$$=\frac{1}{16}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{4}iabx^4\log\left(1+\frac{ic}{x}\right)+\frac{1}{8}b^2x^4\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{16}$$

$$=\frac{1}{24}bcx^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{16}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{8}ib^2c^3x\log\left(1+\frac{ic}{x}\right)-\frac{1}{16}b$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{12}abcx^3+\frac{1}{16}ibc^2x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{24}bcx^3\left(2a+ib\log\left(\right.\right.$$

$$=-\frac{1}{4}abc^3x-\frac{1}{16}ib^2c^3x-\frac{1}{8}iabc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{1}{24}b^2c^4\log$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\log\left(1-\frac{ic}{x}\right)-$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\log\left(1-\frac{ic}{x}\right)-$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\log\left(1-\frac{ic}{x}\right)-$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\log\left(1-\frac{ic}{x}\right)-$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\log\left(1-\frac{ic}{x}\right)-$$

Mathematica [A] time = 0.0895405, size = 111, normalized size = 0.91

$$\frac{1}{12}\left(x\left(3a^2x^3+2abc\left(x^2-3c^2\right)+b^2c^2x\right)+2b\tan^{-1}\left(\frac{c}{x}\right)\left(3a\left(x^4-c^4\right)+bcx\left(x^2-3c^2\right)\right)-4b^2c^4\log\left(c^2+x^2\right)+3b^2\left(x^4-c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (x*(b^2*c^2*x + 3*a^2*x^3 + 2*a*b*c*(-3*c^2 + x^2)) + 2*b*(b*c*x*(-3*c^2 +
x^2) + 3*a*(-c^4 + x^4))*ArcTan[c/x] + 3*b^2*(-c^4 + x^4)*ArcTan[c/x]^2 - 4
*b^2*c^4*Log[c^2 + x^2])/12
```

Maple [A] time = 0.043, size = 157, normalized size = 1.3

$$\frac{a^2x^4}{4}+\frac{b^2x^4}{4}\left(\arctan\left(\frac{c}{x}\right)\right)^2-\frac{c^4b^2}{4}\left(\arctan\left(\frac{c}{x}\right)\right)^2+\frac{b^2cx^3}{6}\arctan\left(\frac{c}{x}\right)-\frac{c^3b^2x}{2}\arctan\left(\frac{c}{x}\right)-\frac{c^4b^2}{3}\ln\left(1+\frac{c^2}{x^2}\right)+\frac{b^2c^2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c/x))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctan(c/x)^2-1/4*c^4*b^2*arctan(c/x)^2+1/6*c*b^2*a
rctan(c/x)*x^3-1/2*c^3*b^2*arctan(c/x)*x-1/3*b^2*c^4*ln(1+c^2/x^2)+1/12*b^2
*c^2*x^2+2/3*c^4*b^2*ln(c/x)+1/2*a*b*x^4*arctan(c/x)+1/2*c^4*a*b*arctan(x/c
)+1/6*a*b*c*x^3-1/2*c^3*a*b*x
```

Maxima [A] time = 1.60361, size = 177, normalized size = 1.45

$$\frac{1}{4}\,b^2x^4\arctan\left(\frac{c}{x}\right)^2+\frac{1}{4}\,a^2x^4+\frac{1}{6}\left(3\,x^4\arctan\left(\frac{c}{x}\right)+\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\right)ab+\frac{1}{12}\left(\left(3\,c^2\arctan\left(x,c\right)^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c/x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c/x) + (3*c^3*a
rctan(x/c) - 3*c^2*x + x^3)*c)*a*b + 1/12*((3*c^2*arctan2(x, c)^2 - 4*c^2*l
og(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arctan(c
/x))*b^2
```

Fricas [A] time = 2.19016, size = 290, normalized size = 2.38

$$\frac{1}{2}\,abc^4\arctan\left(\frac{x}{c}\right)-\frac{1}{3}\,b^2c^4\log\left(c^2+x^2\right)-\frac{1}{2}\,abc^3x+\frac{1}{12}\,b^2c^2x^2+\frac{1}{6}\,abcx^3+\frac{1}{4}\,a^2x^4-\frac{1}{4}\left(b^2c^4-b^2x^4\right)\arctan\left(\frac{c}{x}\right)^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*b*c^4*arctan(x/c) - 1/3*b^2*c^4*log(c^2 + x^2) - 1/2*a*b*c^3*x + 1/12
*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/4*(b^2*c^4 - b^2*x^4)*arctan
(c/x)^2 - 1/6*(3*b^2*c^3*x - b^2*c*x^3 - 3*a*b*x^4)*arctan(c/x)
```

Sympy [A] time = 1.14757, size = 144, normalized size = 1.18

$$\frac{a^2x^4}{4}-\frac{abc^4\operatorname{atan}\left(\frac{c}{x}\right)}{2}-\frac{abc^3x}{2}+\frac{abcx^3}{6}+\frac{abx^4\operatorname{atan}\left(\frac{c}{x}\right)}{2}-\frac{b^2c^4\log\left(c^2+x^2\right)}{3}-\frac{b^2c^4\operatorname{atan}^2\left(\frac{c}{x}\right)}{4}-\frac{b^2c^3x\operatorname{atan}\left(\frac{c}{x}\right)}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c/x))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**4*atan(c/x)/2 - a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**4
*atan(c/x)/2 - b**2*c**4*log(c**2 + x**2)/3 - b**2*c**4*atan(c/x)**2/4 - b*
*2*c**3*x*atan(c/x)/2 + b**2*c**2*x**2/12 + b**2*c*x**3*atan(c/x)/6 + b**2*
x**4*atan(c/x)**2/4
```

Giac [A] time = 1.1762, size = 228, normalized size = 1.87

$$-\frac{1}{4}\,b^2c^4\arctan\left(\frac{c}{x}\right)^2+\frac{1}{4}\,b^2x^4\arctan\left(\frac{c}{x}\right)^2-\frac{1}{4}\,abc^4i\log\left(ix+c\right)+\frac{1}{4}\,abc^4i\log\left(-ix+c\right)-\frac{1}{2}\,b^2c^3x\arctan\left(\frac{c}{x}\right)+\frac{1}{6}\,b^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] -1/4*b^2*c^4*arctan(c/x)^2 + 1/4*b^2*x^4*arctan(c/x)^2 - 1/4*a*b*c^4*i*log(
i*x + c) + 1/4*a*b*c^4*i*log(-i*x + c) - 1/2*b^2*c^3*x*arctan(c/x) + 1/6*b^
2*c*x^3*arctan(c/x) + 1/2*a*b*x^4*arctan(c/x) - 1/3*b^2*c^4*log(i*x + c) -
1/3*b^2*c^4*log(-i*x + c) - 1/2*a*b*c^3*x + 1/12*b^2*c^2*x^2 + 1/6*a*b*c*x^
3 + 1/4*a^2*x^4
```

3.141 $\int x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=152

$$-\frac{1}{3}ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{2}{3}bc^3\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{3}bcx^2\left(a+b\right.$$

```
[Out] (b^2*c^2*x)/3 + (b^2*c^3*ArcCot[x/c])/3 + (b*c*x^2*(a + b*ArcCot[x/c]))/3 -
 (I/3)*c^3*(a + b*ArcCot[x/c])^2 + (x^3*(a + b*ArcCot[x/c])^2)/3 + (2*b*c^3
*(a + b*ArcCot[x/c])*Log[2 - 2/(1 - (I*c)/x)])/3 - (I/3)*b^2*c^3*PolyLog[2,
 -1 + 2/(1 - (I*c)/x)]
```

Rubi [B] time = 1.49016, antiderivative size = 787, normalized size of antiderivative = 5.18, number of steps used = 73, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.125, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$\frac{1}{6}ib^2c^3\text{PolyLog}\left(2,\frac{c-ix}{2c}\right)-\frac{1}{6}ib^2c^3\text{PolyLog}\left(2,\frac{c+ix}{2c}\right)+\frac{1}{6}ib^2c^3\text{PolyLog}\left(2,-\frac{ic}{x}\right)-\frac{1}{6}ib^2c^3\text{PolyLog}\left(2,\frac{ic}{x}\right)-\frac{1}{6}ib^2c$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (-I/3)*a*b*c^2*x + (b^2*c^2*x)/3 + (a*b*c*x^2)/6 - (I/4)*b^2*c^3*Log[I - c/
x] + (b^2*c^2*x*Log[1 - (I*c)/x])/6 + (I/12)*b^2*c*x^2*Log[1 - (I*c)/x] + (
I/6)*b*c^2*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x]) + (b*c*x^2*(2*a + I
*b*Log[1 - (I*c)/x]))/12 + (I/12)*c^3*(2*a + I*b*Log[1 - (I*c)/x])^2 + (x^3
*(2*a + I*b*Log[1 - (I*c)/x])^2)/12 - (I/6)*b^2*c*x^2*Log[1 + (I*c)/x] - (I
/3)*a*b*x^3*Log[1 + (I*c)/x] + (b^2*x^3*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/
6 + (I/12)*b^2*c^3*Log[1 + (I*c)/x]^2 - (b^2*x^3*Log[1 + (I*c)/x]^2)/12 + (
I/12)*b^2*c^3*Log[I + c/x] - (a*b*c^3*Log[c - I*x])/3 + (I/12)*b^2*c^3*Log[
c - I*x] - (I/6)*b^2*c^3*Log[1 - (I*c)/x]*Log[c - I*x] - (I/12)*b^2*c^3*Log
[c + I*x] + (I/6)*b^2*c^3*Log[1 + (I*c)/x]*Log[c + I*x] - (I/6)*b^2*c^3*Log
[(c - I*x)/(2*c)]*Log[c + I*x] + (I/6)*b^2*c^3*Log[c - I*x]*Log[(c + I*x)/(
2*c)] - (a*b*c^3*Log[x])/3 + (I/6)*b^2*c^3*Log[c + I*x]*Log[((-I)*x)/c] - (
I/6)*b^2*c^3*Log[c - I*x]*Log[(I*x)/c] + (I/6)*b^2*c^3*PolyLog[2, (c - I*x)
/(2*c)] - (I/6)*b^2*c^3*PolyLog[2, (c + I*x)/(2*c)] + (I/6)*b^2*c^3*PolyLog
[2, ((-I)*c)/x] - (I/6)*b^2*c^3*PolyLog[2, (I*c)/x] - (I/6)*b^2*c^3*PolyLog
[2, 1 - (I*x)/c] + (I/6)*b^2*c^3*PolyLog[2, 1 + (I*x)/c]
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2dx=\int\left(\frac{1}{4}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{2}bx^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{4}b^2x^2\log^2\left(\right.\right.$$

$$=\frac{1}{4}\int x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2dx+\frac{1}{2}b\int x^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)dx-\frac{1}{4}b$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^2}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(-2iax^2\log\left(1+\frac{ic}{x}\right)+bx^2\log\left(\right.\right.$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{12}b^2x^3\log^2\left(1+\frac{ic}{x}\right)-(iab)\int x^2\log\left(1+\frac{ic}{x}\right)dx+\frac{1}{2}b^2\int$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{3}iabx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{6}b^2x^3\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{12}$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{3}iabx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{6}b^2x^3\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{12}$$

$$=\frac{1}{12}bcx^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{6}b^2c^2x\log\left(1+\frac{ic}{x}\right)-\frac{1}{12}ib$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{6}abcx^2+\frac{1}{6}ibc^2\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{12}bcx^2\left(2a+ib\log\left(1-\frac{ic}{x}\right.\right.$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1\right.$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1\right.$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1\right.$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1\right.$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1\right.$$

$$=-\frac{1}{3}iabc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1\right.$$

Mathematica [A] time = 0.328369, size = 152, normalized size = 1.

$$\frac{1}{3}\left(-ib^2c^3\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)+a^2x^3-abc^3\log\left(\frac{c^2}{x^2}+1\right)+b\tan^{-1}\left(\frac{c}{x}\right)\left(2ax^3+bc\left(c^2+x^2\right)+2bc^3\log\left(1-e^{2i\tan^{-1}(}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*((-I)*c^3 + x^3)*ArcTan[c/x]^2 + b*A
rcTan[c/x]*(2*a*x^3 + b*c*(c^2 + x^2) + 2*b*c^3*Log[1 - E^((2*I)*ArcTan[c/x
])]) - a*b*c^3*Log[1 + c^2/x^2] + 2*a*b*c^3*Log[c/x] - I*b^2*c^3*PolyLog[2,
 E^((2*I)*ArcTan[c/x])])/3
```

Maple [B] time = 0.104, size = 445, normalized size = 2.9

$$\frac{x^3a^2}{3}+\frac{b^2x^3}{3}\left(\arctan\left(\frac{c}{x}\right)\right)^2-\frac{c^3b^2}{3}\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)+\frac{b^2cx^2}{3}\arctan\left(\frac{c}{x}\right)+\frac{2\,c^3b^2}{3}\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)+\frac{i}{6}c^3b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c/x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctan(c/x)^2-1/3*c^3*b^2*arctan(c/x)*ln(1+c^2/x^2)
+1/3*c*b^2*arctan(c/x)*x^2+2/3*c^3*b^2*ln(c/x)*arctan(c/x)+1/6*I*c^3*b^2*ln
(c/x-I)*ln(-1/2*I*(c/x+I))-1/12*I*c^3*b^2*ln(c/x+I)^2-1/6*I*c^3*b^2*dilog(1
/2*I*(c/x-I))+1/3*I*c^3*b^2*dilog(1+I*c/x)-1/6*I*c^3*b^2*ln(1+c^2/x^2)*ln(c
/x-I)-1/3*I*c^3*b^2*ln(c/x)*ln(1-I*c/x)+1/3*I*c^3*b^2*ln(c/x)*ln(1+I*c/x)+1
/6*I*c^3*b^2*dilog(-1/2*I*(c/x+I))-1/3*c^3*b^2*arctan(x/c)+1/3*b^2*c^2*x+1/
6*I*c^3*b^2*ln(1+c^2/x^2)*ln(c/x+I)+1/12*I*c^3*b^2*ln(c/x-I)^2-1/6*I*c^3*b^
2*ln(c/x+I)*ln(1/2*I*(c/x-I))-1/3*I*c^3*b^2*dilog(1-I*c/x)+2/3*a*b*x^3*arct
an(c/x)-1/3*c^3*a*b*ln(1+c^2/x^2)+1/3*a*b*c*x^2+2/3*c^3*a*b*ln(c/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2x^3+\frac{1}{3}\left(2\,x^3\arctan\left(\frac{c}{x}\right)-\left(c^2\log\left(c^2+x^2\right)-x^2\right)c\right)ab+\frac{1}{48}\left(4\,x^3\arctan\left(c,x\right)^2-x^3\log\left(c^2+x^2\right)^2+48\int\frac{36\,c^2x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a*b +
1/48*(4*x^3*arctan2(c, x)^2 - x^3*log(c^2 + x^2)^2 + 48*integrate(1/48*(36*
c^2*x^2*arctan2(c, x)^2 + 36*x^4*arctan2(c, x)^2 + 8*c*x^3*arctan2(c, x) +
4*x^4*log(c^2 + x^2) + 3*(c^2*x^2 + x^4)*log(c^2 + x^2)^2)/(c^2 + x^2), x))
*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2x^2\arctan\left(\frac{c}{x}\right)^2+2\,abx^2\arctan\left(\frac{c}{x}\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c/x)^2 + 2*a*b*x^2*arctan(c/x) + a^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c/x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c/x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2*x^2, x)
```

3.142 $\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2c^2\log\left(\frac{c^2}{x^2}+1\right)+b^2c^2\log(x)$$

```
[Out] b*c*x*(a + b*ArcCot[x/c]) + (c^2*(a + b*ArcCot[x/c])^2)/2 + (x^2*(a + b*Arc
Cot[x/c])^2)/2 + (b^2*c^2*Log[1 + c^2/x^2])/2 + b^2*c^2*Log[x]
```

Rubi [C] time = 1.12636, antiderivative size = 663, normalized size of antiderivative = 8.09, number of steps used = 58, number of rules used = 32, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.286, Rules used = {5035, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2455, 193, 43, 6742, 30, 2557, 12, 2466, 2448, 263, 2462, 260, 2416, 2394, 2393, 2391, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c-ix}{2c}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c+ix}{2c}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,-\frac{ic}{x}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{ic}{x}\right)+\frac{1}{4}b^2c^2\text{I}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (a*b*c*x)/2 + (b^2*c^2*Log[I - c/x])/4 + (I/4)*b^2*c*x*Log[1 - (I*c)/x] + (
b*c*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x]))/4 + (c^2*(2*a + I*b*Log[1
 - (I*c)/x])^2)/8 + (x^2*(2*a + I*b*Log[1 - (I*c)/x])^2)/8 - (I/2)*b^2*c*x*
Log[1 + (I*c)/x] - (I/2)*a*b*x^2*Log[1 + (I*c)/x] + (b^2*x^2*Log[1 - (I*c)/
x]*Log[1 + (I*c)/x])/4 - (b^2*c^2*Log[1 + (I*c)/x]^2)/8 - (b^2*x^2*Log[1 +
(I*c)/x]^2)/8 - (I/2)*a*b*c^2*Log[c - I*x] + (b^2*c^2*Log[c - I*x])/4 + (b^
2*c^2*Log[1 - (I*c)/x]*Log[c - I*x])/4 + (b^2*c^2*Log[c + I*x])/4 + (b^2*c^
2*Log[1 + (I*c)/x]*Log[c + I*x])/4 - (b^2*c^2*Log[(c - I*x)/(2*c)]*Log[c +
I*x])/4 - (b^2*c^2*Log[c - I*x]*Log[(c + I*x)/(2*c)])/4 + (I/2)*a*b*c^2*Log
[x] + (b^2*c^2*Log[x])/2 + (b^2*c^2*Log[c + I*x]*Log[((-I)*x)/c])/4 + (b^2*
c^2*Log[c - I*x]*Log[(I*x)/c])/4 - (b^2*c^2*PolyLog[2, (c - I*x)/(2*c)])/4
- (b^2*c^2*PolyLog[2, (c + I*x)/(2*c)])/4 - (b^2*c^2*PolyLog[2, ((-I)*c)/x]
)/4 - (b^2*c^2*PolyLog[2, (I*c)/x])/4 + (b^2*c^2*PolyLog[2, 1 - (I*x)/c])/4
 + (b^2*c^2*PolyLog[2, 1 + (I*x)/c])/4
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
```

```
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(\frac{1}{4}x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{2}bx\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{4}b^2x\log^2\left(1\right.\right.$$

$$=\frac{1}{4}\int x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2 dx+\frac{1}{2}b\int x\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)dx-\frac{1}{4}b$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^2}{x^3}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(-2iax\log\left(1+\frac{ic}{x}\right)+bx\log\left(\right.\right.$$

$$=\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{8}b^2x^2\log^2\left(1+\frac{ic}{x}\right)-(iab)\int x\log\left(1+\frac{ic}{x}\right)dx+\frac{1}{2}b^2\int$$

$$=\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{2}iabx^2\log\left(1+\frac{ic}{x}\right)+\frac{1}{4}b^2x^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{8}$$

$$=\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{2}iabx^2\log\left(1+\frac{ic}{x}\right)+\frac{1}{4}b^2x^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)-\frac{1}{8}$$

$$=\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{4}ib^2cx\log\left(1+\frac{ic}{x}\right.$$

$$=\frac{1}{2}abcx+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{8}c^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{8}x^2\left(2a+\right.$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)$$

Mathematica [A] time = 0.0535183, size = 73, normalized size = 0.89

$$\frac{1}{2}\left(2b\tan^{-1}\left(\frac{c}{x}\right)\left(a\left(c^2+x^2\right)+bcx\right)+ax(ax+2bc)+b^2c^2\log\left(c^2+x^2\right)+b^2\left(c^2+x^2\right)\tan^{-1}\left(\frac{c}{x}\right)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (a*x*(2*b*c + a*x) + 2*b*(b*c*x + a*(c^2 + x^2))*ArcTan[c/x] + b^2*(c^2 + x
^2)*ArcTan[c/x]^2 + b^2*c^2*Log[c^2 + x^2])/2
```

Maple [A] time = 0.04, size = 116, normalized size = 1.4

$$\frac{a^2x^2}{2}+\frac{b^2x^2}{2}\left(\arctan\left(\frac{c}{x}\right)\right)^2+\frac{c^2b^2}{2}\left(\arctan\left(\frac{c}{x}\right)\right)^2+cb^2\arctan\left(\frac{c}{x}\right)x+\frac{c^2b^2}{2}\ln\left(1+\frac{c^2}{x^2}\right)-c^2b^2\ln\left(\frac{c}{x}\right)+abx^2\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c/x))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*b^2*x^2*arctan(c/x)^2+1/2*c^2*b^2*arctan(c/x)^2+c*b^2*arcta
n(c/x)*x+1/2*b^2*c^2*ln(1+c^2/x^2)-c^2*b^2*ln(c/x)+a*b*x^2*arctan(c/x)-c^2*
a*b*arctan(x/c)+a*b*c*x
```

Maxima [A] time = 1.5402, size = 136, normalized size = 1.66

$$\frac{1}{2}b^2x^2\arctan\left(\frac{c}{x}\right)^2+\frac{1}{2}a^2x^2+\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)ab-\frac{1}{2}\left(\left(\arctan(x,c)^2-\log\left(c^2+x^2\right)\right)c^2+2\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\arctan\left(\frac{c}{x}\right)\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctan(c/x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c/x) - (c*arctan(x/c)
 - x)*c)*a*b - 1/2*((arctan2(x, c)^2 - log(c^2 + x^2))*c^2 + 2*(c*arctan(x/
c) - x)*c*arctan(c/x))*b^2
```

Fricas [A] time = 2.28212, size = 201, normalized size = 2.45

$$-abc^2\arctan\left(\frac{x}{c}\right)+\frac{1}{2}b^2c^2\log\left(c^2+x^2\right)+abcx+\frac{1}{2}a^2x^2+\frac{1}{2}\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2+\left(b^2cx+abx^2\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] -a*b*c^2*arctan(x/c) + 1/2*b^2*c^2*log(c^2 + x^2) + a*b*c*x + 1/2*a^2*x^2 +
 1/2*(b^2*c^2 + b^2*x^2)*arctan(c/x)^2 + (b^2*c*x + a*b*x^2)*arctan(c/x)
```

Sympy [A] time = 0.586846, size = 97, normalized size = 1.18

$$\frac{a^2x^2}{2}+abc^2\operatorname{atan}\left(\frac{c}{x}\right)+abcx+abx^2\operatorname{atan}\left(\frac{c}{x}\right)+\frac{b^2c^2\log\left(c^2+x^2\right)}{2}+\frac{b^2c^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2}+b^2cx\operatorname{atan}\left(\frac{c}{x}\right)+\frac{b^2x^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c/x))**2,x)
```

```
[Out] a**2*x**2/2 + a*b*c**2*atan(c/x) + a*b*c*x + a*b*x**2*atan(c/x) + b**2*c**2
*log(c**2 + x**2)/2 + b**2*c**2*atan(c/x)**2/2 + b**2*c*x*atan(c/x) + b**2*
x**2*atan(c/x)**2/2
```

Giac [A] time = 1.17458, size = 173, normalized size = 2.11

$$\frac{1}{2}b^2c^2\arctan\left(\frac{c}{x}\right)^2+\frac{1}{2}b^2x^2\arctan\left(\frac{c}{x}\right)^2+\frac{1}{2}abc^2i\log\left(ix+c\right)-\frac{1}{2}abc^2i\log\left(-ix+c\right)+b^2cx\arctan\left(\frac{c}{x}\right)+abx^2\operatorname{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*c^2*arctan(c/x)^2 + 1/2*b^2*x^2*arctan(c/x)^2 + 1/2*a*b*c^2*i*log(i
*x + c) - 1/2*a*b*c^2*i*log(-i*x + c) + b^2*c*x*arctan(c/x) + a*b*x^2*arcta
n(c/x) + 1/2*b^2*c^2*log(i*x + c) + 1/2*b^2*c^2*log(-i*x + c) + a*b*c*x + 1
/2*a^2*x^2
```

3.143 $\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx$

Optimal. Leaf size=83

$$ib^2c\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)+ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)$$

```
[Out] I*c*(a + b*ArcCot[x/c])^2 + x*(a + b*ArcCot[x/c])^2 - 2*b*c*(a + b*ArcCot[x
/c])*Log[(2*c)/(c + I*x)] + I*b^2*c*PolyLog[2, 1 - (2*c)/(c + I*x)]
```

Rubi [B] time = 0.440043, antiderivative size = 478, normalized size of antiderivative = 5.76, number of steps used = 31, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {5029, 2448, 263, 31, 2449, 2391, 2556, 12, 2462, 260, 2416, 2394, 2393, 2315}

$$-\frac{1}{2}ib^2c\text{PolyLog}\left(2,\frac{c-ix}{2c}\right)+\frac{1}{2}ib^2c\text{PolyLog}\left(2,\frac{c+ix}{2c}\right)-\frac{1}{2}ib^2c\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib^2c\text{PolyLog}\left(2,\frac{ic}{x}\right)+\frac{1}{2}ib^2c\text{Poly}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c/x])^2,x]
```

```
[Out] a^2*x + I*a*b*x*Log[1 - (I*c)/x] + (b^2*(I*c - x)*Log[1 - (I*c)/x]^2)/4 - I
*a*b*x*Log[1 + (I*c)/x] + (b^2*x*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/2 - (b^
2*(I*c + x)*Log[1 + (I*c)/x]^2)/4 - (I/2)*b^2*c*Log[1 + (I*c)/x]*Log[-c - I
*x] + a*b*c*Log[c - I*x] + (I/2)*b^2*c*Log[-c - I*x]*Log[(c - I*x)/(2*c)] +
 (I/2)*b^2*c*Log[1 - (I*c)/x]*Log[-c + I*x] + a*b*c*Log[c + I*x] - (I/2)*b^
2*c*Log[-c + I*x]*Log[(c + I*x)/(2*c)] - (I/2)*b^2*c*Log[-c - I*x]*Log[((-I
)*x)/c] + (I/2)*b^2*c*Log[-c + I*x]*Log[(I*x)/c] - (I/2)*b^2*c*PolyLog[2, (
c - I*x)/(2*c)] + (I/2)*b^2*c*PolyLog[2, (c + I*x)/(2*c)] - (I/2)*b^2*c*Pol
yLog[2, ((-I)*c)/x] + (I/2)*b^2*c*PolyLog[2, (I*c)/x] + (I/2)*b^2*c*PolyLog
[2, 1 - (I*x)/c] - (I/2)*b^2*c*PolyLog[2, 1 + (I*x)/c]
```

Rule 5029

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x],
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(a^2+iab\log\left(1-\frac{ic}{x}\right)-\frac{1}{4}b^2\log^2\left(1-\frac{ic}{x}\right)-iab\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right.\right.$$
$$= a^2x+(iab)\int\log\left(1-\frac{ic}{x}\right)dx-(iab)\int\log\left(1+\frac{ic}{x}\right)dx-\frac{1}{4}b^2\int\log^2\left(1-\frac{ic}{x}\right)dx-\frac{1}{4}b^2\int \mathrm{l}$$
$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2x\log\left(1-\frac{ic}{x}\right)\mathrm{lo}$$
$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2x\log\left(1-\frac{ic}{x}\right)\mathrm{lo}$$
$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2x\log\left(1-\frac{ic}{x}\right)\mathrm{lo}$$
$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2x\log\left(1-\frac{ic}{x}\right)\mathrm{lo}$$
$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2x\log\left(1-\frac{ic}{x}\right)\mathrm{lo}$$
$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2x\log\left(1-\frac{ic}{x}\right)\mathrm{lo}$$
$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2x\log\left(1-\frac{ic}{x}\right)\mathrm{lo}$$
$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2x\log\left(1-\frac{ic}{x}\right)\mathrm{lo}$$

Mathematica [A] time = 0.106135, size = 105, normalized size = 1.27

$$ib^2c\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)+a\left(ax+bc\log\left(\frac{c^2}{x^2}+1\right)-2bc\log\left(\frac{c}{x}\right)\right)+2b\tan^{-1}\left(\frac{c}{x}\right)\left(ax-bc\log\left(1-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)\right)+b^2($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^2,x]
```

```
[Out] b^2*(I*c + x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*x - b*c*Log[1 - E^((2*I)*A
rcTan[c/x])]) + a*(a*x + b*c*Log[1 + c^2/x^2] - 2*b*c*Log[c/x]) + I*b^2*c*P
olyLog[2, E^((2*I)*ArcTan[c/x])]
```

Maple [B] time = 0.09, size = 357, normalized size = 4.3

$$a^2x+b^2x\left(\arctan\left(\frac{c}{x}\right)\right)^2+cb^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)-2\,cb^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)+icb^2\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)+icb^2\mathrm{dilog}\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2,x)
```

```
[Out] a^2*x+b^2*x*arctan(c/x)^2+c*b^2*arctan(c/x)*ln(1+c^2/x^2)-2*c*b^2*ln(c/x)*a
rctan(c/x)+I*c*b^2*ln(c/x)*ln(1-I*c/x)+I*c*b^2*dilog(1-I*c/x)-I*c*b^2*dilog
```

```
(1+I*c/x)-1/2*I*c*b^2*dilog(-1/2*I*(c/x+I))-I*c*b^2*ln(c/x)*ln(1+I*c/x)+1/4
*I*c*b^2*ln(c/x+I)^2+1/2*I*c*b^2*dilog(1/2*I*(c/x-I))+1/2*I*c*b^2*ln(c/x+I)
*ln(1/2*I*(c/x-I))-1/2*I*c*b^2*ln(c/x-I)*ln(-1/2*I*(c/x+I))-1/4*I*c*b^2*ln(
c/x-I)^2-1/2*I*c*b^2*ln(1+c^2/x^2)*ln(c/x+I)+1/2*I*c*b^2*ln(1+c^2/x^2)*ln(c
/x-I)+2*a*b*x*arctan(c/x)+c*a*b*ln(1+c^2/x^2)-2*c*a*b*ln(c/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,x\arctan\left(\frac{c}{x}\right)+c\log\left(c^2+x^2\right)\right)ab+\frac{1}{16}\left(12\,c\arctan\left(\frac{c}{x}\right)^2\arctan\left(\frac{x}{c}\right)+4\left(\frac{3\arctan\left(\frac{c}{x}\right)\arctan\left(\frac{x}{c}\right)^2}{c}+\frac{\arctan\left(\frac{x}{c}\right.}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] (2*x*arctan(c/x) + c*log(c^2 + x^2))*a*b + 1/16*(12*c*arctan(c/x)^2*arctan(
x/c) + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*c^2 + 4*x*arctan
2(c, x)^2 + 16*c^2*integrate(1/16*log(c^2 + x^2)^2/(c^2 + x^2), x) - x*log(
c^2 + x^2)^2 + 128*c*integrate(1/16*x*arctan(c/x)/(c^2 + x^2), x) + 192*int
egrate(1/16*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 16*integrate(1/16*x^2*log(c
^2 + x^2)^2/(c^2 + x^2), x) + 64*integrate(1/16*x^2*log(c^2 + x^2)/(c^2 + x
^2), x))*b^2 + a^2*x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2\arctan\left(\frac{c}{x}\right)^2+2\,ab\arctan\left(\frac{c}{x}\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2,x)
```

```
[Out] Integral((a + b*atan(c/x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2, x)
```

3.144 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=148

$$ib\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-ib\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{}{1}\right.$$

```
[Out] -2*(a + b*ArcCot[x/c])^2*ArcTanh[1 - 2/(1 + (I*c)/x)] + I*b*(a + b*ArcCot[x
/c])*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - I*b*(a + b*ArcCot[x/c])*PolyLog[2, -
1 + 2/(1 + (I*c)/x)] + (b^2*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/2 - (b^2*PolyL
og[3, -1 + 2/(1 + (I*c)/x)])/2
```

Rubi [A] time = 0.293581, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5031, 4850, 4988, 4884, 4994, 6610}

$$ib\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-ib\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{}{1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcCot[x/c])^2*ArcTanh[1 - 2/(1 + (I*c)/x)] + I*b*(a + b*ArcCot[x
/c])*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - I*b*(a + b*ArcCot[x/c])*PolyLog[2, -
1 + 2/(1 + (I*c)/x)] + (b^2*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/2 - (b^2*PolyL
og[3, -1 + 2/(1 + (I*c)/x)])/2
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+(4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\tanh^{-1}\left(1-\frac{}{1}\right.}{1+c^2x^2}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)-(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right)-ib\left(a+b\,\text{c}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right)-ib\left(a+b\,\text{c}\right.$$

Mathematica [A] time = 0.0853304, size = 148, normalized size = 1.

$$\frac{1}{2}b\left(2i\text{PolyLog}\left(2,\frac{c+ix}{c-ix}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-2i\text{PolyLog}\left(2,\frac{x-ic}{x+ic}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+b\left(\text{PolyLog}\left(3,\frac{c+ix}{c-ix}\right)-\text{Pol}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcTan[c/x])^2*ArcTanh[(c + I*x)/(c - I*x)] + (b*((2*I)*(a + b*Ar
cTan[c/x])*PolyLog[2, (c + I*x)/(c - I*x)] - (2*I)*(a + b*ArcTan[c/x])*Poly
Log[2, ((-I)*c + x)/(I*c + x)] + b*(PolyLog[3, (c + I*x)/(c - I*x)] - PolyL
og[3, ((-I)*c + x)/(I*c + x)])))/2
```

Maple [C] time = 0.394, size = 1249, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2/x,x)
```

```
[Out] -b^2*ln(c/x)*arctan(c/x)^2+b^2*arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-
b^2*arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-b^2*arctan(c/x)^2*ln(1+
(1+I*c/x)/(1+c^2/x^2)^(1/2))+1/2*I*b^2*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1
))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan
(c/x)^2+1/2*I*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2
/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*a
rctan(c/x)^2+1/2*b^2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))-1/2*I*b^2*Pi*csgn(
I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*(
(1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^2-1/2*I
*b^2*Pi*arctan(c/x)^2-I*a*b*dilog(1+I*c/x)+I*a*b*dilog(1-I*c/x)-2*a*b*ln(c/
x)*arctan(c/x)-I*b^2*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))+2*I*b^
2*arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))+2*I*b^2*arctan(c/x)*po
lylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-I*a*b*ln(c/x)*ln(1+I*c/x)+I*a*b*ln(c/
x)*ln(1-I*c/x)-1/2*I*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2
/(1+c^2/x^2)+1))^3*arctan(c/x)^2+1/2*I*b^2*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)
-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-1/2*I*b^2*Pi*csgn(((1+I*c/
x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^2-a^2*ln(c/x
)-2*b^2*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))-2*b^2*polylog(3,-(1+I*c/x)/(
1+c^2/x^2)^(1/2))+1/2*I*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*(
(1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-1/2
*I*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*c
sgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \frac{1}{16}\int \frac{12\, b^2 \arctan(c, x)^2 + b^2 \log\left(c^2 + x^2\right)^2 + 32\, ab \arctan(c, x)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan2(c, x)^2 + b^2*log(c^2 + x^2)^2
+ 32*a*b*arctan2(c, x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan\left(\frac{c}{x}\right)^2 + 2\, ab \arctan\left(\frac{c}{x}\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c/x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(\frac{c}{x}\right) + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2/x, x)
```

3.145 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=96

$$-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c}-\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x}-\frac{2b\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c}$$

```
[Out] ((-I)*(a + b*ArcCot[x/c])^2)/c - (a + b*ArcCot[x/c])^2/x - (2*b*(a + b*ArcC
ot[x/c])*Log[2/(1 + (I*c)/x)])/c - (I*b^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)])/
c
```

Rubi [B] time = 0.527944, antiderivative size = 259, normalized size of antiderivative = 2.7, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$\frac{ib^2\text{PolyLog}\left(2,-\frac{-x+ic}{2x}\right)}{2c}-\frac{ib^2\text{PolyLog}\left(2,\frac{x+ic}{2x}\right)}{2c}+\frac{b\log\left(1+\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)}{2x}-\frac{ib\log\left(\frac{x+ic}{2x}\right)\left(2ia-b\log\left(1-\right.\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c/x])^2/x^2,x]
```

```
[Out] ((-I/4)*(1 - (I*c)/x)*(2*a + I*b*Log[1 - (I*c)/x])^2)/c + (b*((2*I)*a - b*L
og[1 - (I*c)/x])*Log[1 + (I*c)/x])/(2*x) - ((I/4)*b^2*(1 + (I*c)/x)*Log[1 +
 (I*c)/x]^2)/c - ((I/2)*b^2*Log[1 + (I*c)/x]*Log[-(I*c - x)/(2*x)])/c - ((I
/2)*b*((2*I)*a - b*Log[1 - (I*c)/x])*Log[(I*c + x)/(2*x)])/c + ((I/2)*b^2*P
olyLog[2, -(I*c - x)/(2*x)])/c - ((I/2)*b^2*PolyLog[2, (I*c + x)/(2*x)])/c
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4x^2} + \frac{b\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x^2} - \frac{b^2\log^2\left(1+\frac{ic}{x}\right)}{4x^2}\right)dx$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{x^2}\,dx + \frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{x^2}\,dx - \frac{1}{4}b^2\int \frac{\log \ldots}{}$$

$$= -\left(\frac{1}{4}\operatorname{Subst}\left(\int (2a+ib\log(1-icx))^2\,dx, x, \frac{1}{x}\right)\right) - \frac{1}{2}b\operatorname{Subst}\left(\int (-2ia+b\log(1-icx))\log \ldots\right.$$

$$= \frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x} - \frac{i\operatorname{Subst}\left(\int (2a+ib\log(x))^2\,dx, x, 1-\frac{ic}{x}\right)}{4c} - \frac{(ib^2)\operatorname{Sub}\ldots}{}$$

$$= -\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c} + \frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x} - \frac{ib^2\left(1+\frac{ic}{x}\right)\log^2 \ldots}{4c}$$

$$= \frac{iab}{x} + \frac{b^2}{2x} - \frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c} + \frac{ib^2\left(1+\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{2c} + \frac{b\left(2ia-b\log\left(1 \ldots\right.\right.}{2\ldots}$$

$$= \frac{b^2}{x} - \frac{ib^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{2c} - \frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c} + \frac{ib^2\left(1+\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{2c} \ldots$$

$$= \frac{b^2}{2x} - \frac{ib^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{2c} - \frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c} + \frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\ldots}{2x}$$

$$= -\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c} + \frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x} - \frac{ib^2\left(1+\frac{ic}{x}\right)\log^2 \ldots}{4c}$$

Mathematica [A] time = 0.111112, size = 107, normalized size = 1.11

$$-\frac{-ib^2x\operatorname{PolyLog}\left(2,-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right) + a\left(ac+2bx\log\left(\frac{1}{\sqrt{\frac{c^2}{x^2}+1}}\right)\right) + 2b\tan^{-1}\left(\frac{c}{x}\right)\left(ac+bx\log\left(1+e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)\right) + b^2(c-ix\ldots}{cx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x^2,x]
```

```
[Out] -((b^2*(c - I*x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*c + b*x*Log[1 + E^((2*I
)*ArcTan[c/x])]) + a*(a*c + 2*b*x*Log[1/Sqrt[1 + c^2/x^2]]) - I*b^2*x*PolyL
og[2, -E^((2*I)*ArcTan[c/x])])/(c*x))
```

Maple [A] time = 0.088, size = 147, normalized size = 1.5

$$-\frac{a^2}{x} + \frac{ib^2}{c}\left(\arctan\left(\frac{c}{x}\right)\right)^2 - \frac{b^2}{x}\left(\arctan\left(\frac{c}{x}\right)\right)^2 + \frac{ib^2}{c}\operatorname{polylog}\left(2,-\left(1+\frac{ic}{x}\right)^2\left(1+\frac{c^2}{x^2}\right)^{-1}\right) - 2\frac{b^2}{c}\arctan\left(\frac{c}{x}\right)\ln\left(\left(1+\frac{ic}{x}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2/x^2,x)
```

```
[Out] -a^2/x+I/c*arctan(c/x)^2*b^2-1/x*b^2*arctan(c/x)^2+I/c*polylog(2,-(1+I*c/x)
^2/(1+c^2/x^2))*b^2-2/c*arctan(c/x)*ln((1+I*c/x)^2/(1+c^2/x^2)+1)*b^2-2/x*a
*b*arctan(c/x)+1/c*a*b*ln(1+c^2/x^2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\arctan\left(\frac{c}{x}\right)^2+2\,ab\arctan\left(\frac{c}{x}\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c/x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2/x^2, x)
```

3.146 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=84

$$-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}+\frac{ab}{cx}-\frac{b^2\log\left(\frac{c^2}{x^2}+1\right)}{2c^2}+\frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx}$$

```
[Out] (a*b)/(c*x) + (b^2*ArcCot[x/c])/(c*x) - (a + b*ArcCot[x/c])^2/(2*c^2) - (a
+ b*ArcCot[x/c])^2/(2*x^2) - (b^2*Log[1 + c^2/x^2])/(2*c^2)
```

Rubi [C] time = 1.29363, antiderivative size = 836, normalized size of antiderivative = 9.95, number of steps used = 66, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 6742, 30, 2557, 12, 2466, 2462, 260, 2416, 2394, 2393, 2391, 2315}

$$-\frac{\left(1-\frac{ic}{x}\right)^2 b^2}{16c^2}-\frac{\left(\frac{ic}{x}+1\right)^2 b^2}{16c^2}-\frac{\left(\frac{ic}{x}+1\right)^2\log^2\left(\frac{ic}{x}+1\right)b^2}{8c^2}+\frac{\left(\frac{ic}{x}+1\right)\log^2\left(\frac{ic}{x}+1\right)b^2}{4c^2}+\frac{\log\left(i-\frac{c}{x}\right)b^2}{8c^2}-\frac{3\left(1-\frac{ic}{x}\right)\log\left(1\right.}{4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c/x])^2/x^3,x]
```

```
[Out] -(b^2*(1 - (I*c)/x)^2)/(16*c^2) - (b^2*(1 + (I*c)/x)^2)/(16*c^2) - ((I/4)*a
*b)/x^2 - b^2/(8*x^2) + (3*a*b)/(2*c*x) + ((I/2)*a*b*Log[I - c/x])/c^2 + (b
^2*Log[I - c/x])/(8*c^2) - (3*b^2*(1 - (I*c)/x)*Log[1 - (I*c)/x])/(4*c^2) +
 (b^2*Log[1 - (I*c)/x])/(8*x^2) - ((I/8)*b*(1 - (I*c)/x)^2*(2*a + I*b*Log[1
 - (I*c)/x]))/c^2 - ((1 - (I*c)/x)*(2*a + I*b*Log[1 - (I*c)/x])^2)/(4*c^2)
+ ((1 - (I*c)/x)^2*(2*a + I*b*Log[1 - (I*c)/x])^2)/(8*c^2) - (3*b^2*(1 + (I
*c)/x)*Log[1 + (I*c)/x])/(4*c^2) + (b^2*(1 + (I*c)/x)^2*Log[1 + (I*c)/x])/(
8*c^2) + ((I/2)*a*b*Log[1 + (I*c)/x])/x^2 + (b^2*Log[1 + (I*c)/x])/(8*x^2)
- (b^2*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/(4*x^2) + (b^2*(1 + (I*c)/x)*Log[
1 + (I*c)/x]^2)/(4*c^2) - (b^2*(1 + (I*c)/x)^2*Log[1 + (I*c)/x]^2)/(8*c^2)
+ (b^2*Log[I + c/x])/(8*c^2) - (b^2*Log[1 - (I*c)/x]*Log[c - I*x])/(4*c^2)
- (b^2*Log[1 + (I*c)/x]*Log[c + I*x])/(4*c^2) + (b^2*Log[(c - I*x)/(2*c)]*L
og[c + I*x])/(4*c^2) + (b^2*Log[c - I*x]*Log[(c + I*x)/(2*c)])/(4*c^2) - (b
^2*Log[c + I*x]*Log[((-I)*x)/c])/(4*c^2) - (b^2*Log[c - I*x]*Log[(I*x)/c])/
(4*c^2) + (b^2*PolyLog[2, (c - I*x)/(2*c)])/(4*c^2) + (b^2*PolyLog[2, (c +
I*x)/(2*c)])/(4*c^2) + (b^2*PolyLog[2, ((-I)*c)/x])/(4*c^2) + (b^2*PolyLog[
2, (I*c)/x])/(4*c^2) - (b^2*PolyLog[2, 1 - (I*x)/c])/(4*c^2) - (b^2*PolyLog
[2, 1 + (I*x)/c])/(4*c^2)
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4x^3} + \frac{b\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x^3} - \frac{b^2\log^2\left(1+\frac{ic}{x}\right)}{4x^3}\right) dx$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{x^3}\,dx + \frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{x^3}\,dx - \frac{1}{4}b^2\int \frac{\log}{}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int x(2a+ib\log(1-icx))^2\,dx, x, \frac{1}{x}\right)\right) + \frac{1}{2}b\int \left(-\frac{2ia\log\left(1+\frac{ic}{x}\right)}{x^3} + \frac{b\log\left(1-\right.}{}\right.$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \left(-\frac{i(2a+ib\log(1-icx))^2}{c} + \frac{i(1-icx)(2a+ib\log(1-icx))^2}{c}\right) dx, x, \frac{1}{x}\right)\right) -$$

$$= -\frac{b^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} + (iab)\,\text{Subst}\left(\int x\log(1+icx)\,dx, x, \frac{1}{x}\right) - \frac{1}{2}b^2\int \frac{c\log\left(1-\right.}{2(c-ix)x}$$

$$= \frac{iab\log\left(1+\frac{ic}{x}\right)}{2x^2} - \frac{b^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} - \frac{\text{Subst}\left(\int(2a+ib\log(x))^2\,dx, x, 1-\frac{ic}{x}\right)}{4c^2} + $$

$$= -\frac{\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c^2} + \frac{\left(1-\frac{ic}{x}\right)^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c^2} + \frac{iab\log\left(1+\frac{ic}{x}\right)}{2x^2} - \frac{b^2}{}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{ib^2}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{ib\left(1-\frac{ic}{x}\right)^2\left(2a+ib\log\right.}{8c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{b^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{2c^2} - \frac{ib\left(\right.}{}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{b^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{2c^2} + \frac{b^2}{}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{3b^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{4c^2} + \frac{b^2}{}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} - \frac{b^2}{8x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} + \frac{b^2\log\left(i-\frac{c}{x}\right)}{8c^2} - \frac{3b^2\left(\right.}{}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} - \frac{b^2}{8x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} + \frac{b^2\log\left(i-\frac{c}{x}\right)}{8c^2} - \frac{3b^2\left(\right.}{}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} - \frac{b^2}{8x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} + \frac{b^2\log\left(i-\frac{c}{x}\right)}{8c^2} - \frac{3b^2\left(\right.}{}$$

Mathematica [A] time = 0.073959, size = 99, normalized size = 1.18

$$-\frac{a^2c^2 - 2abx^2\tan^{-1}\left(\frac{x}{c}\right) - 2abcx + 2bc\tan^{-1}\left(\frac{c}{x}\right)(ac-bx) + b^2x^2\log\left(c^2+x^2\right) + b^2\left(c^2+x^2\right)\tan^{-1}\left(\frac{c}{x}\right)^2 - 2b^2x^2\log(}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x^3,x]
```

```
[Out] -(a^2*c^2 - 2*a*b*c*x + 2*b*c*(a*c - b*x)*ArcTan[c/x] + b^2*(c^2 + x^2)*Arc
Tan[c/x]^2 - 2*a*b*x^2*ArcTan[x/c] - 2*b^2*x^2*Log[x] + b^2*x^2*Log[c^2 + x
```

```
^2])/(2*c^2*x^2)
```

Maple [A] time = 0.034, size = 110, normalized size = 1.3

$$-\frac{a^2}{2x^2}-\frac{b^2}{2x^2}\left(\arctan\left(\frac{c}{x}\right)\right)^2-\frac{b^2}{2c^2}\left(\arctan\left(\frac{c}{x}\right)\right)^2+\frac{b^2}{cx}\arctan\left(\frac{c}{x}\right)-\frac{b^2}{2c^2}\ln\left(1+\frac{c^2}{x^2}\right)-\frac{ab}{x^2}\arctan\left(\frac{c}{x}\right)+\frac{ab}{c^2}\arctan\left(\frac{x}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2/x^2*b^2*arctan(c/x)^2-1/2/c^2*b^2*arctan(c/x)^2+1/c*b^2*ar
ctan(c/x)/x-1/2*b^2*ln(1+c^2/x^2)/c^2-b/x^2*a*arctan(c/x)+1/c^2*a*b*arctan(
x/c)+a*b/c/x
```

Maxima [A] time = 1.62002, size = 158, normalized size = 1.88

$$\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)-\frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)ab+\frac{1}{2}\left(2c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)\arctan\left(\frac{c}{x}\right)+\frac{\arctan\left(x,c\right)^2-\log\left(c^2+x^2\right)+2\log}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="maxima")
```

```
[Out] (c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a*b + 1/2*(2*c*(arctan(
x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan2(x, c)^2 - log(c^2 + x^2) + 2*l
og(x))/c^2)*b^2 - 1/2*b^2*arctan(c/x)^2/x^2 - 1/2*a^2/x^2
```

Fricas [A] time = 2.31232, size = 239, normalized size = 2.85

$$\frac{2abx^2\arctan\left(\frac{x}{c}\right)-b^2x^2\log\left(c^2+x^2\right)+2b^2x^2\log\left(x\right)-a^2c^2+2abcx-\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2-2\left(abc^2-b^2cx\right)\operatorname{arct}}{2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x^2*arctan(x/c) - b^2*x^2*log(c^2 + x^2) + 2*b^2*x^2*log(x) - a^
2*c^2 + 2*a*b*c*x - (b^2*c^2 + b^2*x^2)*arctan(c/x)^2 - 2*(a*b*c^2 - b^2*c*
x)*arctan(c/x))/(c^2*x^2)
```

Sympy [A] time = 1.43797, size = 117, normalized size = 1.39

$$\begin{cases}-\frac{a^2}{2x^2}-\frac{ab\operatorname{atan}\left(\frac{c}{x}\right)}{x^2}+\frac{ab}{cx}-\frac{ab\operatorname{atan}\left(\frac{c}{x}\right)}{c^2}-\frac{b^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2x^2}+\frac{b^2\operatorname{atan}\left(\frac{c}{x}\right)}{cx}+\frac{b^2\log\left(x\right)}{c^2}-\frac{b^2\log\left(c^2+x^2\right)}{2c^2}-\frac{b^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2c^2} & \text{for}\ c\neq 0\\-\frac{a^2}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*atan(c/x)/x**2 + a*b/(c*x) - a*b*atan(c/x)/
c**2 - b**2*atan(c/x)**2/(2*x**2) + b**2*atan(c/x)/(c*x) + b**2*log(x)/c**2
 - b**2*log(c**2 + x**2)/(2*c**2) - b**2*atan(c/x)**2/(2*c**2), Ne(c, 0)),
(-a**2/(2*x**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(\frac{c}{x}\right) + a\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2/x^3, x)
```

3.147 $\int x^3 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 \, dx$

Optimal. Leaf size=214

$$-ib^3c^4\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)+\frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+2b^2c^4\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{4}c^4\left(a+b\cot\right.$$

```
[Out] (b^3*c^3*x)/4 + (b^3*c^4*ArcCot[x/c])/4 + (b^2*c^2*x^2*(a + b*ArcCot[x/c]))
/4 - I*b*c^4*(a + b*ArcCot[x/c])^2 - (3*b*c^3*x*(a + b*ArcCot[x/c])^2)/4 +
(b*c*x^3*(a + b*ArcCot[x/c])^2)/4 - (c^4*(a + b*ArcCot[x/c])^3)/4 + (x^4*(a
 + b*ArcCot[x/c])^3)/4 + 2*b^2*c^4*(a + b*ArcCot[x/c])*Log[2 - 2/(1 - (I*c)
/x)] - I*b^3*c^4*PolyLog[2, -1 + 2/(1 - (I*c)/x)]
```

Rubi [F] time = 4.62858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (-3*a^2*b*c^3*x)/8 - ((5*I)/16)*a*b^2*c^3*x + (b^3*c^3*x)/16 - ((3*I)/16)*a
^2*b*c^2*x^2 + (3*a*b^2*c^2*x^2)/16 + (a^2*b*c*x^3)/8 - (11*a*b^2*c^4*Log[I
 - c/x])/16 - (I/32)*b^3*c^4*Log[I - c/x] - ((3*I)/8)*a*b^2*c^3*x*Log[1 - (
I*c)/x] + (3*a*b^2*c^2*x^2*Log[1 - (I*c)/x])/16 + (I/8)*a*b^2*c*x^3*Log[1 -
 (I*c)/x] + ((5*I)/32)*b^2*c^3*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x])
 + (b^2*c^2*x^2*(2*a + I*b*Log[1 - (I*c)/x]))/32 + ((5*I)/64)*b*c^4*(2*a +
I*b*Log[1 - (I*c)/x])^2 - (3*b*c^3*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)
/x])^2)/32 + ((3*I)/64)*b*c^2*x^2*(2*a + I*b*Log[1 - (I*c)/x])^2 + (b*c*x^3
*(2*a + I*b*Log[1 - (I*c)/x])^2)/32 - (c^4*(2*a + I*b*Log[1 - (I*c)/x])^3)/
32 + (x^4*(2*a + I*b*Log[1 - (I*c)/x])^3)/32 + ((3*I)/4)*a*b^2*c^3*x*Log[1
+ (I*c)/x] - (5*b^3*c^3*(1 + (I*c)/x)*x*Log[1 + (I*c)/x])/32 - (I/32)*b^3*c
^2*x^2*Log[1 + (I*c)/x] - (I/4)*a*b^2*c*x^3*Log[1 + (I*c)/x] - ((3*I)/8)*a^
2*b*x^4*Log[1 + (I*c)/x] + (3*a*b^2*x^4*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/
8 + (3*a*b^2*c^4*Log[1 + (I*c)/x]^2)/16 + ((5*I)/64)*b^3*c^4*Log[1 + (I*c)/
x]^2 + (3*b^3*c^3*(1 + (I*c)/x)*x*Log[1 + (I*c)/x]^2)/32 + ((3*I)/64)*b^3*c
^2*x^2*Log[1 + (I*c)/x]^2 - (b^3*c*x^3*Log[1 + (I*c)/x]^2)/32 - (3*a*b^2*x^
4*Log[1 + (I*c)/x]^2)/16 - (I/32)*b^3*c^4*Log[1 + (I*c)/x]^3 + (I/32)*b^3*x
^4*Log[1 + (I*c)/x]^3 + (I/32)*b^3*c^4*Log[I + c/x] + ((3*I)/8)*a^2*b*c^4*L
og[c - I*x] - (5*a*b^2*c^4*Log[c - I*x])/16 - (3*a*b^2*c^4*Log[1 - (I*c)/x]
*Log[c - I*x])/8 - (5*a*b^2*c^4*Log[c + I*x])/16 - (3*a*b^2*c^4*Log[1 + (I*
c)/x]*Log[c + I*x])/8 + (3*a*b^2*c^4*Log[(c - I*x)/(2*c)]*Log[c + I*x])/8 +
 (3*a*b^2*c^4*Log[c - I*x]*Log[(c + I*x)/(2*c)])/8 + ((3*I)/32)*b^3*c^4*Log
[1 + (I*c)/x]^2*Log[((-I)*c)/x] + ((3*I)/32)*b*c^4*(2*a + I*b*Log[1 - (I*c)
/x])^2*Log[(I*c)/x] - (11*a*b^2*c^4*Log[x])/8 - (3*a*b^2*c^4*Log[c + I*x]*L
og[((-I)*x)/c])/8 - (3*a*b^2*c^4*Log[c - I*x]*Log[(I*x)/c])/8 - (3*b^2*c^4*
(2*a + I*b*Log[1 - (I*c)/x])*PolyLog[2, 1 - (I*c)/x])/16 + ((3*I)/16)*b^3*c
^4*Log[1 + (I*c)/x]*PolyLog[2, 1 + (I*c)/x] + (3*a*b^2*c^4*PolyLog[2, (c -
I*x)/(2*c)])/8 + (3*a*b^2*c^4*PolyLog[2, (c + I*x)/(2*c)])/8 + (3*a*b^2*c^4
*PolyLog[2, ((-I)*c)/x])/8 + ((11*I)/32)*b^3*c^4*PolyLog[2, ((-I)*c)/x] - (
(11*I)/32)*b^3*c^4*PolyLog[2, (I*c)/x] - (3*a*b^2*c^4*PolyLog[2, 1 - (I*x)/
c])/8 - (3*a*b^2*c^4*PolyLog[2, 1 + (I*x)/c])/8 + ((3*I)/16)*b^3*c^4*PolyLo
g[3, 1 - (I*c)/x] - ((3*I)/16)*b^3*c^4*PolyLog[3, 1 + (I*c)/x] + ((3*I)/8)*
```

```
b^3*Defer[Int][x^3*Log[1 - (I*c)/x]^2*Log[1 + (I*c)/x], x] - ((3*I)/8)*b^3*
Defer[Int][x^3*Log[1 - (I*c)/x]*Log[1 + (I*c)/x]^2, x]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int\left(\frac{1}{8}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{3}{8}ibx^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)-\frac{3}{8}ib^2x^3$$

$$=\frac{1}{8}\int x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3 dx+\frac{1}{8}(3ib)\int x^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^3}{x^5}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3ib)\int\left(-4a^2x^3\log\left(1+\frac{ic}{x}\right)-4i\right.$$

$$=\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{1}{32}ib^3x^4\log^3\left(1+\frac{ic}{x}\right)-\frac{1}{2}\left(3ia^2b\right)\int x^3\log\left(1+\frac{ic}{x}\right)dx$$

$$=\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{8}ia^2bx^4\log\left(1+\frac{ic}{x}\right)+\frac{3}{8}ab^2x^4\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)$$

$$=\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{8}ia^2bx^4\log\left(1+\frac{ic}{x}\right)+\frac{3}{8}ab^2x^4\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)$$

$$=\frac{1}{32}bcx^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{8}ia^2bx^4\log\left(1+\frac{ic}{x}\right)+$$

$$=-\frac{3}{8}a^2bc^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{8}a^2bcx^3+\frac{3}{64}ibc^2x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{32}bcx^3\left(2a+\right.$$

$$=-\frac{3}{8}a^2bc^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}iab^2c^3x\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}ab^2c^2x^2\log\left(1-\frac{ic}{x}\right)+$$

$$=-\frac{3}{8}a^2bc^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}iab^2c^3x\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}ab^2c^2x^2\log\left(1-\frac{ic}{x}\right)+$$

$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-\frac{11}{16}ab^2c^4\log$$

$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-\frac{11}{16}ab^2c^4\log$$

$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-\frac{11}{16}ab^2c^4\log$$

$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-\frac{11}{16}ab^2c^4\log$$

Mathematica [A] time = 0.590986, size = 253, normalized size = 1.18

$$\frac{1}{4}\left(-4ib^3c^4\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)+b\tan^{-1}\left(\frac{c}{x}\right)\left(3a^2\left(x^4-c^4\right)+2abcx\left(x^2-3c^2\right)+b^2c^2\left(c^2+x^2\right)+8b^2c^4\log\left(1-e\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (a*b^2*c^4 - 3*a^2*b*c^3*x + b^3*c^3*x + a*b^2*c^2*x^2 + a^2*b*c*x^3 + a^3*
x^4 + b^2*(b*c*((-4*I)*c^3 - 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*ArcTan[c/x]
^2 + b^3*(-c^4 + x^4)*ArcTan[c/x]^3 + b*ArcTan[c/x]*(2*a*b*c*x*(-3*c^2 + x^
2) + b^2*c^2*(c^2 + x^2) + 3*a^2*(-c^4 + x^4) + 8*b^2*c^4*Log[1 - E^((2*I)*
ArcTan[c/x])]) + 8*a*b^2*c^4*Log[c/(Sqrt[1 + c^2/x^2]*x)] - (4*I)*b^3*c^4*P
olyLog[2, E^((2*I)*ArcTan[c/x])])/4
```

Maple [B] time = 0.108, size = 608, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c/x))^3,x)
```

```
[Out] 1/2*I*c^4*b^3*ln(1+c^2/x^2)*ln(c/x+I)+1/2*I*c^4*b^3*ln(c/x-I)*ln(-1/2*I*(c/
x+I))-I*c^4*b^3*ln(c/x)*ln(1-I*c/x)+I*c^4*b^3*ln(c/x)*ln(1+I*c/x)+1/2*c*a*b
^2*arctan(c/x)*x^3-3/2*c^3*a*b^2*arctan(c/x)*x-1/2*I*c^4*b^3*ln(c/x+I)*ln(1
/2*I*(c/x-I))-1/4*I*c^4*b^3*ln(c/x+I)^2-1/2*I*c^4*b^3*dilog(1/2*I*(c/x-I))-
1/2*I*c^4*b^3*ln(1+c^2/x^2)*ln(c/x-I)+1/4*b^3*x^4*arctan(c/x)^3-1/4*c^4*b^3
*arctan(c/x)^3-1/4*c^4*b^3*arctan(x/c)+1/4*b^3*c^3*x-3/4*c^3*b^3*arctan(c/x
)^2*x+1/4*c^2*b^3*arctan(c/x)*x^2+3/4*a^2*b*x^4*arctan(c/x)+3/4*a*b^2*x^4*a
rctan(c/x)^2+1/2*I*c^4*b^3*dilog(-1/2*I*(c/x+I))+1/4*I*c^4*b^3*ln(c/x-I)^2+
1/4*c*b^3*arctan(c/x)^2*x^3-3/4*c^3*a^2*b*x-c^4*b^3*arctan(c/x)*ln(1+c^2/x^
2)+2*c^4*b^3*ln(c/x)*arctan(c/x)-c^4*a*b^2*ln(1+c^2/x^2)+I*c^4*b^3*dilog(1+
I*c/x)+3/4*c^4*a^2*b*arctan(x/c)-3/4*c^4*a*b^2*arctan(c/x)^2+2*c^4*a*b^2*ln
(c/x)+1/4*x^4*a^3-I*c^4*b^3*dilog(1-I*c/x)+1/4*c*a^2*b*x^3+1/4*c^2*x^2*a*b^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctan(c/x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c/x) + (3*c^3
*arctan(x/c) - 3*c^2*x + x^3)*c)*a^2*b + 1/4*((3*c^2*arctan2(x, c)^2 - 4*c^
2*log(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arcta
n(c/x))*a*b^2 - 1/64*(12*c^4*arctan(c/x)^2*arctan(x/c) + 8*c^4*arctan2(c, x
)^3 - 8*x^4*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c
)^3/c)*c^5 + 12*c^3*x*arctan2(c, x)^2 - 4*c*x^3*arctan2(c, x)^2 + 192*c^5*i
ntegrate(1/64*log(c^2 + x^2)^2/(c^2 + x^2), x) + 1536*c^4*integrate(1/64*x*
arctan(c/x)/(c^2 + x^2), x) + 768*c^3*integrate(1/64*x^2*log(c^2 + x^2)/(c^
2 + x^2), x) - 2048*c^2*integrate(1/64*x^3*arctan(c/x)^3/(c^2 + x^2), x) -
512*c^2*integrate(1/64*x^3*arctan(c/x)/(c^2 + x^2), x) - (3*c^3*x - c*x^3)*
log(c^2 + x^2)^2 - 768*c*integrate(1/64*x^4*arctan(c/x)^2/(c^2 + x^2), x) -
 192*c*integrate(1/64*x^4*log(c^2 + x^2)^2/(c^2 + x^2), x) - 256*c*integrat
e(1/64*x^4*log(c^2 + x^2)/(c^2 + x^2), x) - 2048*integrate(1/64*x^5*arctan(
c/x)^3/(c^2 + x^2), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^3\arctan\left(\frac{c}{x}\right)^3 + 3\,ab^2x^3\arctan\left(\frac{c}{x}\right)^2 + 3\,a^2bx^3\arctan\left(\frac{c}{x}\right) + a^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c/x)^3 + 3*a*b^2*x^3*arctan(c/x)^2 + 3*a^2*b*x^3*ar
ctan(c/x) + a^3*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a + b\,\text{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c/x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atan(c/x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(\frac{c}{x}\right) + a\right)^3 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3*x^3, x)
```

3.148 $\int x^2 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=229

$$-ib^2c^3\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^3c^3\text{PolyLog}\left(3,-1+\frac{2}{1-\frac{ic}{x}}\right)+b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{3}ic^3\left(a\right.$$

```
[Out] b^2*c^2*x*(a + b*ArcCot[x/c]) + (b*c^3*(a + b*ArcCot[x/c])^2)/2 + (b*c*x^2*
(a + b*ArcCot[x/c])^2)/2 - (I/3)*c^3*(a + b*ArcCot[x/c])^3 + (x^3*(a + b*Ar
cCot[x/c])^3)/3 + b*c^3*(a + b*ArcCot[x/c])^2*Log[2 - 2/(1 - (I*c)/x)] + (b
^3*c^3*Log[1 + c^2/x^2])/2 + b^3*c^3*Log[x] - I*b^2*c^3*(a + b*ArcCot[x/c])
*PolyLog[2, -1 + 2/(1 - (I*c)/x)] + (b^3*c^3*PolyLog[3, -1 + 2/(1 - (I*c)/x
)])/2
```

Rubi [F] time = 3.52024, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (-I/2)*a^2*b*c^2*x + (3*a*b^2*c^2*x)/4 + (a^2*b*c*x^2)/4 - ((3*I)/4)*a*b^2*
c^3*Log[I - c/x] + (a*b^2*c^2*x*Log[1 - (I*c)/x])/2 + (I/4)*a*b^2*c*x^2*Log
[1 - (I*c)/x] + (b^2*c^2*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x]))/8 +
(b*c^3*(2*a + I*b*Log[1 - (I*c)/x])^2)/16 + (I/8)*b*c^2*(1 - (I*c)/x)*x*(2*
a + I*b*Log[1 - (I*c)/x])^2 + (b*c*x^2*(2*a + I*b*Log[1 - (I*c)/x])^2)/16 +
 (I/24)*c^3*(2*a + I*b*Log[1 - (I*c)/x])^3 + (x^3*(2*a + I*b*Log[1 - (I*c)/
x])^3)/24 - (I/8)*b^3*c^2*(1 + (I*c)/x)*x*Log[1 + (I*c)/x] - (I/2)*a*b^2*c*
x^2*Log[1 + (I*c)/x] - (I/2)*a^2*b*x^3*Log[1 + (I*c)/x] + (a*b^2*x^3*Log[1
- (I*c)/x]*Log[1 + (I*c)/x])/2 + (I/4)*a*b^2*c^3*Log[1 + (I*c)/x]^2 - (b^3*
c^3*Log[1 + (I*c)/x]^2)/16 + (I/8)*b^3*c^2*(1 + (I*c)/x)*x*Log[1 + (I*c)/x]
^2 - (b^3*c*x^2*Log[1 + (I*c)/x]^2)/16 - (a*b^2*x^3*Log[1 + (I*c)/x]^2)/4 +
 (b^3*c^3*Log[1 + (I*c)/x]^3)/24 + (I/24)*b^3*x^3*Log[1 + (I*c)/x]^3 - (a^2
*b*c^3*Log[c - I*x])/2 + (I/4)*a*b^2*c^3*Log[c - I*x] - (I/2)*a*b^2*c^3*Log
[1 - (I*c)/x]*Log[c - I*x] - (I/4)*a*b^2*c^3*Log[c + I*x] + (I/2)*a*b^2*c^3
*Log[1 + (I*c)/x]*Log[c + I*x] - (I/2)*a*b^2*c^3*Log[(c - I*x)/(2*c)]*Log[c
 + I*x] + (I/2)*a*b^2*c^3*Log[c - I*x]*Log[(c + I*x)/(2*c)] - (b^3*c^3*Log[
1 + (I*c)/x]^2*Log[((-I)*c)/x])/8 + (b*c^3*(2*a + I*b*Log[1 - (I*c)/x])^2*L
og[(I*c)/x])/8 + (b^3*c^3*Log[x])/4 + (I/2)*a*b^2*c^3*Log[c + I*x]*Log[((-I
)*x)/c] - (I/2)*a*b^2*c^3*Log[c - I*x]*Log[(I*x)/c] + (I/4)*b^2*c^3*(2*a +
I*b*Log[1 - (I*c)/x])*PolyLog[2, 1 - (I*c)/x] - (b^3*c^3*Log[1 + (I*c)/x]*P
olyLog[2, 1 + (I*c)/x])/4 + (I/2)*a*b^2*c^3*PolyLog[2, (c - I*x)/(2*c)] - (
I/2)*a*b^2*c^3*PolyLog[2, (c + I*x)/(2*c)] + (I/2)*a*b^2*c^3*PolyLog[2, ((-
I)*c)/x] - (3*b^3*c^3*PolyLog[2, ((-I)*c)/x])/8 - (3*b^3*c^3*PolyLog[2, (I*
c)/x])/8 - (I/2)*a*b^2*c^3*PolyLog[2, 1 - (I*x)/c] + (I/2)*a*b^2*c^3*PolyLo
g[2, 1 + (I*x)/c] + (b^3*c^3*PolyLog[3, 1 - (I*c)/x])/4 + (b^3*c^3*PolyLog[
3, 1 + (I*c)/x])/4 + ((3*I)/8)*b^3*Defer[Int][x^2*Log[1 - (I*c)/x]^2*Log[1
+ (I*c)/x], x] - ((3*I)/8)*b^3*Defer[Int][x^2*Log[1 - (I*c)/x]*Log[1 + (I*c
)/x]^2, x]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3dx = \int\left(\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{3}{8}ibx^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)-\frac{3}{8}ib^2x^2\right.$$

$$= \frac{1}{8}\int x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3dx+\frac{1}{8}(3ib)\int x^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^3}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3ib)\int\left(-4a^2x^2\log\left(1+\frac{ic}{x}\right)-4i\right.$$

$$= \frac{1}{24}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{1}{24}ib^3x^3\log^3\left(1+\frac{ic}{x}\right)-\frac{1}{2}\left(3ia^2b\right)\int x^2\log\left(1+\frac{ic}{x}\right)dx$$

$$= \frac{1}{24}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{1}{2}ia^2bx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}ab^2x^3\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)$$

$$= \frac{1}{24}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{1}{2}ia^2bx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}ab^2x^3\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)$$

$$= \frac{1}{16}bcx^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{24}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{1}{2}ia^2bx^3\log\left(1+\frac{ic}{x}\right)+$$

$$= -\frac{1}{2}ia^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{8}ibc^2\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{16}bcx^2\left(2a+ib\log\right.$$

$$= -\frac{1}{2}ia^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}iab^2cx^2\log\left(1-\frac{ic}{x}\right)+\frac{1}{8}b^2c^2\left(1-\frac{ic}{x}\right)$$

$$= -\frac{1}{2}ia^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}iab^2cx^2\log\left(1-\frac{ic}{x}\right)+\frac{1}{8}b^2c^2\left(1-\frac{ic}{x}\right)$$

$$= -\frac{1}{2}ia^2bc^2x+\frac{1}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2-\frac{3}{4}iab^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{2}ab^2c^2x\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}iab^2$$

$$= -\frac{1}{2}ia^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2-\frac{3}{4}iab^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{2}ab^2c^2x\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}iab^2$$

$$= -\frac{1}{2}ia^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2-\frac{3}{4}iab^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{2}ab^2c^2x\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}iab^2$$

$$= -\frac{1}{2}ia^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2-\frac{3}{4}iab^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{2}ab^2c^2x\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}iab^2$$

<u>Mathematica</u> [A] time = 0.678251, size = 330, normalized size = 1.44

$$ab^2\left(-ic^3\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)+\left(x^3-ic^3\right)\tan^{-1}\left(\frac{c}{x}\right)^2+c\tan^{-1}\left(\frac{c}{x}\right)\left(2c^2\log\left(1-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)+c^2+x^2\right)+c^2x\right)+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (a^2*b*c*x^2)/2 + (a^3*x^3)/3 + a^2*b*x^3*ArcTan[c/x] - (a^2*b*c^3*Log[c^2
+ x^2])/2 + a*b^2*(c^2*x + ((-I)*c^3 + x^3)*ArcTan[c/x]^2 + c*ArcTan[c/x]*(
c^2 + x^2 + 2*c^2*Log[1 - E^((2*I)*ArcTan[c/x])]) - I*c^3*PolyLog[2, E^((2*
```

```
I)*ArcTan[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTan[c/x] + 12*c^3*Arc
Tan[c/x]^2 + 12*c*x^2*ArcTan[c/x]^2 + (8*I)*c^3*ArcTan[c/x]^3 + 8*x^3*ArcTa
n[c/x]^3 + 24*c^3*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c/x])] - 24*c^3*Lo
g[c/(Sqrt[1 + c^2/x^2]*x)] + (24*I)*c^3*ArcTan[c/x]*PolyLog[2, E^((-2*I)*Ar
cTan[c/x])] + 12*c^3*PolyLog[3, E^((-2*I)*ArcTan[c/x])]))/24
```

Maple [C] time = 1.027, size = 6441, normalized size = 28.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c/x))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}\,b^3x^3\arctan\left(c,x\right)^3-\frac{1}{32}\,b^3x^3\arctan\left(c,x\right)\log\left(c^2+x^2\right)^2+\frac{1}{3}\,a^3x^3+\frac{1}{2}\left(2\,x^3\arctan\left(\frac{c}{x}\right)-\left(c^2\log\left(c^2+x^2\right)-x^2\right)c\right)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan2(c, x)^3 - 1/32*b^3*x^3*arctan2(c, x)*log(c^2 + x^2)^2
+ 1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a^2*
b + integrate(1/32*(4*b^3*c*x^3*arctan2(c, x)^2 + 4*b^3*x^4*arctan2(c, x)*l
og(c^2 + x^2) + 4*(7*b^3*arctan2(c, x)^3 + 24*a*b^2*arctan2(c, x)^2)*x^4 +
4*(7*b^3*c^2*arctan2(c, x)^3 + 24*a*b^2*c^2*arctan2(c, x)^2)*x^2 + (3*b^3*c
^2*x^2*arctan2(c, x) + 3*b^3*x^4*arctan2(c, x) - b^3*c*x^3)*log(c^2 + x^2)^
2)/(c^2 + x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^2\arctan\left(\frac{c}{x}\right)^3+3\,ab^2x^2\arctan\left(\frac{c}{x}\right)^2+3\,a^2bx^2\arctan\left(\frac{c}{x}\right)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctan(c/x)^3 + 3*a*b^2*x^2*arctan(c/x)^2 + 3*a^2*b*x^2*ar
ctan(c/x) + a^3*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\,\text{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c/x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atan(c/x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3*x^2, x)
```

3.149 $\int x \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 \, dx$

Optimal. Leaf size=145

$$\frac{3}{2}ib^3c^2\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)-3b^2c^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}c^2\left(a+b\cot^{-1}\right.$$

```
[Out] ((3*I)/2)*b*c^2*(a + b*ArcCot[x/c])^2 + (3*b*c*x*(a + b*ArcCot[x/c])^2)/2 +
 (c^2*(a + b*ArcCot[x/c])^3)/2 + (x^2*(a + b*ArcCot[x/c])^3)/2 - 3*b^2*c^2*
(a + b*ArcCot[x/c])*Log[2 - 2/(1 - (I*c)/x)] + ((3*I)/2)*b^3*c^2*PolyLog[2,
 -1 + 2/(1 - (I*c)/x)]
```

Rubi [F] time = 2.35191, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (3*a^2*b*c*x)/4 + (3*a*b^2*c^2*Log[I - c/x])/4 + ((3*I)/4)*a*b^2*c*x*Log[1
- (I*c)/x] + (3*b*c*(1 - (I*c)/x)*x*(2*a + I*b*Log[1 - (I*c)/x])^2)/16 + (c
^2*(2*a + I*b*Log[1 - (I*c)/x])^3)/16 + (x^2*(2*a + I*b*Log[1 - (I*c)/x])^3
)/16 - ((3*I)/2)*a*b^2*c*x*Log[1 + (I*c)/x] - ((3*I)/4)*a^2*b*x^2*Log[1 + (
I*c)/x] + (3*a*b^2*x^2*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/4 - (3*a*b^2*c^2*
Log[1 + (I*c)/x]^2)/8 - (3*b^3*c*(1 + (I*c)/x)*x*Log[1 + (I*c)/x]^2)/16 - (
3*a*b^2*x^2*Log[1 + (I*c)/x]^2)/8 + (I/16)*b^3*c^2*Log[1 + (I*c)/x]^3 + (I/
16)*b^3*x^2*Log[1 + (I*c)/x]^3 - ((3*I)/4)*a^2*b*c^2*Log[c - I*x] + (3*a*b^
2*c^2*Log[c - I*x])/4 + (3*a*b^2*c^2*Log[1 - (I*c)/x]*Log[c - I*x])/4 + (3*
a*b^2*c^2*Log[c + I*x])/4 + (3*a*b^2*c^2*Log[1 + (I*c)/x]*Log[c + I*x])/4 -
 (3*a*b^2*c^2*Log[(c - I*x)/(2*c)]*Log[c + I*x])/4 - (3*a*b^2*c^2*Log[c - I
*x]*Log[(c + I*x)/(2*c)])/4 - ((3*I)/16)*b^3*c^2*Log[1 + (I*c)/x]^2*Log[((-
I)*c)/x] - ((3*I)/16)*b*c^2*(2*a + I*b*Log[1 - (I*c)/x])^2*Log[(I*c)/x] + (
3*a*b^2*c^2*Log[x])/2 + (3*a*b^2*c^2*Log[c + I*x]*Log[((-I)*x)/c])/4 + (3*a
*b^2*c^2*Log[c - I*x]*Log[(I*x)/c])/4 + (3*b^2*c^2*(2*a + I*b*Log[1 - (I*c)
/x])*PolyLog[2, 1 - (I*c)/x])/8 - ((3*I)/8)*b^3*c^2*Log[1 + (I*c)/x]*PolyLo
g[2, 1 + (I*c)/x] - (3*a*b^2*c^2*PolyLog[2, (c - I*x)/(2*c)])/4 - (3*a*b^2*
c^2*PolyLog[2, (c + I*x)/(2*c)])/4 - (3*a*b^2*c^2*PolyLog[2, ((-I)*c)/x])/4
 - ((3*I)/8)*b^3*c^2*PolyLog[2, ((-I)*c)/x] + ((3*I)/8)*b^3*c^2*PolyLog[2,
(I*c)/x] + (3*a*b^2*c^2*PolyLog[2, 1 - (I*x)/c])/4 + (3*a*b^2*c^2*PolyLog[2
, 1 + (I*x)/c])/4 - ((3*I)/8)*b^3*c^2*PolyLog[3, 1 - (I*c)/x] + ((3*I)/8)*b
^3*c^2*PolyLog[3, 1 + (I*c)/x] + ((3*I)/8)*b^3*Defer[Int][x*Log[1 - (I*c)/x
]^2*Log[1 + (I*c)/x], x] - ((3*I)/8)*b^3*Defer[Int][x*Log[1 - (I*c)/x]*Log[
1 + (I*c)/x]^2, x]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(\frac{1}{8}x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{3}{8}ibx\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)-\frac{3}{8}ib^2x\left(-2\right.\right.$$

$$=\frac{1}{8}\int x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3 dx+\frac{1}{8}(3ib)\int x\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)dx$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^3}{x^3}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3ib)\int\left(-4a^2x\log\left(1+\frac{ic}{x}\right)-4iab\right.$$

$$=\frac{1}{16}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{1}{16}ib^3x^2\log^3\left(1+\frac{ic}{x}\right)-\frac{1}{2}\left(3ia^2b\right)\int x\log\left(1+\frac{ic}{x}\right)dx-$$

$$=\frac{1}{16}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{4}ia^2bx^2\log\left(1+\frac{ic}{x}\right)+\frac{3}{4}ab^2x^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)+$$

$$=\frac{1}{16}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{4}ia^2bx^2\log\left(1+\frac{ic}{x}\right)+\frac{3}{4}ab^2x^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)-$$

$$=\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{4}ia^2bx^2\log\left(1+\right.$$

$$=\frac{3}{4}a^2bcx+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{4}ia^2b$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{16}c^2\left(2a+ib\right.$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{16}c^2\left(2a+ib\right.$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\left(i-\frac{c}{x}\right)+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\left(i-\frac{c}{x}\right)+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\left(i-\frac{c}{x}\right)+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$

$$=\frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\left(i-\frac{c}{x}\right)+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\right.\right.$$

Mathematica [A] time = 0.274443, size = 174, normalized size = 1.2

$$\frac{1}{2}\left(3ib^3c^2\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)+a\left(ax(ax+3bc)-6b^2c^2\log\left(\frac{c}{x\sqrt{\frac{c^2}{x^2}+1}}\right)\right)+3b^2\tan^{-1}\left(\frac{c}{x}\right)^2\left(a\left(c^2+x^2\right)+bc(x+i\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (3*b^2*(b*c*(I*c + x) + a*(c^2 + x^2))*ArcTan[c/x]^2 + b^3*(c^2 + x^2)*ArcT
an[c/x]^3 + 3*b*ArcTan[c/x]*(a*(2*b*c*x + a*(c^2 + x^2)) - 2*b^2*c^2*Log[1
- E^((2*I)*ArcTan[c/x])]) + a*(a*x*(3*b*c + a*x) - 6*b^2*c^2*Log[c/(Sqrt[1
+ c^2/x^2]*x)]) + (3*I)*b^3*c^2*PolyLog[2, E^((2*I)*ArcTan[c/x])])/2
```

Maple [B] time = 0.104, size = 507, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c/x))^3,x)
```

```
[Out] 3/2*I*c^2*b^3*dilog(1-I*c/x)+3/2*c^2*a*b^2*ln(1+c^2/x^2)+3/2*c*x*a^2*b+3/2*
c^2*b^3*arctan(c/x)*ln(1+c^2/x^2)-3*c^2*b^3*ln(c/x)*arctan(c/x)+3/2*a^2*b*x
^2*arctan(c/x)+3/2*a*b^2*arctan(c/x)^2*x^2+3/8*I*c^2*b^3*ln(c/x+I)^2-3/4*I*
c^2*b^3*dilog(-1/2*I*(c/x+I))-3/8*I*c^2*b^3*ln(c/x-I)^2-3/2*I*c^2*b^3*dilog
(1+I*c/x)+3/4*I*c^2*b^3*dilog(1/2*I*(c/x-I))-3/2*c^2*a^2*b*arctan(x/c)+3/2*
c^2*a*b^2*arctan(c/x)^2-3*c^2*a*b^2*ln(c/x)+3/2*c*b^3*arctan(c/x)^2*x+3/4*I
*c^2*b^3*ln(c/x-I)*ln(1+c^2/x^2)-3/4*I*c^2*b^3*ln(c/x+I)*ln(1+c^2/x^2)-3/2*
I*c^2*b^3*ln(c/x)*ln(1+I*c/x)-3/4*I*c^2*b^3*ln(c/x-I)*ln(-1/2*I*(c/x+I))+3/
2*I*c^2*b^3*ln(c/x)*ln(1-I*c/x)+3/4*I*c^2*b^3*ln(c/x+I)*ln(1/2*I*(c/x-I))+3
*c*a*b^2*arctan(c/x)*x+1/2*b^3*x^2*arctan(c/x)^3+1/2*c^2*b^3*arctan(c/x)^3+
1/2*x^2*a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{2}ab^2x^2\arctan\left(\frac{c}{x}\right)^2 + \frac{1}{2}a^3x^2 + \frac{3}{2}\left(x^2\arctan\left(\frac{c}{x}\right) - \left(c\arctan\left(\frac{x}{c}\right) - x\right)c\right)a^2b - \frac{3}{2}\left(\left(\arctan(x,c)^2 - \log\left(c^2+x^2\right)\right)c^2 + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctan(c/x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c/x) - (c*arcta
n(x/c) - x)*c)*a^2*b - 3/2*((arctan2(x, c)^2 - log(c^2 + x^2))*c^2 + 2*(c*a
rctan(x/c) - x)*c*arctan(c/x))*a*b^2 + 1/32*(12*c^2*arctan(c/x)^2*arctan(x/
c) + 8*c^2*arctan2(c, x)^3 + 8*x^2*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arcta
n(x/c)^2/c + arctan(x/c)^3/c)*c^3 + 12*c*x*arctan2(c, x)^2 + 96*c^3*integra
te(1/32*log(c^2 + x^2)^2/(c^2 + x^2), x) - 3*c*x*log(c^2 + x^2)^2 + 512*c^2
*integrate(1/32*x*arctan(c/x)^3/(c^2 + x^2), x) + 768*c^2*integrate(1/32*x*
arctan(c/x)/(c^2 + x^2), x) + 384*c*integrate(1/32*x^2*arctan(c/x)^2/(c^2 +
 x^2), x) + 96*c*integrate(1/32*x^2*log(c^2 + x^2)^2/(c^2 + x^2), x) + 384*
c*integrate(1/32*x^2*log(c^2 + x^2)/(c^2 + x^2), x) + 512*integrate(1/32*x^
3*arctan(c/x)^3/(c^2 + x^2), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x\arctan\left(\frac{c}{x}\right)^3 + 3\,ab^2x\arctan\left(\frac{c}{x}\right)^2 + 3\,a^2bx\arctan\left(\frac{c}{x}\right) + a^3x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c/x)^3 + 3*a*b^2*x*arctan(c/x)^2 + 3*a^2*b*x*arctan(c
/x) + a^3*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c/x))**3,x)
```

```
[Out] Integral(x*(a + b*atan(c/x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3*x, x)
```

3.150 $\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=119

$$3ib^2c\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{3}{2}b^3c\text{PolyLog}\left(3,1-\frac{2c}{c+ix}\right)+ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\cot^{-1}\left(\right.\right.$$

```
[Out] I*c*(a + b*ArcCot[x/c])^3 + x*(a + b*ArcCot[x/c])^3 - 3*b*c*(a + b*ArcCot[x
/c])^2*Log[(2*c)/(c + I*x)] + (3*I)*b^2*c*(a + b*ArcCot[x/c])*PolyLog[2, 1
- (2*c)/(c + I*x)] - (3*b^3*c*PolyLog[3, 1 - (2*c)/(c + I*x)])/2
```

Rubi [F] time = 0.738651, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c/x])^3,x]
```

```
[Out] a^3*x + ((3*I)/2)*a^2*b*x*Log[1 - (I*c)/x] + (3*a*b^2*(I*c - x)*Log[1 - (I*
c)/x]^2)/4 + (I/8)*b^3*(I*c - x)*Log[1 - (I*c)/x]^3 - ((3*I)/2)*a^2*b*x*Log
[1 + (I*c)/x] + (3*a*b^2*x*Log[1 - (I*c)/x]*Log[1 + (I*c)/x])/2 - (3*a*b^2*
(I*c + x)*Log[1 + (I*c)/x]^2)/4 + (I/8)*b^3*(I*c + x)*Log[1 + (I*c)/x]^3 -
((3*I)/2)*a*b^2*c*Log[1 + (I*c)/x]*Log[-c - I*x] + (3*a^2*b*c*Log[c - I*x])
/2 + ((3*I)/2)*a*b^2*c*Log[-c - I*x]*Log[(c - I*x)/(2*c)] + ((3*I)/2)*a*b^2
*c*Log[1 - (I*c)/x]*Log[-c + I*x] + (3*a^2*b*c*Log[c + I*x])/2 - ((3*I)/2)*
a*b^2*c*Log[-c + I*x]*Log[(c + I*x)/(2*c)] + (3*b^3*c*Log[1 + (I*c)/x]^2*Lo
g[((-I)*c)/x])/8 + (3*b^3*c*Log[1 - (I*c)/x]^2*Log[(I*c)/x])/8 - ((3*I)/2)*
a*b^2*c*Log[-c - I*x]*Log[((-I)*x)/c] + ((3*I)/2)*a*b^2*c*Log[-c + I*x]*Log
[(I*x)/c] + (3*b^3*c*Log[1 - (I*c)/x]*PolyLog[2, 1 - (I*c)/x])/4 + (3*b^3*c
*Log[1 + (I*c)/x]*PolyLog[2, 1 + (I*c)/x])/4 - ((3*I)/2)*a*b^2*c*PolyLog[2,
 (c - I*x)/(2*c)] + ((3*I)/2)*a*b^2*c*PolyLog[2, (c + I*x)/(2*c)] - ((3*I)/
2)*a*b^2*c*PolyLog[2, ((-I)*c)/x] + ((3*I)/2)*a*b^2*c*PolyLog[2, (I*c)/x] +
 ((3*I)/2)*a*b^2*c*PolyLog[2, 1 - (I*x)/c] - ((3*I)/2)*a*b^2*c*PolyLog[2, 1
 + (I*x)/c] - (3*b^3*c*PolyLog[3, 1 - (I*c)/x])/4 - (3*b^3*c*PolyLog[3, 1 +
 (I*c)/x])/4 + ((3*I)/8)*b^3*Defer[Int][Log[1 - (I*c)/x]^2*Log[1 + (I*c)/x]
, x] - ((3*I)/8)*b^3*Defer[Int][Log[1 - (I*c)/x]*Log[1 + (I*c)/x]^2, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(a^3+\frac{3}{2}ia^2b\log\left(1-\frac{ic}{x}\right)-\frac{3}{4}ab^2\log^2\left(1-\frac{ic}{x}\right)-\frac{1}{8}ib^3\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2b\log\left(1+\frac{ic}{x}\right)\right.$$

$$= a^3x+\frac{1}{2}\left(3ia^2b\right)\int \log\left(1-\frac{ic}{x}\right)dx-\frac{1}{2}\left(3ia^2b\right)\int \log\left(1+\frac{ic}{x}\right)dx-\frac{1}{4}\left(3ab^2\right)\int \log^2\left(1-\right.$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}ia^2$$

Mathematica [A] time = 0.256732, size = 215, normalized size = 1.81

$$-3ab^2\left(-ic\text{PolyLog}\left(2,e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)-(x+ic)\tan^{-1}\left(\frac{c}{x}\right)^2+2c\tan^{-1}\left(\frac{c}{x}\right)\log\left(1-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)\right)-\frac{1}{8}b^3\left(24ic\tan^{-1}\left(\frac{c}{x}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTan[c/x] + (3*a^2*b*c*Log[c^2 + x^2])/2 - 3*a*b^2*(-((
I*c + x)*ArcTan[c/x]^2) + 2*c*ArcTan[c/x]*Log[1 - E^((2*I)*ArcTan[c/x])] -
I*c*PolyLog[2, E^((2*I)*ArcTan[c/x])]) - (b^3*((-I)*c*Pi^3 + (8*I)*c*ArcTan
[c/x]^3 - 8*x*ArcTan[c/x]^3 + 24*c*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c
/x])] + (24*I)*c*ArcTan[c/x]*PolyLog[2, E^((-2*I)*ArcTan[c/x])] + 12*c*Poly
Log[3, E^((-2*I)*ArcTan[c/x])]))/8
```

Maple [C] time = 0.33, size = 2363, normalized size = 19.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3,x)
```

```
[Out] -3/2*I*c*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/((1+I*c/x)^2/(1+
c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1)
)*arctan(c/x)^2+3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2
)+1)^2)*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+
I*c/x)^2/(1+c^2/x^2)+1)^2)+x*a^3+b^3*x*arctan(c/x)^3-6*c*b^3*polylog(3,(1+I
*c/x)/(1+c^2/x^2)^(1/2))-6*c*b^3*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-3*
c*b^3*arctan(c/x)^2*ln(2)+3*a^2*b*x*arctan(c/x)+3*a*b^2*x*arctan(c/x)^2-3*c
*b^3*arctan(c/x)^2*ln(1+(1+I*c/x)/(1+c^2/x^2)^(1/2))-3*c*b^3*arctan(c/x)^2*
ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))+3/2*c*b^3*arctan(c/x)^2*ln(1+c^2/x^2)-3*c
*b^3*ln(c/x)*arctan(c/x)^2-3*c*b^3*arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(
1/2))+3*c*b^3*arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)+I*c*b^3*arctan(c/
x)^3-3*c*a^2*b*ln(c/x)+3/2*c*a^2*b*ln(1+c^2/x^2)+6*I*c*b^3*arctan(c/x)*poly
log(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))+6*I*c*b^3*arctan(c/x)*polylog(2,(1+I*c/
x)/(1+c^2/x^2)^(1/2))-3/2*I*c*b^3*Pi*arctan(c/x)^2+3/4*I*c*a*b^2*ln(c/x+I)^
2+3/2*I*c*a*b^2*dilog(1/2*I*(c/x-I))-3*I*c*a*b^2*dilog(1+I*c/x)+3*I*c*a*b^2
*dilog(1-I*c/x)-3/4*I*c*a*b^2*ln(c/x-I)^2-3/2*I*c*a*b^2*dilog(-1/2*I*(c/x+I
))+3*c*a*b^2*arctan(c/x)*ln(1+c^2/x^2)-6*c*a*b^2*arctan(c/x)*ln(c/x)-3/2*I*
c*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*cs
gn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^2+3
/2*I*c*b^3*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^
2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2+3/2*I*c*b^3*Pi*csgn(
I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x
)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2+3/2*I*c*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c
^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((
1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-3/4*I*c*b^3*arctan(c/x)^2*Pi*csg
n(I*((1+I*c/x)^2/(1+c^2/x^2)+1))^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)+3/
4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))^2*csgn(I*(1+
I*c/x)^2/(1+c^2/x^2))-3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I/((1+I*c/x)^2/(1+c
^2/x^2)+1)^2)*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)
^2-3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*csgn(I*(1+I
*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2-3/2*I*c*b^3*arctan(c/x
)^2*Pi*csgn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^
2+3/2*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*(
(1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2+3/2*I*c*a*b^2*ln(c/x+I)*ln(1/2*I*(c/x-I))-3
*I*c*a*b^2*ln(c/x)*ln(1+I*c/x)+3*I*c*a*b^2*ln(c/x)*ln(1-I*c/x)+3/2*I*c*b^3*
Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c
/x)^2-3/2*I*c*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2
/x^2)+1))^3*arctan(c/x)^2-3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*((1+I*c/x)^2/
(1+c^2/x^2)+1)^2)^3-3/2*I*c*b^3*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c
/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^2+3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*(
1+I*c/x)^2/(1+c^2/x^2))^3+3/4*I*c*b^3*arctan(c/x)^2*Pi*csgn(I*(1+I*c/x)^2/(
1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^3+3/2*I*c*a*b^2*ln(1+c^2/x^2)*ln(
c/x-I)-3/2*I*c*a*b^2*ln(c/x-I)*ln(-1/2*I*(c/x+I))-3/2*I*c*a*b^2*ln(1+c^2/x^
2)*ln(c/x+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{7}{8}\,b^3c\arctan\left(\frac{c}{x}\right)^3\arctan\left(\frac{x}{c}\right)+3\,ab^2c\arctan\left(\frac{c}{x}\right)^2\arctan\left(\frac{x}{c}\right)+\frac{1}{8}\,b^3x\arctan\left(c,x\right)^3-\frac{3}{32}\,b^3x\arctan\left(c,x\right)\log\left(c^2+x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c*arctan(c/x)^3*arctan(x/c) + 3*a*b^2*c*arctan(c/x)^2*arctan(x/c) +
 1/8*b^3*x*arctan2(c, x)^3 - 3/32*b^3*x*arctan2(c, x)*log(c^2 + x^2)^2 + (3
```

```
*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*a*b^2*c^2 + 7/32*(6*arctan(
c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^3/c + arctan(x/c)^4/c)*b
^3*c^2 + 3*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2 + x^2),
 x) + 12*b^3*c*integrate(1/32*x*arctan(c/x)^2/(c^2 + x^2), x) - 3*b^3*c*int
egrate(1/32*x*log(c^2 + x^2)^2/(c^2 + x^2), x) + 3/2*(2*x*arctan(c/x) + c*l
og(c^2 + x^2))*a^2*b + a^3*x + 28*b^3*integrate(1/32*x^2*arctan(c/x)^3/(c^2
 + x^2), x) + 3*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)^2/(c^2 +
x^2), x) + 96*a*b^2*integrate(1/32*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 12*b
^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)/(c^2 + x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3\arctan\left(\frac{c}{x}\right)^3 + 3\,ab^2\arctan\left(\frac{c}{x}\right)^2 + 3\,a^2b\arctan\left(\frac{c}{x}\right) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
a^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3,x)
```

```
[Out] Integral((a + b*atan(c/x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\arctan\left(\frac{c}{x}\right) + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3, x)
```

3.151 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

Optimal. Leaf size=230

$$\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{3}{2}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}ib\text{PolyLog}\left(2,1-\frac{}{1}\right.$$

```
[Out] -2*(a + b*ArcCot[x/c])^3*ArcTanh[1 - 2/(1 + (I*c)/x)] + ((3*I)/2)*b*(a + b*
ArcCot[x/c])^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - ((3*I)/2)*b*(a + b*ArcCot[
x/c])^2*PolyLog[2, -1 + 2/(1 + (I*c)/x)] + (3*b^2*(a + b*ArcCot[x/c])*PolyL
og[3, 1 - 2/(1 + (I*c)/x)])/2 - (3*b^2*(a + b*ArcCot[x/c])*PolyLog[3, -1 +
2/(1 + (I*c)/x)])/2 - ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + (I*c)/x)] + ((3*I
)/4)*b^3*PolyLog[4, -1 + 2/(1 + (I*c)/x)]
```

Rubi [A] time = 0.494497, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5031, 4850, 4988, 4884, 4994, 4998, 6610}

$$\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{3}{2}b^2\text{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}ib\text{PolyLog}\left(2,1-\frac{}{1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^3/x,x]
```


```
[Out] -2*(a + b*ArcCot[x/c])^3*ArcTanh[1 - 2/(1 + (I*c)/x)] + ((3*I)/2)*b*(a + b*
ArcCot[x/c])^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - ((3*I)/2)*b*(a + b*ArcCot[
x/c])^2*PolyLog[2, -1 + 2/(1 + (I*c)/x)] + (3*b^2*(a + b*ArcCot[x/c])*PolyL
og[3, 1 - 2/(1 + (I*c)/x)])/2 - (3*b^2*(a + b*ArcCot[x/c])*PolyLog[3, -1 +
2/(1 + (I*c)/x)])/2 - ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + (I*c)/x)] + ((3*I
)/4)*b^3*PolyLog[4, -1 + 2/(1 + (I*c)/x)]
```


Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4850

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTan[c*x])^(p - 1)*ArcTanh[1 - 2/(1 + I*c*x)])/(1 + c^2*x^2), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4988

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTan[c*x])^p)/(d + e
*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTan[c*x])^p)/(d + e*x^2
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 4998

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+(6bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1\right.}{1+c^2x^2}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)-(3bc)\,\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right.}{1+c^2x^2}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right)-\frac{3}{2}ib$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right)-\frac{3}{2}ib$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\frac{2}{1+\frac{ic}{x}}\right)-\frac{3}{2}ib$$

Mathematica [A] time = 0.207111, size = 219, normalized size = 0.95

$$\frac{3}{4}ib\left(2\text{PolyLog}\left(2,\frac{c+ix}{c-ix}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2-2\text{PolyLog}\left(2,\frac{x-ic}{x+ic}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2+b\left(-2i\text{PolyLog}\left(3,\frac{c+ix}{c-ix}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcTan[c/x])^3*ArcTanh[(c + I*x)/(c - I*x)] + ((3*I)/4)*b*(2*(a +
 b*ArcTan[c/x])^2*PolyLog[2, (c + I*x)/(c - I*x)] - 2*(a + b*ArcTan[c/x])^2
*PolyLog[2, ((-I)*c + x)/(I*c + x)] + b*((-2*I)*(a + b*ArcTan[c/x])*PolyLog
[3, (c + I*x)/(c - I*x)] + (2*I)*(a + b*ArcTan[c/x])*PolyLog[3, ((-I)*c + x
)/(I*c + x)] + b*(-PolyLog[4, (c + I*x)/(c - I*x)] + PolyLog[4, ((-I)*c + x
)/(I*c + x)])))
```

Maple [C] time = 0.276, size = 2542, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3/x,x)
```

```
[Out] -6*I*b^3*polylog(4,(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*I*b^3*polylog(4,-(1+I*c/x
)/(1+c^2/x^2)^(1/2))+3/2*a*b^2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))-6*a*b^2*
polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*a*b^2*polylog(3,-(1+I*c/x)/(1+c^2/
x^2)^(1/2))-b^3*ln(c/x)*arctan(c/x)^3+b^3*arctan(c/x)^3*ln((1+I*c/x)^2/(1+c
^2/x^2)-1)-b^3*arctan(c/x)^3*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*b^3*arctan
(c/x)*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))-b^3*arctan(c/x)^3*ln(1+(1+I*c/
x)/(1+c^2/x^2)^(1/2))-6*b^3*arctan(c/x)*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1
/2))+3/2*b^3*arctan(c/x)*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))+3/4*I*b^3*poly
log(4,-(1+I*c/x)^2/(1+c^2/x^2))-a^3*ln(c/x)-3/2*I*a*b^2*Pi*csgn(I*((1+I*c/x
)^2/(1+c^2/x^2)-1))*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2
/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^2-1/2*I*b^3*Pi*csg
n(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^
3-3/2*I*a*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2
)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(
c/x)^2+3/2*I*a*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^
2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*
arctan(c/x)^2+3/2*I*a*b^2*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1
+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2+3/2*I
*a*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^
2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^2-1/2*I*b^3*Pi*csgn(I*((1+
I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c
/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x)^3+1/2*I*b^3*P
i*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/
x)^3+6*I*a*b^2*arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))-3*I*a*b^2
*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))+6*I*a*b^2*arctan(c/x)*poly
log(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-3/2*I*a*b^2*Pi*arctan(c/x)^2-3/2*I*a^2*
b*ln(c/x)*ln(1+I*c/x)+3/2*I*a^2*b*ln(c/x)*ln(1-I*c/x)-1/2*I*b^3*Pi*csgn(((1
+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^3-1/2*I
*b^3*Pi*arctan(c/x)^3+3*I*b^3*arctan(c/x)^2*polylog(2,(1+I*c/x)/(1+c^2/x^2)
^(1/2))-3/2*I*a^2*b*dilog(1+I*c/x)+3/2*I*a^2*b*dilog(1-I*c/x)+1/2*I*b^3*Pi*
csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+
I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^3-3/2*I*a*b^2*Pi*csgn(I*((1+I*c/x)^2
/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^2+3/2*I*a*b^2*Pi
*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x
)^2+1/2*I*b^3*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1
+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arctan(c/x)^3-3/2*I*a*b^2*Pi*cs
gn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*arctan(c/x)^2
-1/2*I*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1
))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*arctan(c/x
)^3+1/2*I*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2
)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*arcta
n(c/x)^3-3*a^2*b*ln(c/x)*arctan(c/x)-3*a*b^2*ln(c/x)*arctan(c/x)^2+3*a*b^2*
```

```
arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-3*a*b^2*arctan(c/x)^2*ln(1-(1+I
*c/x)/(1+c^2/x^2)^(1/2))-3*a*b^2*arctan(c/x)^2*ln(1+(1+I*c/x)/(1+c^2/x^2)^(
1/2))-3/2*I*b^3*arctan(c/x)^2*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))+3*I*b^3*a
rctan(c/x)^2*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3\log(x)+\frac{1}{32}\int\frac{28\,b^3\arctan(c,x)^3+3\,b^3\arctan(c,x)\log\left(c^2+x^2\right)^2+96\,ab^2\arctan(c,x)^2+96\,a^2b\arctan(c,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan2(c, x)^3 + 3*b^3*arctan2(c, x)*l
og(c^2 + x^2)^2 + 96*a*b^2*arctan2(c, x)^2 + 96*a^2*b*arctan2(c, x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3\arctan\left(\frac{c}{x}\right)^3+3\,ab^2\arctan\left(\frac{c}{x}\right)^2+3\,a^2b\arctan\left(\frac{c}{x}\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3/x,x)
```

```
[Out] Integral((a + b*atan(c/x))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3/x, x)
```

3.152 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=136

$$-\frac{3ib^2\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c}-\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)}{2c}-\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x}-$$

```
[Out] ((-I)*(a + b*ArcCot[x/c])^3)/c - (a + b*ArcCot[x/c])^3/x - (3*b*(a + b*ArcC
ot[x/c])^2*Log[2/(1 + (I*c)/x)])/c - ((3*I)*b^2*(a + b*ArcCot[x/c])*PolyLog
[2, 1 - 2/(1 + (I*c)/x)])/c - (3*b^3*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/(2*c)
```

Rubi [B] time = 2.35871, antiderivative size = 551, normalized size of antiderivative = 4.05, number of steps used = 82, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {5035, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$\frac{3b^2\text{PolyLog}\left(2,-\frac{-x+ic}{2x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,-\frac{-x+ic}{2x}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,\frac{x+ic}{2x}\right)}{2c}-\frac{3b^3\log\left(1+\frac{ic}{x}\right)\text{Poly}}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTan[c/x])^3/x^2,x]
```

```
[Out] (-3*b*(1 - (I*c)/x)*((2*I)*a - b*Log[1 - (I*c)/x])^2)/(8*c) - (3*b*(1 - (I*
c)/x)*(2*a + I*b*Log[1 - (I*c)/x])^2)/(8*c) - ((I/8)*(1 - (I*c)/x)*(2*a + I
*b*Log[1 - (I*c)/x])^3)/c + (3*b*((2*I)*a - b*Log[1 - (I*c)/x])^2*Log[1 + (
I*c)/x])/(8*c) - (((3*I)/8)*b*((2*I)*a - b*Log[1 - (I*c)/x])^2*Log[1 + (I*c
)/x])/x - (3*b^2*((2*I)*a - b*Log[1 - (I*c)/x])*Log[1 + (I*c)/x]^2)/(8*c) -
 (((3*I)/8)*b^2*((2*I)*a - b*Log[1 - (I*c)/x])*Log[1 + (I*c)/x]^2)/x - (b^3
*(1 + (I*c)/x)*Log[1 + (I*c)/x]^3)/(8*c) - (3*b^3*Log[1 + (I*c)/x]^2*Log[-(
I*c - x)/(2*x)])/(4*c) - (3*b*((2*I)*a - b*Log[1 - (I*c)/x])^2*Log[(I*c + x
)/(2*x)])/(4*c) + (3*b^2*((2*I)*a - b*Log[1 - (I*c)/x])*PolyLog[2, -(I*c -
x)/(2*x)])/(2*c) - (3*b^3*Log[1 + (I*c)/x]*PolyLog[2, (I*c + x)/(2*x)])/(2*
c) + (3*b^3*PolyLog[3, -(I*c - x)/(2*x)])/(2*c) + (3*b^3*PolyLog[3, (I*c +
x)/(2*x)])/(2*c)
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8x^2}+\frac{3ib\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x^2}-\frac{3ib^2\left(-2ia+b\log\right.}{}\right.$$

$$=\frac{1}{8}\int\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{x^2}\,dx+\frac{1}{8}(3ib)\int\frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{x^2}\,dx-\frac{1}{8}\left(3i\right.$$

$$=-\left(\frac{1}{8}\operatorname{Subst}\left(\int(2a+ib\log(1-icx))^3\,dx,x,\frac{1}{x}\right)\right)-\frac{1}{8}(3ib)\operatorname{Subst}\left(\int(-2ia+b\log(1-icx))\right.$$

$$=-\frac{3ib\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x}-\frac{3ib^2\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log^2\left(1+\frac{ic}{x}\right)}{8x}-\frac{i\operatorname{Subs}}{}$$

$$=-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c}-\frac{3ib\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x}-\frac{3ib^2\left(2ia-b\right.}{}$$

$$=-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c}-\frac{3ib\left(2ia-b\log\left(1\right.\right.}{8}$$

$$=\frac{3ab^2}{2x}+\frac{3ib^3}{4x}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c}-\frac{3b^3}{}$$

$$=\frac{3ab^2}{2x}-\frac{3b^3\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{4c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\right.}{8c}$$

$$=-\frac{3ib^3}{4x}-\frac{3b^3\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{4c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\right.}{8c}$$

$$=-\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+i\right.}{8c}$$

$$=-\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+i\right.}{8c}$$

Mathematica [A] time = 0.118877, size = 222, normalized size = 1.63

$$-\frac{-6ib^2x\text{PolyLog}\left(2,-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+3b^3x\text{PolyLog}\left(3,-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right)-3a^2bx\log\left(\frac{c^2}{x^2}+1\right)+6a^2bc\tan^{-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x^2,x]
```

```
[Out] -(2*a^3*c + 6*a^2*b*c*ArcTan[c/x] + 6*a*b^2*c*ArcTan[c/x]^2 - (6*I)*a*b^2*x
*ArcTan[c/x]^2 + 2*b^3*c*ArcTan[c/x]^3 - (2*I)*b^3*x*ArcTan[c/x]^3 + 12*a*b
^2*x*ArcTan[c/x]*Log[1 + E^((2*I)*ArcTan[c/x])] + 6*b^3*x*ArcTan[c/x]^2*Log
[1 + E^((2*I)*ArcTan[c/x])] - 3*a^2*b*x*Log[1 + c^2/x^2] - (6*I)*b^2*x*(a +
 b*ArcTan[c/x])*PolyLog[2, -E^((2*I)*ArcTan[c/x])] + 3*b^3*x*PolyLog[3, -E^
((2*I)*ArcTan[c/x])])/(2*c*x)
```

Maple [B] time = 0.125, size = 306, normalized size = 2.3

$$-\frac{a^3}{x}+\frac{ib^3}{c}\left(\arctan\left(\frac{c}{x}\right)\right)^3-\frac{b^3}{x}\left(\arctan\left(\frac{c}{x}\right)\right)^3-3\frac{b^3}{c}\left(\arctan\left(\frac{c}{x}\right)\right)^2\ln\left(\left(1+\frac{ic}{x}\right)^2\left(1+\frac{c^2}{x^2}\right)^{-1}+1\right)+\frac{3\,ib^3}{c}\arctan\left(\frac{c}{x}\right)\text{po}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3/x^2,x)
```

```
[Out] -a^3/x+I/c*b^3*arctan(c/x)^3-b^3*arctan(c/x)^3/x-3/c*b^3*arctan(c/x)^2*ln((
1+I*c/x)^2/(1+c^2/x^2)+1)+3*I/c*b^3*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c
^2/x^2))-3/2/c*b^3*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))+3*I/c*arctan(c/x)^2*
a*b^2-3*arctan(c/x)^2/x*a*b^2-6/c*arctan(c/x)*ln((1+I*c/x)^2/(1+c^2/x^2)+1)
*a*b^2+3*I/c*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))*a*b^2-3*a^2*b/x*arctan(c/x
)+3/2/c*a^2*b*ln(1+c^2/x^2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\arctan\left(\frac{c}{x}\right)^3+3\,ab^2\arctan\left(\frac{c}{x}\right)^2+3\,a^2b\arctan\left(\frac{c}{x}\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atan}{\left(\frac{c}{x} \right)}\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3/x**2,x)
```

```
[Out] Integral((a + b*atan(c/x))**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(\frac{c}{x}\right) + a\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3/x^2, x)
```

3.153 $\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=147

$$\frac{3ib^3\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{2c^2}+\frac{3b^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c^2}+\frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\cot\right.}{2x^2}$$

```
[Out] (((3*I)/2)*b*(a + b*ArcCot[x/c])^2)/c^2 + (3*b*(a + b*ArcCot[x/c])^2)/(2*c*
x) - (a + b*ArcCot[x/c])^3/(2*c^2) - (a + b*ArcCot[x/c])^3/(2*x^2) + (3*b^2
*(a + b*ArcCot[x/c])*Log[2/(1 + (I*c)/x)])/c^2 + (((3*I)/2)*b^3*PolyLog[2,
1 - 2/(1 + (I*c)/x)])/c^2
```

Rubi [F] time = 2.25451, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c/x])^3/x^3,x]
```

```
[Out] (((3*I)/64)*b^3*(1 - (I*c)/x)^2)/c^2 - (3*a*b^2*(1 + (I*c)/x)^2)/(16*c^2) -
 (((3*I)/64)*b^3*(1 + (I*c)/x)^2)/c^2 - (((3*I)/8)*a^2*b)/x^2 - (3*a*b^2)/(
8*x^2) + (3*a^2*b)/(4*c*x) - (3*b^3)/(2*c*x) + (((3*I)/4)*a^2*b*Log[I - c/x
])/c^2 + (3*a*b^2*Log[I - c/x])/(8*c^2) - (3*a*b^2*(1 - (I*c)/x)*Log[1 - (I
*c)/x])/(4*c^2) + (((3*I)/4)*b^3*(1 - (I*c)/x)*Log[1 - (I*c)/x])/c^2 + (3*a
*b^2*Log[1 - (I*c)/x])/(8*x^2) - (3*b^2*(1 - (I*c)/x)^2*(2*a + I*b*Log[1 -
(I*c)/x]))/(32*c^2) + (((3*I)/8)*b*(1 - (I*c)/x)*(2*a + I*b*Log[1 - (I*c)/x
])^2)/c^2 - (((3*I)/32)*b*(1 - (I*c)/x)^2*(2*a + I*b*Log[1 - (I*c)/x])^2)/c
^2 - ((1 - (I*c)/x)*(2*a + I*b*Log[1 - (I*c)/x])^3)/(8*c^2) + ((1 - (I*c)/x
)^2*(2*a + I*b*Log[1 - (I*c)/x])^3)/(16*c^2) - (9*a*b^2*(1 + (I*c)/x)*Log[1
 + (I*c)/x])/(4*c^2) - (((3*I)/4)*b^3*(1 + (I*c)/x)*Log[1 + (I*c)/x])/c^2 +
 (3*a*b^2*(1 + (I*c)/x)^2*Log[1 + (I*c)/x])/(8*c^2) + (((3*I)/32)*b^3*(1 +
(I*c)/x)^2*Log[1 + (I*c)/x])/c^2 + (((3*I)/4)*a^2*b*Log[1 + (I*c)/x])/x^2 +
 (3*a*b^2*Log[1 + (I*c)/x])/(8*x^2) - (3*a*b^2*Log[1 - (I*c)/x]*Log[1 + (I*
c)/x])/(4*x^2) + (3*a*b^2*(1 + (I*c)/x)*Log[1 + (I*c)/x]^2)/(4*c^2) + (((3*
I)/8)*b^3*(1 + (I*c)/x)*Log[1 + (I*c)/x]^2)/c^2 - (3*a*b^2*(1 + (I*c)/x)^2*
Log[1 + (I*c)/x]^2)/(8*c^2) - (((3*I)/32)*b^3*(1 + (I*c)/x)^2*Log[1 + (I*c)
/x]^2)/c^2 - ((I/8)*b^3*(1 + (I*c)/x)*Log[1 + (I*c)/x]^3)/c^2 + ((I/16)*b^3
*(1 + (I*c)/x)^2*Log[1 + (I*c)/x]^3)/c^2 + (3*a*b^2*Log[I + c/x])/(8*c^2) -
 (3*a*b^2*Log[1 - (I*c)/x]*Log[c - I*x])/(4*c^2) - (3*a*b^2*Log[1 + (I*c)/x
]*Log[c + I*x])/(4*c^2) + (3*a*b^2*Log[(c - I*x)/(2*c)]*Log[c + I*x])/(4*c^
2) + (3*a*b^2*Log[c - I*x]*Log[(c + I*x)/(2*c)])/(4*c^2) - (3*a*b^2*Log[c +
 I*x]*Log[((-I)*x)/c]))/(4*c^2) - (3*a*b^2*Log[c - I*x]*Log[(I*x)/c])/(4*c^2
) + (3*a*b^2*PolyLog[2, (c - I*x)/(2*c)])/(4*c^2) + (3*a*b^2*PolyLog[2, (c
+ I*x)/(2*c)])/(4*c^2) + (3*a*b^2*PolyLog[2, ((-I)*c)/x])/(4*c^2) + (3*a*b^
2*PolyLog[2, (I*c)/x])/(4*c^2) - (3*a*b^2*PolyLog[2, 1 - (I*x)/c])/(4*c^2)
- (3*a*b^2*PolyLog[2, 1 + (I*x)/c])/(4*c^2) + ((3*I)/8)*b^3*Defer[Int][(Log
[1 - (I*c)/x]^2*Log[1 + (I*c)/x])/x^3, x] - ((3*I)/8)*b^3*Defer[Int][(Log[1
 - (I*c)/x]*Log[1 + (I*c)/x]^2)/x^3, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8x^3} + \frac{3ib\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x^3} - \frac{3ib^2\left(-2ia+b\log\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{x^3}dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{x^3}dx - \frac{1}{8}(3i$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int x(2a+ib\log(1-icx))^3\,dx, x, \frac{1}{x}\right)\right) + \frac{1}{8}(3ib)\int \left(-\frac{4a^2\log\left(1+\frac{ic}{x}\right)}{x^3} - \frac{4iab\log}{}\right.$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int \left(-\frac{i(2a+ib\log(1-icx))^3}{c} + \frac{i(1-icx)(2a+ib\log(1-icx))^3}{c}\right)dx, x, \frac{1}{x}\right)\right) -$$

$$= -\frac{3ab^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} + \frac{1}{2}\left(3ia^2b\right)\text{Subst}\left(\int x\log(1+icx)\,dx, x, \frac{1}{x}\right) + \frac{1}{4}\left(3ab^2\right)\text{S}$$

$$= \frac{3ia^2b\log\left(1+\frac{ic}{x}\right)}{4x^2} - \frac{3ab^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} + \frac{1}{4}\left(3ab^2\right)\text{Subst}\left(\int\left(\frac{i\log^2(1+icx)}{c} - \right.\right.$$

$$= -\frac{\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c^2} + \frac{\left(1-\frac{ic}{x}\right)^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{16c^2} + \frac{3ia^2b\log\left(1+\frac{ic}{x}\right)}{4x^2} -$$

$$= -\frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} + \frac{3ia^2b\log\left(i-\frac{c}{x}\right)}{4c^2} + \frac{3ib\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c^2} - \frac{3ib\left(1-\frac{ic}{x}\right)^2\left(2a\right.}{}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} - \frac{3iab^2}{2cx} - \frac{3b^3}{4cx} + \frac{3ia^2b\log\left(i-\frac{c}{x}\right)}{4c^2} - \frac{3b^2\left(1\right.}{}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ia^2b\log\left(i-\frac{c}{x}\right)}{4c^2}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ia^2b\log\left(i-\frac{c}{x}\right)}{4c^2}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} - \frac{3ab^2}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ia^2b\log}{4c}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} - \frac{3ab^2}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ia^2b\log}{4c}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} - \frac{3ab^2}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ia^2b\log}{4c}$$

Mathematica [A] time = 0.282954, size = 178, normalized size = 1.21

$$\frac{-3ib^3x^2\text{PolyLog}\left(2,-e^{2i\tan^{-1}\left(\frac{c}{x}\right)}\right) + a\left(ac(3bx-ac) + 6b^2x^2\log\left(\frac{1}{\sqrt{\frac{c^2}{x^2}+1}}\right)\right) - 3b\tan^{-1}\left(\frac{c}{x}\right)\left(a\left(a\left(c^2+x^2\right)-2bcx\right)-2b\right.}{2c^2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x^3,x]
```

```
[Out] (3*b^2*(c - I*x)*(-(a*(c + I*x)) + b*x)*ArcTan[c/x]^2 - b^3*(c^2 + x^2)*Arc
Tan[c/x]^3 - 3*b*ArcTan[c/x]*(a*(-2*b*c*x + a*(c^2 + x^2)) - 2*b^2*x^2*Log[
1 + E^((2*I)*ArcTan[c/x])]) + a*(a*c*(-(a*c) + 3*b*x) + 6*b^2*x^2*Log[1/Sqr
t[1 + c^2/x^2]]) - (3*I)*b^3*x^2*PolyLog[2, -E^((2*I)*ArcTan[c/x])])/(2*c^2
*x^2)
```

Maple [B] time = 0.095, size = 396, normalized size = 2.7

$$-\frac{a^3}{2x^2}-\frac{b^3}{2x^2}\left(\arctan\left(\frac{c}{x}\right)\right)^3-\frac{b^3}{2c^2}\left(\arctan\left(\frac{c}{x}\right)\right)^3+\frac{3b^3}{2cx}\left(\arctan\left(\frac{c}{x}\right)\right)^2-\frac{3b^3}{2c^2}\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\frac{3i}{4}b^3}{c^2}\ln\left(\frac{c}{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3/x^3,x)
```

```
[Out] -1/2*a^3/x^2-1/2/x^2*b^3*arctan(c/x)^3-1/2/c^2*b^3*arctan(c/x)^3+3/2/c*b^3*
arctan(c/x)^2/x-3/2/c^2*b^3*arctan(c/x)*ln(1+c^2/x^2)-3/4*I/c^2*b^3*ln(c/x-
I)*ln(1+c^2/x^2)+3/4*I/c^2*b^3*ln(-1/2*I*(c/x+I))*ln(c/x-I)-3/4*I/c^2*b^3*l
n(c/x+I)*ln(1/2*I*(c/x-I))-3/8*I/c^2*b^3*ln(c/x+I)^2+3/8*I/c^2*b^3*ln(c/x-I
)^2+3/4*I/c^2*b^3*dilog(-1/2*I*(c/x+I))+3/4*I/c^2*b^3*ln(c/x+I)*ln(1+c^2/x^
2)-3/4*I/c^2*b^3*dilog(1/2*I*(c/x-I))-3/2*a^2*b/x^2*arctan(c/x)+3/2*a^2*b/c
/x+3/2/c^2*a^2*b*arctan(x/c)-3/2/x^2*a*b^2*arctan(c/x)^2-3/2/c^2*a*b^2*arct
an(c/x)^2+3/c*a*b^2/x*arctan(c/x)-3/2/c^2*a*b^2*ln(1+c^2/x^2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\arctan\left(\frac{c}{x}\right)^3+3\,ab^2\arctan\left(\frac{c}{x}\right)^2+3\,a^2b\arctan\left(\frac{c}{x}\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3/x**3,x)
```

```
[Out] Integral((a + b*atan(c/x))**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(\frac{c}{x}\right) + a\right)^3}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3/x^3, x)
```

3.154 $\int x^2 \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=51

$$-\frac{x^{5/2}}{15}+\frac{x^{3/2}}{9}+\frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -Sqrt[x]/3 + x^(3/2)/9 - x^(5/2)/15 + ArcTan[Sqrt[x]]/3 + (x^3*ArcTan[Sqrt[
x]])/3
```

Rubi [A] time = 0.0146102, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5033, 50, 63, 203}

$$-\frac{x^{5/2}}{15}+\frac{x^{3/2}}{9}+\frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x]/3 + x^(3/2)/9 - x^(5/2)/15 + ArcTan[Sqrt[x]]/3 + (x^3*ArcTan[Sqrt[
x]])/3
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tan^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{1+x} dx \\
&= -\frac{x^{5/2}}{15} + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) + \frac{1}{6}\int \frac{x^{3/2}}{1+x} dx \\
&= \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{\sqrt{x}}{1+x} dx \\
&= -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) + \frac{1}{6}\int \frac{1}{\sqrt{x}(1+x)} dx \\
&= -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{x}\right) \\
&= -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}\tan^{-1}\left(\sqrt{x}\right) + \frac{1}{3}x^3 \tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0132214, size = 34, normalized size = 0.67

$$\frac{1}{45}\left(\sqrt{x}\left(-3x^2+5x-15\right)+15\left(x^3+1\right)\tan^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Sqrt[x]],x]
```

```
[Out] (Sqrt[x]*(-15 + 5*x - 3*x^2) + 15*(1 + x^3)*ArcTan[Sqrt[x]])/45
```

Maple [A] time = 0.023, size = 32, normalized size = 0.6

$$\frac{1}{9}x^{\frac{3}{2}} - \frac{1}{15}x^{\frac{5}{2}} + \frac{1}{3}\arctan\left(\sqrt{x}\right) + \frac{x^3}{3}\arctan\left(\sqrt{x}\right) - \frac{1}{3}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x^(1/2)),x)
```

```
[Out] 1/9*x^(3/2)-1/15*x^(5/2)+1/3*arctan(x^(1/2))+1/3*x^3*arctan(x^(1/2))-1/3*x^
(1/2)
```

Maxima [A] time = 1.5056, size = 42, normalized size = 0.82

$$\frac{1}{3}x^3\arctan\left(\sqrt{x}\right) - \frac{1}{15}x^{\frac{5}{2}} + \frac{1}{9}x^{\frac{3}{2}} - \frac{1}{3}\sqrt{x} + \frac{1}{3}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*ar
ctan(sqrt(x))
```

Fricas [A] time = 2.21294, size = 88, normalized size = 1.73

$$\frac{1}{3}\left(x^3+1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{45}\left(3\,x^2-5\,x+15\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*arctan(sqrt(x)) - 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

Sympy [A] time = 4.15935, size = 39, normalized size = 0.76

$$-\frac{x^{\frac{5}{2}}}{15}+\frac{x^{\frac{3}{2}}}{9}-\frac{\sqrt{x}}{3}+\frac{x^3\operatorname{atan}\left(\sqrt{x}\right)}{3}+\frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x**(1/2)),x)
```

```
[Out] -x**(5/2)/15 + x**(3/2)/9 - sqrt(x)/3 + x**3*atan(sqrt(x))/3 + atan(sqrt(x)
)/3
```

Giac [A] time = 1.1379, size = 42, normalized size = 0.82

$$\frac{1}{3}\,x^3\arctan\left(\sqrt{x}\right)-\frac{1}{15}\,x^{\frac{5}{2}}+\frac{1}{9}\,x^{\frac{3}{2}}-\frac{1}{3}\,\sqrt{x}+\frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*ar
ctan(sqrt(x))
```

3.155 $\int x \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=42

$$-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]/2 - x^(3/2)/6 - ArcTan[Sqrt[x]]/2 + (x^2*ArcTan[Sqrt[x]])/2
```

Rubi [A] time = 0.0098464, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5033, 50, 63, 203}

$$-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Sqrt[x]],x]
```

```
[Out] Sqrt[x]/2 - x^(3/2)/6 - ArcTan[Sqrt[x]]/2 + (x^2*ArcTan[Sqrt[x]])/2
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}\left(\sqrt{x}\right)dx &= \frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x^{3/2}}{1+x}dx\\ &= -\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{\sqrt{x}}{1+x}dx\\ &= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{1}{\sqrt{x}(1+x)}dx\\ &= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}dx,x,\sqrt{x}\right)\\ &= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01061, size = 28, normalized size = 0.67

$$\frac{1}{6}\left(3\left(x^2-1\right)\tan^{-1}\left(\sqrt{x}\right)-(x-3)\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Sqrt[x]],x]
```

```
[Out] (-((-3 + x)*Sqrt[x]) + 3*(-1 + x^2)*ArcTan[Sqrt[x]])/6
```

Maple [A] time = 0.025, size = 27, normalized size = 0.6

$$-\frac{1}{6}x^{\frac{3}{2}}-\frac{1}{2}\arctan\left(\sqrt{x}\right)+\frac{x^2}{2}\arctan\left(\sqrt{x}\right)+\frac{1}{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x^(1/2)),x)
```

```
[Out] -1/6*x^(3/2)-1/2*arctan(x^(1/2))+1/2*x^2*arctan(x^(1/2))+1/2*x^(1/2)
```

Maxima [A] time = 1.49418, size = 35, normalized size = 0.83

$$\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)-\frac{1}{6}x^{\frac{3}{2}}+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Fricas [A] time = 2.20084, size = 72, normalized size = 1.71

$$\frac{1}{2}\left(x^2-1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{6}(x-3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 - 1)*arctan(sqrt(x)) - 1/6*(x - 3)*sqrt(x)
```

Sympy [A] time = 2.02125, size = 32, normalized size = 0.76

$$-\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{x^2\operatorname{atan}\left(\sqrt{x}\right)}{2}-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x**(1/2)),x)
```

```
[Out] -x**(3/2)/6 + sqrt(x)/2 + x**2*atan(sqrt(x))/2 - atan(sqrt(x))/2
```

Giac [A] time = 1.11712, size = 35, normalized size = 0.83

$$\frac{1}{2}\,x^2\arctan\left(\sqrt{x}\right)-\frac{1}{6}\,x^{\frac{3}{2}}+\frac{1}{2}\,\sqrt{x}-\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

3.156 $\int \tan^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0058701, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5027, 50, 63, 203}

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]],x]

[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}\left(\sqrt{x}\right)dx &= x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{1+x}dx\\ &= -\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int \frac{1}{\sqrt{x}(1+x)}dx\\ &= -\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}dx,x,\sqrt{x}\right)\\ &= -\sqrt{x}+\tan^{-1}\left(\sqrt{x}\right)+x\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0058068, size = 18, normalized size = 0.82

$$(x+1)\tan^{-1}\left(\sqrt{x}\right)-\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

Maple [A] time = 0.02, size = 17, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2)),x)
```

```
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Maxima [A] time = 1.46798, size = 22, normalized size = 1.

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Fricas [A] time = 2.20887, size = 47, normalized size = 2.14

$$(x+1)\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*arctan(sqrt(x)) - sqrt(x)
```

Sympy [A] time = 1.35766, size = 19, normalized size = 0.86

$$-\sqrt{x} + x\operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2)),x)
```

```
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

Giac [A] time = 1.10477, size = 22, normalized size = 1.

$$x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

3.157 $\int \frac{\tan^{-1}(\sqrt{x})}{x} dx$

Optimal. Leaf size=31

$$i\text{PolyLog}\left(2,-i\sqrt{x}\right)-i\text{PolyLog}\left(2,i\sqrt{x}\right)$$

```
[Out] I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

Rubi [A] time = 0.0353228, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5031, 4848, 2391}

$$i\text{PolyLog}\left(2,-i\sqrt{x}\right)-i\text{PolyLog}\left(2,i\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x,x]
```

```
[Out] I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x} dx &= 2\,\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x} dx, x, \sqrt{x}\right)\\ &= i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, \sqrt{x}\right) - i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, \sqrt{x}\right)\\ &= i\text{Li}_2\left(-i\sqrt{x}\right) - i\text{Li}_2\left(i\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0044137, size = 31, normalized size = 1.

$$i\text{PolyLog}\left(2,-i\sqrt{x}\right)-i\text{PolyLog}\left(2,i\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x,x]
```

```
[Out] I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

Maple [B] time = 0.036, size = 61, normalized size = 2.

$$\ln(x)\arctan\left(\sqrt{x}\right) + \frac{i}{2}\ln(x)\ln\left(1+i\sqrt{x}\right) - \frac{i}{2}\ln(x)\ln\left(1-i\sqrt{x}\right) + i\mathrm{dilog}\left(1+i\sqrt{x}\right) - i\mathrm{dilog}\left(1-i\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x,x)
```

```
[Out] ln(x)*arctan(x^(1/2))+1/2*I*ln(x)*ln(1+I*x^(1/2))-1/2*I*ln(x)*ln(1-I*x^(1/2
))+I*dilog(1+I*x^(1/2))-I*dilog(1-I*x^(1/2))
```

Maxima [B] time = 1.53405, size = 47, normalized size = 1.52

$$-\frac{1}{2}\pi\log(x+1) + \arctan\left(\sqrt{x}\right)\log(x) - i\,\mathrm{Li}_2\left(i\sqrt{x}+1\right) + i\,\mathrm{Li}_2\left(-i\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] -1/2*pi*log(x + 1) + arctan(sqrt(x))*log(x) - I*dilog(I*sqrt(x) + 1) + I*di
log(-I*sqrt(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(sqrt(x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{atan}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x,x)
```

```
[Out] Integral(atan(sqrt(x))/x, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(x))/x, x)
```

3.158 $\int \frac{\tan^{-1}(\sqrt{x})}{x^2} dx$

Optimal. Leaf size=27

$$-\frac{1}{\sqrt{x}} - \frac{\tan^{-1}(\sqrt{x})}{x} - \tan^{-1}(\sqrt{x})$$

```
[Out] -(1/Sqrt[x]) - ArcTan[Sqrt[x]] - ArcTan[Sqrt[x]]/x
```

Rubi [A] time = 0.011758, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5033, 51, 63, 203}

$$-\frac{1}{\sqrt{x}} - \frac{\tan^{-1}(\sqrt{x})}{x} - \tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^2,x]

[Out] -(1/Sqrt[x]) - ArcTan[Sqrt[x]] - ArcTan[Sqrt[x]]/x
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\tan^{-1}\left(\sqrt{x}\right)}{x} + \frac{1}{2}\int \frac{1}{x^{3/2}(1+x)}\,dx \\ &= -\frac{1}{\sqrt{x}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\int \frac{1}{\sqrt{x}(1+x)}\,dx \\ &= -\frac{1}{\sqrt{x}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{x} - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\ &= -\frac{1}{\sqrt{x}} - \tan^{-1}\left(\sqrt{x}\right) - \frac{\tan^{-1}\left(\sqrt{x}\right)}{x}\end{aligned}$$

Mathematica [C] time = 0.0095062, size = 30, normalized size = 1.11

$$-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-x\right)}{\sqrt{x}} - \frac{\tan^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^2,x]
```

```
[Out] -(ArcTan[Sqrt[x]]/x) - Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

Maple [A] time = 0.026, size = 22, normalized size = 0.8

$$-\arctan\left(\sqrt{x}\right) - \frac{1}{x}\arctan\left(\sqrt{x}\right) - \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^2,x)
```

```
[Out] -arctan(x^(1/2))-arctan(x^(1/2))/x-1/x^(1/2)
```

Maxima [A] time = 1.46563, size = 28, normalized size = 1.04

$$-\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

Fricas [A] time = 2.13151, size = 54, normalized size = 2.

$$-\frac{(x+1)\arctan\left(\sqrt{x}\right) + \sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -((x + 1)*arctan(sqrt(x)) + sqrt(x))/x
```

Sympy [B] time = 3.14029, size = 94, normalized size = 3.48

$$-\frac{x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{x^2}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**2,x)
```

```
[Out] -x**(5/2)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*atan(sqrt(x))/(x
**(5/2) + x**(3/2)) - sqrt(x)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - x**2/(x
**(5/2) + x**(3/2)) - x/(x**(5/2) + x**(3/2))
```

Giac [A] time = 1.34476, size = 28, normalized size = 1.04

$$-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}-\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

3.159 $\int \frac{\tan^{-1}(\sqrt{x})}{x^3} dx$

Optimal. Leaf size=42

$$-\frac{1}{6x^{3/2}} - \frac{\tan^{-1}(\sqrt{x})}{2x^2} + \frac{1}{2\sqrt{x}} + \frac{1}{2}\tan^{-1}(\sqrt{x})$$

```
[Out] -1/(6*x^(3/2)) + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 - ArcTan[Sqrt[x]]/(2*x^2
)
```

Rubi [A] time = 0.0138852, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5033, 51, 63, 203}

$$-\frac{1}{6x^{3/2}} - \frac{\tan^{-1}(\sqrt{x})}{2x^2} + \frac{1}{2\sqrt{x}} + \frac{1}{2}\tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^3,x]
```

```
[Out] -1/(6*x^(3/2)) + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 - ArcTan[Sqrt[x]]/(2*x^2
)
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{x^{5/2}(1+x)}\,dx\\ &= -\frac{1}{6x^{3/2}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{4}\int \frac{1}{x^{3/2}(1+x)}\,dx\\ &= -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)-\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}\end{aligned}$$

Mathematica [C] time = 0.0110679, size = 34, normalized size = 0.81

$$-\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x\right)}{6x^{3/2}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^3,x]
```

```
[Out] -ArcTan[Sqrt[x]]/(2*x^2) - Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2))
```

Maple [A] time = 0.027, size = 27, normalized size = 0.6

$$-\frac{1}{6}x^{-\frac{3}{2}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)-\frac{1}{2\,x^2}\arctan\left(\sqrt{x}\right)+\frac{1}{2}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^3,x)
```

```
[Out] -1/6/x^(3/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2+1/2/x^(1/2)
```

Maxima [A] time = 1.45289, size = 35, normalized size = 0.83

$$\frac{3\,x-1}{6\,x^{\frac{3}{2}}}-\frac{\arctan\left(\sqrt{x}\right)}{2\,x^2}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

Fricas [A] time = 2.19196, size = 80, normalized size = 1.9

$$\frac{3\left(x^2-1\right)\arctan\left(\sqrt{x}\right)+(3\,x-1)\sqrt{x}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(x^2 - 1)*arctan(sqrt(x)) + (3*x - 1)*sqrt(x))/x^2
```

Sympy [B] time = 8.43446, size = 160, normalized size = 3.81

$$\frac{3x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^3}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{2x^2}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**3,x)
```

```
[Out] 3*x**(7/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*atan(sqrt(x
))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**
(5/2)) - 3*sqrt(x)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**3/(6*x**(
7/2) + 6*x**(5/2)) + 2*x**2/(6*x**(7/2) + 6*x**(5/2)) - x/(6*x**(7/2) + 6*x
**(5/2))
```

Giac [A] time = 1.24529, size = 35, normalized size = 0.83

$$\frac{3\,x-1}{6\,x^{\frac{3}{2}}}-\frac{\arctan\left(\sqrt{x}\right)}{2\,x^2}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

3.160 $\int x^{3/2} \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=36

$$-\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)+\frac{x}{5}-\frac{1}{5}\log(x+1)$$

```
[Out] x/5 - x^2/10 + (2*x^(5/2)*ArcTan[Sqrt[x]])/5 - Log[1 + x]/5
```

Rubi [A] time = 0.0150146, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 43}

$$-\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)+\frac{x}{5}-\frac{1}{5}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTan[Sqrt[x]],x]
```

```
[Out] x/5 - x^2/10 + (2*x^(5/2)*ArcTan[Sqrt[x]])/5 - Log[1 + x]/5
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tan^{-1}\left(\sqrt{x}\right)dx &= \frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\frac{x^2}{1+x}\,dx\\ &= \frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\left(-1+x+\frac{1}{1+x}\right)dx\\ &= \frac{x}{5}-\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0152966, size = 30, normalized size = 0.83

$$\frac{1}{10}\left(4x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-(x-2)x-2\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTan[Sqrt[x]],x]
```

```
[Out] (-((-2 + x)*x) + 4*x^(5/2)*ArcTan[Sqrt[x]] - 2*Log[1 + x])/10
```

Maple [A] time = 0.023, size = 25, normalized size = 0.7

$$\frac{x}{5} - \frac{x^2}{10} + \frac{2}{5}x^{\frac{5}{2}} \arctan\left(\sqrt{x}\right) - \frac{\ln\left(x+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctan(x^(1/2)),x)
```

```
[Out] 1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(x+1)
```

Maxima [A] time = 0.977442, size = 32, normalized size = 0.89

$$\frac{2}{5}x^{\frac{5}{2}} \arctan\left(\sqrt{x}\right) - \frac{1}{10}x^2 + \frac{1}{5}x - \frac{1}{5}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Fricas [A] time = 2.2254, size = 88, normalized size = 2.44

$$\frac{2}{5}x^{\frac{5}{2}} \arctan\left(\sqrt{x}\right) - \frac{1}{10}x^2 + \frac{1}{5}x - \frac{1}{5}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Sympy [B] time = 7.67426, size = 92, normalized size = 2.56

$$\frac{4x^{\frac{7}{2}} \operatorname{atan}\left(\sqrt{x}\right)}{10x+10} + \frac{4x^{\frac{5}{2}} \operatorname{atan}\left(\sqrt{x}\right)}{10x+10} - \frac{x^3}{10x+10} + \frac{x^2}{10x+10} - \frac{2x\log\left(x+1\right)}{10x+10} + \frac{x}{10x+10} - \frac{2\log\left(x+1\right)}{10x+10} - \frac{1}{10x+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atan(x**(1/2)),x)
```

```
[Out] 4*x**(7/2)*atan(sqrt(x))/(10*x + 10) + 4*x**(5/2)*atan(sqrt(x))/(10*x + 10)
 - x**3/(10*x + 10) + x**2/(10*x + 10) - 2*x*log(x + 1)/(10*x + 10) + x/(10
*x + 10) - 2*log(x + 1)/(10*x + 10) - 1/(10*x + 10)
```

Giac [A] time = 1.10913, size = 32, normalized size = 0.89

$$\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}\,x^{2}+\frac{1}{5}\,x-\frac{1}{5}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

3.161 $\int \sqrt{x}\tan^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=29

$$\frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

```
[Out] -x/3 + (2*x^(3/2)*ArcTan[Sqrt[x]])/3 + Log[1 + x]/3
```

Rubi [A] time = 0.012381, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 43}

$$\frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

```
[Out] -x/3 + (2*x^(3/2)*ArcTan[Sqrt[x]])/3 + Log[1 + x]/3
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tan^{-1}\left(\sqrt{x}\right)\,dx&=\frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int\frac{x}{1+x}\,dx\\&=\frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int\left(1+\frac{1}{-1-x}\right)dx\\&=-\frac{x}{3}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0100218, size = 25, normalized size = 0.86

$$\frac{1}{3}\left(2x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-x+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

```
[Out] (-x + 2*x^(3/2)*ArcTan[Sqrt[x]] + Log[1 + x])/3
```

Maple [A] time = 0.023, size = 20, normalized size = 0.7

$$-\frac{x}{3}+\frac{2}{3}x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)+\frac{\ln\left(x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctan(x^(1/2)),x)
```

```
[Out] -1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(x+1)
```

Maxima [A] time = 0.976734, size = 26, normalized size = 0.9

$$\frac{2}{3}x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{3}x+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Fricas [A] time = 2.17916, size = 73, normalized size = 2.52

$$\frac{2}{3}x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{3}x+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Sympy [A] time = 1.6332, size = 24, normalized size = 0.83

$$\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{3}-\frac{x}{3}+\frac{\log\left(x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atan(x**(1/2)),x)
```

```
[Out] 2*x**(3/2)*atan(sqrt(x))/3 - x/3 + log(x + 1)/3
```

Giac [A] time = 1.18162, size = 26, normalized size = 0.9

$$\frac{2}{3}x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{3}x+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

3.162 $\int \frac{\tan^{-1}(\sqrt{x})}{\sqrt{x}} \, dx$

Optimal. Leaf size=20

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right) - \log(x+1)$$

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rubi [A] time = 0.0079356, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 31}

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} \, dx &= 2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right) - \int \frac{1}{1+x} \, dx\\ &= 2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right) - \log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0066829, size = 20, normalized size = 1.

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Maple [A] time = 0.023, size = 17, normalized size = 0.9

$$-\ln(x+1)+2\sqrt{x}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(1/2),x)
```

```
[Out] -ln(x+1)+2*x^(1/2)*arctan(x^(1/2))
```

Maxima [A] time = 0.96609, size = 22, normalized size = 1.1

$$2\sqrt{x}\,\arctan\left(\sqrt{x}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Fricas [A] time = 2.16347, size = 54, normalized size = 2.7

$$2\sqrt{x}\,\arctan\left(\sqrt{x}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Sympy [A] time = 0.451744, size = 17, normalized size = 0.85

$$2\sqrt{x}\,\operatorname{atan}\left(\sqrt{x}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

Giac [A] time = 1.11763, size = 22, normalized size = 1.1

$$2\sqrt{x}\,\arctan\left(\sqrt{x}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

3.163 $\int \frac{\tan^{-1}(\sqrt{x})}{x^{3/2}}\,dx$

Optimal. Leaf size=22

$$\log(x)-\log(x+1)-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

```
[Out] (-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Rubi [A] time = 0.0087961, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5033, 36, 29, 31}

$$\log(x)-\log(x+1)-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx &= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{x(1+x)}\,dx\\ &= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{x}\,dx-\int \frac{1}{1+x}\,dx\\ &= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0109185, size = 22, normalized size = 1.

$$\log(x) - \log(x+1) - \frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Maple [A] time = 0.026, size = 19, normalized size = 0.9

$$\ln(x) - \ln(x+1) - 2\,\frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(3/2),x)
```

```
[Out] ln(x)-ln(x+1)-2*arctan(x^(1/2))/x^(1/2)
```

Maxima [A] time = 1.14953, size = 24, normalized size = 1.09

$$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

Fricas [A] time = 2.18452, size = 78, normalized size = 3.55

$$-\frac{x\log(x+1) - x\log(x) + 2\sqrt{x}\arctan\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] -(x*log(x + 1) - x*log(x) + 2*sqrt(x)*arctan(sqrt(x)))/x
```

Sympy [A] time = 1.91852, size = 20, normalized size = 0.91

$$\log(x) - \log(x+1) - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(3/2),x)
```

```
[Out] log(x) - log(x + 1) - 2*atan(sqrt(x))/sqrt(x)
```

Giac [A] time = 1.11018, size = 24, normalized size = 1.09

$$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log\left(x+1\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

3.164 $\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{1}{3x}-\frac{\log(x)}{3}+\frac{1}{3}\log(x+1)$$

```
[Out] -1/(3*x) - (2*ArcTan[Sqrt[x]])/(3*x^(3/2)) - Log[x]/3 + Log[1 + x]/3
```

Rubi [A] time = 0.0137589, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 44}

$$-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{1}{3x}-\frac{\log(x)}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^(5/2),x]
```

```
[Out] -1/(3*x) - (2*ArcTan[Sqrt[x]])/(3*x^(3/2)) - Log[x]/3 + Log[1 + x]/3
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx &= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{1}{3}\int \frac{1}{x^2(1+x)}\,dx\\
&= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{1}{3}\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx\\
&= -\frac{1}{3x}-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{\log(x)}{3}+\frac{1}{3}\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.0179356, size = 31, normalized size = 0.84

$$\frac{1}{3}\left(-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{1}{x}-\log(x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^(5/2),x]
```

```
[Out] (-x^(-1) - (2*ArcTan[Sqrt[x]])/x^(3/2) - Log[x] + Log[1 + x])/3
```

Maple [A] time = 0.029, size = 26, normalized size = 0.7

$$-\frac{1}{3x}-\frac{2}{3}\arctan\left(\sqrt{x}\right)x^{-\frac{3}{2}}-\frac{\ln\left(x\right)}{3}+\frac{\ln\left(x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(5/2),x)
```

```
[Out] -1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(x+1)
```

Maxima [A] time = 0.995832, size = 34, normalized size = 0.92

$$-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{1}{3x}+\frac{1}{3}\log\left(x+1\right)-\frac{1}{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*arctan(sqrt(x))/x^(3/2) - 1/3/x + 1/3*log(x + 1) - 1/3*log(x)
```

Fricas [A] time = 2.19221, size = 96, normalized size = 2.59

$$\frac{x^{2}\log\left(x+1\right)-x^{2}\log\left(x\right)-2\sqrt{x}\arctan\left(\sqrt{x}\right)-x}{3x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2*log(x + 1) - x^2*log(x) - 2*sqrt(x)*arctan(sqrt(x)) - x)/x^2
```

Sympy [B] time = 8.49852, size = 143, normalized size = 3.86

$$-\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{3x^{3}+3x^{2}}-\frac{2\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{3x^{3}+3x^{2}}-\frac{x^{3}\log\left(x\right)}{3x^{3}+3x^{2}}+\frac{x^{3}\log\left(x+1\right)}{3x^{3}+3x^{2}}+\frac{x^{3}}{3x^{3}+3x^{2}}-\frac{x^{2}\log\left(x\right)}{3x^{3}+3x^{2}}+\frac{x^{2}\log\left(x+1\right)}{3x^{3}+3x^{2}}-\frac{x}{3x^{3}+3x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(5/2),x)
```

```
[Out] -2*x**(3/2)*atan(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*atan(sqrt(x))/(3*x*
*3 + 3*x**2) - x**3*log(x)/(3*x**3 + 3*x**2) + x**3*log(x + 1)/(3*x**3 + 3*
```

```
x**2) + x**3/(3*x**3 + 3*x**2) - x**2*log(x)/(3*x**3 + 3*x**2) + x**2*log(x
 + 1)/(3*x**3 + 3*x**2) - x/(3*x**3 + 3*x**2)
```

Giac [A] time = 1.15805, size = 38, normalized size = 1.03

$$\frac{x-1}{3\,x} - \frac{2\,\arctan\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}} + \frac{1}{3}\,\log\left(x+1\right) - \frac{1}{3}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x - 1)/x - 2/3*arctan(sqrt(x))/x^(3/2) + 1/3*log(x + 1) - 1/3*log(x)
```

3.165 $\int \frac{\tan^{-1}(ax^5)}{x} \, dx$

Optimal. Leaf size=33

$$\frac{1}{10}i\text{PolyLog}\left(2,-iax^5\right)-\frac{1}{10}i\text{PolyLog}\left(2,iax^5\right)$$

```
[Out] (I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

Rubi [A] time = 0.036191, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5031, 4848, 2391}

$$\frac{1}{10}i\text{PolyLog}\left(2,-iax^5\right)-\frac{1}{10}i\text{PolyLog}\left(2,iax^5\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x^5]/x,x]

[Out] (I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(ax^5\right)}{x} \, dx &= \frac{1}{5}\text{Subst}\left(\int \frac{\tan^{-1}(ax)}{x} \, dx, x, x^5\right)\\
&= \frac{1}{10}i\,\text{Subst}\left(\int \frac{\log(1-iax)}{x} \, dx, x, x^5\right)-\frac{1}{10}i\,\text{Subst}\left(\int \frac{\log(1+iax)}{x} \, dx, x, x^5\right)\\
&= \frac{1}{10}i\text{Li}_2\left(-iax^5\right)-\frac{1}{10}i\text{Li}_2\left(iax^5\right)
\end{aligned}$$

Mathematica [A] time = 0.0061189, size = 33, normalized size = 1.

$$\frac{1}{10}i\text{PolyLog}\left(2,-iax^5\right)-\frac{1}{10}i\text{PolyLog}\left(2,iax^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x^5]/x,x]
```

```
[Out] (I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

Maple [C] time = 0.088, size = 57, normalized size = 1.7

$$\ln(x)\arctan\left(ax^5\right)-\frac{1}{2a}\sum_{\_R1=\mathrm{RootOf}\left(a^2\_Z^{10}+1\right)}\frac{1}{\_R1^5}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x^5)/x,x)
```

```
[Out] ln(x)*arctan(a*x^5)-1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1)),_R1=RootOf(_Z^10*a^2+1))
```

Maxima [B] time = 1.46912, size = 84, normalized size = 2.55

$$\frac{1}{5}i\arctan\left(ax^5\right)\arctan(0,a)-\frac{1}{20}\pi\log\left(a^2x^{10}+1\right)+\frac{1}{5}\arctan\left(ax^5\right)\log\left(x^5|a|\right)-\frac{1}{10}i\,\mathrm{Li}_2\left(i\,ax^5+1\right)+\frac{1}{10}i\,\mathrm{Li}_2\left(-i\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] 1/5*I*arctan(a*x^5)*arctan2(0, a) - 1/20*pi*log(a^2*x^10 + 1) + 1/5*arctan(
a*x^5)*log(x^5*abs(a)) - 1/10*I*dilog(I*a*x^5 + 1) + 1/10*I*dilog(-I*a*x^5
+ 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\arctan\left(ax^5\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x^5)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x**5)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x^5)/x, x)
```

3.166 $\int \frac{\tan^{-1}(ax^n)}{x} dx$

Optimal. Leaf size=39

$$\frac{i\text{PolyLog}(2,-iax^n)}{2n}-\frac{i\text{PolyLog}(2,iax^n)}{2n}$$

```
[Out] ((I/2)*PolyLog[2, (-I)*a*x^n])/n - ((I/2)*PolyLog[2, I*a*x^n])/n
```

Rubi [A] time = 0.0358367, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5031, 4848, 2391}

$$\frac{i\text{PolyLog}(2,-iax^n)}{2n}-\frac{i\text{PolyLog}(2,iax^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x^n]/x,x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*a*x^n])/n - ((I/2)*PolyLog[2, I*a*x^n])/n
```

Rule 5031

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(ax^n)}{x} dx &= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(ax)}{x} dx, x, x^n\right)}{n}\\ &= \frac{i\,\text{Subst}\left(\int \frac{\log(1-iax)}{x} dx, x, x^n\right)}{2n}-\frac{i\,\text{Subst}\left(\int \frac{\log(1+iax)}{x} dx, x, x^n\right)}{2n}\\ &= \frac{i\text{Li}_2(-iax^n)}{2n}-\frac{i\text{Li}_2(iax^n)}{2n}\end{aligned}$$

Mathematica [A] time = 0.0131773, size = 32, normalized size = 0.82

$$\frac{i\left(\text{PolyLog}(2,-iax^n)-\text{PolyLog}(2,iax^n)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x^n]/x,x]
```

```
[Out] ((I/2)*(PolyLog[2, (-I)*a*x^n] - PolyLog[2, I*a*x^n]))/n
```

Maple [B] time = 0.033, size = 94, normalized size = 2.4

$$\frac{\ln(ax^n)\arctan(ax^n)}{n}+\frac{\frac{i}{2}\ln(ax^n)\ln(1+iax^n)}{n}-\frac{\frac{i}{2}\ln(ax^n)\ln(1-iax^n)}{n}+\frac{\frac{i}{2}\mathrm{dilog}(1+iax^n)}{n}-\frac{\frac{i}{2}\mathrm{dilog}(1-iax^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x^n)/x,x)
```

```
[Out] 1/n*ln(a*x^n)*arctan(a*x^n)+1/2*I/n*ln(a*x^n)*ln(1+I*a*x^n)-1/2*I/n*ln(a*x^
n)*ln(1-I*a*x^n)+1/2*I/n*dilog(1+I*a*x^n)-1/2*I/n*dilog(1-I*a*x^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-an\int\frac{x^n\log(x)}{a^2xx^{2n}+x}\,dx+\arctan(ax^n)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(a*x^n)*log(x)
```

Fricas [B] time = 2.56211, size = 181, normalized size = 4.64

$$\frac{2\,n\arctan(ax^n)\log(x)+i\,n\log(i\,ax^n+1)\log(x)-i\,n\log(-i\,ax^n+1)\log(x)-i\,\mathrm{Li}_2(i\,ax^n)+i\,\mathrm{Li}_2(-i\,ax^n)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*n*arctan(a*x^n)*log(x) + I*n*log(I*a*x^n + 1)*log(x) - I*n*log(-I*a*
x^n + 1)*log(x) - I*dilog(I*a*x^n) + I*dilog(-I*a*x^n))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atan}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x**n)/x,x)
```

```
[Out] Integral(atan(a*x**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(ax^n)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x^n)/x, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
   :
            return expn in expn.parent().base_ring() or expn in expn.parent().
   gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
   Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
   args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
   )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
   Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
   Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
   expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
   [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
   isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
   .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```